

# Bit Error Rate Performance of QPSK Modulation and OFDM-QPSK with AWGN and Rayleigh Multipath Channel

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**Abstract:** In this paper Bit Error Rate performance of QPSK modulation and OFDM-QPSK System over AWGN and Rayleigh fading channel is analyzed. The performance of BER of QPSK over AWGN and Rayleigh channel is compared. OFDM is an orthogonal frequency division multiplexing to reduce intersymbol interference problem. Simulation of QPSK signals is carried with both AWGN and Rayleigh channel. Finally simulations of OFDM signals are carried with Rayleigh faded signals to understand the effect of channel fading and to obtain optimum value of Bit Error Rate (BER) and Signal to noise ratio (SNR). The simulation results show that the simulated bit error rate is in good agreement with the theoretical bit error rate for QPSK modulation

**Keywords:** QPSK, BER, OFDM, AWGN, Rayleigh fading channel

## 1. Introduction

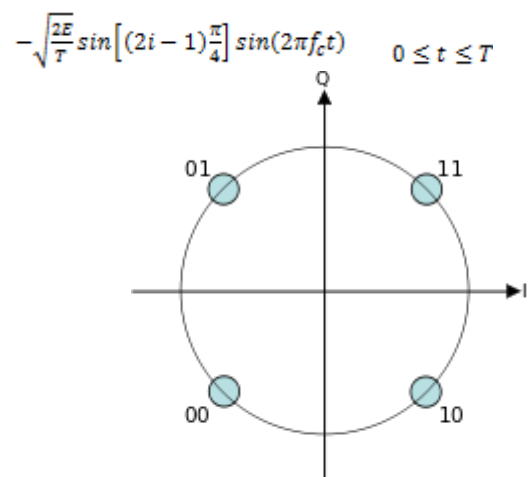
In digital modulation techniques a set of basis functions are chosen for a particular modulation scheme. Generally the basis functions are orthogonal to each other. Basis functions can be derived using 'Gram Schmidt orthogonalization' procedure. Once the basis function are chosen, any vector in the signal space can be represented as a linear combination of the basis functions. In Quadrature Phase Shift Keying (QPSK) two sinusoids (sin and cos) are taken as basis functions for modulation. Modulation is achieved by varying the phase of the basis functions depending on the message symbols. In QPSK, modulation is symbol based, where one symbol contains 2 bits [2] [3]. Orthogonal Frequency Division Multiplexing (OFDM) is a widely used technique in wireless communication systems. It's also the base for fourth generation mobile communication. OFDM provides high spectral efficiency and high data rates. In this paper bit error rate (BER) of QPSK and QPSK based OFDM is analyzed.

## 2. Page Bit Error Rate (BER) performance of the QPSK

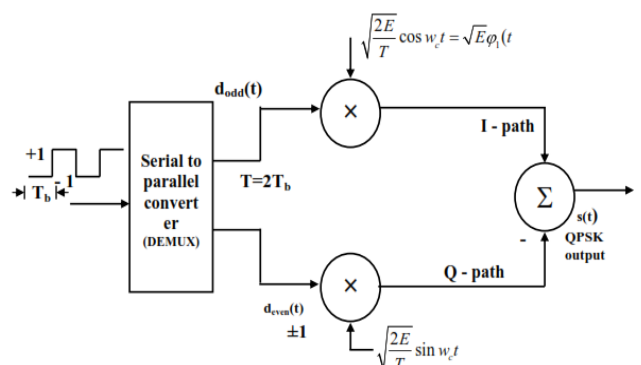
In a popular variation of BPSK, quadrature PSK (QPSK), the modulator produces two sine carriers 90° apart. The binary data modulates each phase, producing four unique sine signals shifted by 45° from one another. The two phases are added together to produce the final signal. Each unique pair of bits generates a carrier with a different phase [4]. QPSK is an expanded version from binary PSK where in a symbol consists of two bits and two orthonormal basis functions are used. A group of two bits is often called a 'dibit'. So, four dibits are possible. Each symbol carries same energy [1].

