Fuzzy Inventory Model for Power Demand Pattern with Inlation, Shortages under Partially Backlogging

Wasim Akram Mandal[*], Sahidul Islam[**]

[1]Beldanga D.H.Sr.Madrasah, Beldanga-742189, Murshidabad , WB, India

[2]Department of mathematics, University of kalyani, kalyani, W.B, India

Abstract: *In this paper fuzzy inventory model for non deteriorating item with power demand pattern, shortage under partially backlogged , and inflation is formulated and solved. Fixed cost, shortages cost, holding cost, opportunity cost are the costs function considered in this model. Fuzziness is applying by allowing the cost components (holding cost, shortage cost, etc) and inflation. In fuzzy environment it considered all required parameter to be pentagonal fuzzy numbers. The purpose of the model is to minimize total cost function.*

Keywords: Inventory, Power demand pattern, Fuzzy number, Shortages, Inflation, Pentagonal fuzzy number

1. Introduction

An inventory deal with decision that minimum the total average cost or maximize The total average profit. For this purpose the task is to construct a mathematical model of the real life Inventory system, such a mathematical model is based on various assumption and approximation

In a inventory model shortages is very important condition. There are several type of customer. At shortage period some customers are waiting for actual product and others do not it. For this it consider partially- backlogging. In ordinary inventory model it consider all parameter like shortage cost, holding cost, unit cost as fixed. But in real life situation it will have some little fluctuations . so consideration of fuzzy variables is more realistic.

The study of inventory model where demand rates varies with time is the last decades. Datta and pal investigated an inventory system with power demand pattern for item which variable rate deterioration. Park and Wang studied shortages and partial backlogging of items. Friedman(1978) presented continuous time inventory model with time varying demand. Ritchie(1984) studied in inventory model with linear increasing demand. Goswami, Chaudhuri(1991) discussed an inventory model with shortage.Gen et. Al. (1997) considered classical inventory model with Triangular fuzzy number.Yao and Lee(1998) considered an economic production quantity model in the fuzzy sense. Sujit Kumar De, P.K.Kundu and A.Goswami(2003) presented an economic production quantity inventory model involving fuzzy demand rate. J.K.Syde and L.A.Aziz(2007) applied sign distance method to fuzzy inventory model without shortage . D.Datta and Pravin Kumar published several paper of fuzzy inventory with or without shortage.

In this paper we first consider crisp inventory model with power demand where shortage are allowed and partially backlogged. Thereafter we developed fuzzy inventory model with fuzzy power demand rate under partially backlogged. All inventory cost parameters are fuzzyfied as pentagonal fuzzy number.

2. Preliminaries

For graded representation method to defuzzyfy, we need the following definitions,

Definition2.1: A fuzzy set $\tilde{A}$ on the given universal set X is a set of order pairs,

$\tilde{A}$={(x,μ_A(x)): xϵX} where μ_A(x)→[0,1] is called a membership function.

Definition2.2:The α-cut of $\tilde{A}$, is defined by $A_α$={x: μ_A(x)=α, α≥0}

Definition2.3:$\tilde{A}$ is normal if there exists xϵX such that μ_A(x)=1

Definition2.4:A pentagonal fuzzy number $\tilde{A}$=(a,b,c,d,e) is represented with membership function $\tilde{A}$

$\tilde{A}$ is defined as,

$$\mu_A(X)=\begin{cases} L_1(x)=\frac{x-a}{b-a}, a \le x \le b \\ L_2(x)=\frac{x-b}{c-b}, b \le x \le c \\ 1, x=c \\ R_1(x)=\frac{d-x}{d-c}, c \le x \le d \\ R_2(x)=\frac{e-x}{e-d}, d \le x \le e \\ 0, otherwise \end{cases}$$

The α-cut 0f $\tilde{A}$=(a,b,c,d,e), 0≤α≤1 is A(α)=[$A_L(α)$,$A_R(α)$]

Where A_{L_1} (α)=a+(b-a)α=$L_1^{-1}(α)$

A_{L_2}(α)=b+(c-d)α=$L_2^{-1}(α)$

And A_{R_1}(α)=d-(d-c)α=$R_1^{-1}(α)$

A_{R_2}(α)=e-(e-d)α=$R_2^{-1}(α)$

So $L^{-1}(α)$= $\frac{1}{2}$[$L_1^{-1}(α)$+$L_2^{-1}(α)$]

=$\frac{1}{2}$[a+b+(c-a)α]

$R^{-1}(α)$=$\frac{1}{2}$[$R_1^{-1}(α)$+$R_2^{-1}(α)$]

=$\frac{1}{2}$[d+e-(e-c)α]