On simple trigonometric expansion, the QPSK modulated signal  $s(t)$  can be expressed as:

$$s(t) = \sqrt{\frac{2E}{T}} \cos\left[(2i-1)\frac{\pi}{4}\right] \cos(2\pi f_c t)$$



**Figure 1:** Constellation diagram for QPSK with Gray coding. Each adjacent symbol only differs by one bit.



**Figure 2:** Schematic diagram of a QPSK modulator

The bit error probability for the QPSK is given as

$$\frac{1}{2} \operatorname{erfc}\left[\frac{E_B}{\eta}\right]^{1/2}$$

### 2.1 Additive white Gaussian noise (AWGN)

AWGN is a basic noise model used in Information theory to mimic the effect of many random processes that occur in

nature. *Additive* because it is added to any noise that might be intrinsic to the information system. *White* refers to the idea that it has uniform power across the frequency band for the information system. It is an analogy to the color white which has uniform emissions at all frequencies in the visible spectrum. *Gaussian* because it has a normal distribution in the time domain with an average time domain value of zero.

### 2.2 Rayleigh fading

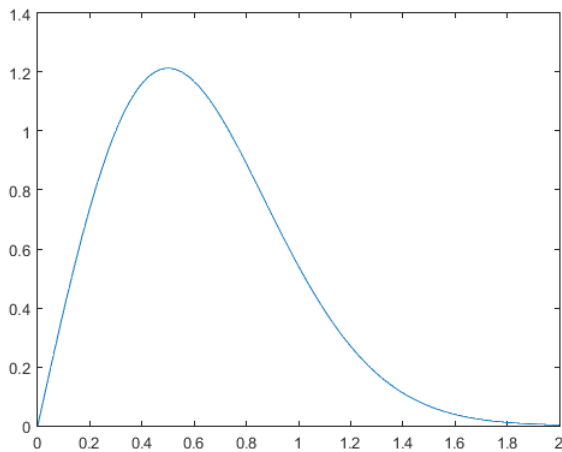
Rayleigh fading is a statistical model for the effect of a propagation environment on a radio signal, such as that used by wireless devices.

Rayleigh fading models assume that the magnitude of a signal that has passed through such a transmission medium (also called a communications channel) will vary randomly, or fade, according to a Rayleigh distribution, the radial component of the sum of two uncorrelated Gaussian random variables.

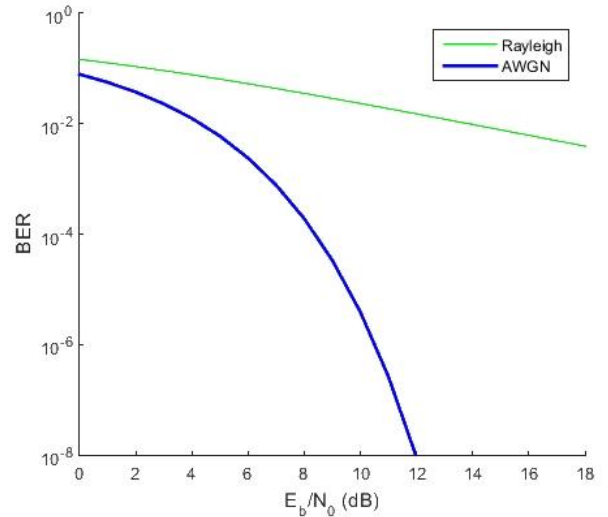
The Rayleigh pdf is

$$y = f(x|b) = \frac{x}{b^2} e^{\left(\frac{-x^2}{2b^2}\right)}$$

The Rayleigh distribution is a special case of the Weibull distribution [5].



**Figure 3:** Rayleigh Distribution pdf

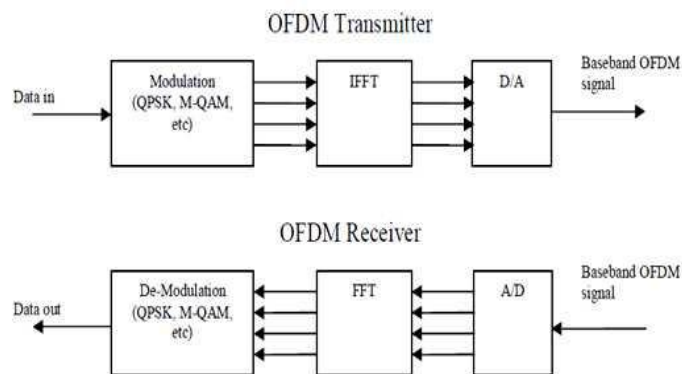


**Figure 4:** BER of QPSK in AWGN and Rayleigh Channel

### 2.3 Orthogonal Frequency Division Multiplexing

Orthogonal Frequency Division Multiplexing (OFDM) has proven to be the most promising technique for high speed data transmission over a dispersive channel [6]. It provides high spectral efficiency, low implementation complexity [7], less vulnerability to echoes and non-linear distortion [8].

Orthogonal frequency division multiplexing (OFDM) transmission scheme is a type of a multichannel system. It does not require individual band limited filters and oscillators for each sub-channel and furthermore the spectra of the subcarriers are overlapped for bandwidth efficiency. Overlapped multiple orthogonal subcarrier signals can be produced by generalizing the single carrier Nyquist criterion into multi-carrier criterion.



**Figure 5:** OFDM General Block Diagram

### 3. BER Performance of OFDM-QPSK

The analytical BER expressions for M-ary QAM in AWGN and Rayleigh channels are respectively given as

$$P_e = \frac{2(M-1)}{M \log_2 M} Q \left( \sqrt{\frac{6E_b}{N_o} \frac{\log_2 M}{M^2 - 1}} \right)$$

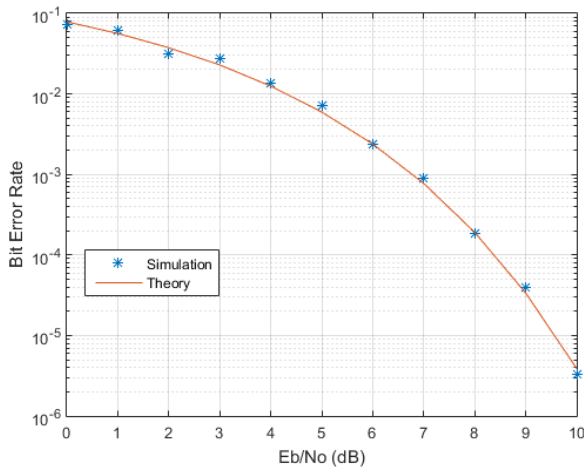
$$P_e = \frac{(M-1)}{M \log_2 M} Q \left( 1 - \sqrt{\frac{3\gamma \log_2 M / (M^2 - 1)}{3\gamma \log_2 M / (M^2 - 1) + 1}} \right)$$

Where  $\gamma$  and M denote  $E_b/N_o$  and the modulation order, respectively. Q(.) is a standard Q-function.

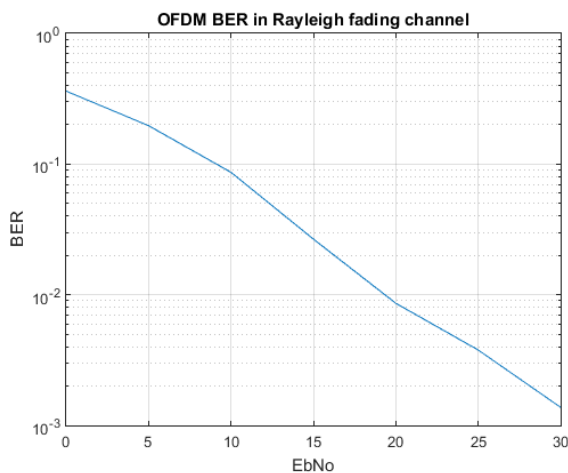
$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt$$

OFDM parameters are as follows:

Number of subcarriers	128
Cyclic Prefix Length	16
Channel Sampling Period	1e-3
No of Pilot symbols	4
Number of OFDM Frames	10 <sup>3</sup>



**Figure 6:** BER Performance of QPSK-OFDM in AWGN channel



**Figure 7:** BER of OFDM in Rayleigh fading channel

### 4. Conclusion

In this paper, the performance of QPSK with OFDM AWGN and Rayleigh fading distribution was evaluated. Graphical

results show the improvement in QPSK with Rayleigh fading channel compared to its performance in AWGN channel. The graphical results prove that simulated BER of QPSK-OFDM is same as that of theoretical BER of QPSK-OFDM. The reported BER can be further reduced by using channel estimation or suitable diversity scheme.

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