Definition2.5: If $\tilde{A}$=(a,b,c,d,e) is a pentagonal fuzzy number then the graded mean integration of $\tilde{A}$ is defined as,

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$$P(\tilde{A}) = \frac{\int_0^{W_A} \left(\frac{L^{-1}(h) + R^{-1}(h)}{2}\right) dh}{\int_0^{W_A} h \, dh}, (0 \leq h \leq W_A \text{ and } 0 \leq W_A \leq 1)$$

$$P(\tilde{A}) = \frac{\int_0^1 \left[\frac{a+b+(c-a)h+d+e-(e-c)h\right] dh}{2}}{\int_0^1 h \, dh}$$

$$= \frac{a + 3b + 4c + 3d + e}{12}$$

Suppose $\tilde{a} = (a_1, a_2, a_3, a_4, a_5)$ and $\tilde{b} = (b_1, b_2, b_3, b_4, b_5)$ are two fuzzy pentagonal number then

(1) $\tilde{a} + \tilde{b} = (a_1, a_2, a_3, a_4, a_5) + (b_1, b_2, b_3, b_4, b_5) = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4, a_5+b_5)$.

(2) $\tilde{a} - \tilde{b} = (a_1, a_2, a_3, a_4, a_5) - (b_1, b_2, b_3, b_4, b_5) = (a_1-b_1, a_2-b_2, a_3-b_3, a_4-b_4, a_5-b_5)$.

(3) $\tilde{a} \times \tilde{b} = (a_1, a_2, a_3, a_4, a_5) \times (b_1, b_2, b_3, b_4, b_5) = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4, a_5 b_5)$.

(4) $\frac{1}{\tilde{b}} = (\frac{1}{b_5}, \frac{1}{b_4}, \frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1})$ where b_5, b_4, b_3, b_2, b_1 are all non zero positive real number, then
$\frac{\tilde{a}}{\tilde{b}} = (\frac{a_1}{b_5}, \frac{a_2}{b_4}, \frac{a_3}{b_3}, \frac{a_4}{b_2}, \frac{a_5}{b_1})$.

(5) Let $k \in R$, then $k\tilde{a} = k(a_1, a_2, a_3, a_4, a_5) = (ka_1, ka_2, ka_3, ka_4, ka_5)$ for $k \geq 0$.

3. Notation

$I(t)$=Inventory level at any time, $t \geq 0$.
T: Cycle of length.
t_1: Duration of inventory cycle when there is positive inventory.
c_1: Fixed cost per cycle.
c_2: Shortages cost per unit per unit time.
c_3: Opportunity cost due to lost sales per unit per unit time.
c_4: Holding cost per unit, which is variable.
i: Inflation rate per unit time.
r: The discount rate of money.
$TAC(t_1)$: Total average cost per unit.
$\tilde{c}_1$ = Fuzzy fixed cost.
$\tilde{c}_2$ = Fuzzy shortage cost per unit.
$\tilde{c}_3$ = Fuzzy opportunity cost due to lost sales.
$\tilde{c}_4$ = Fuzzy holding cost, which is variable.
$\widetilde{TAC}(t_1)$ = Fuzzy total cost per unit.

3.1 Assumption

a) The inventory system involves only one item.
b) The replenishment occur instantaneously at infinite rate.
c) The lead time is negligible.
d) Demand rate is power demand, we assume it $d(t/T)^{1/n}$ where d is constant. During the fixed time T, $\frac{dt^{(1-n)/n}}{nT^{1/n}}$ is demand rate at time t.
e) Backlogging rate is $\frac{1}{1+\delta(T-t)}$, $t_1 \leq t \leq T$.
f) Holding cost variable as $e^{\theta t}$, $0 \leq \theta < 1$.

3.2 Model Development (CRISP MODEL)
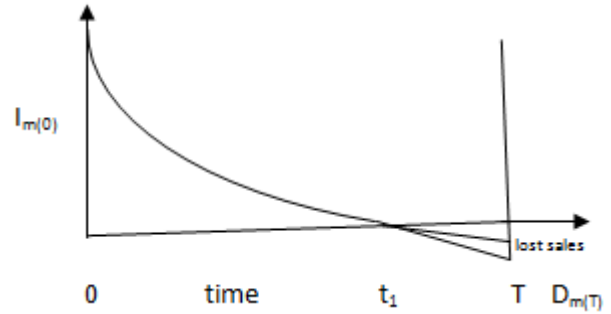
The differential equation describing $I(t)$ as follows

$$\frac{dI(t)}{dt} = -\frac{dt^{\frac{1-n}{n}}}{nT^{1/n}}, 0 \leq t \leq t_1 \tag{3.1}$$

$$\frac{dI(t)}{dt} = -\frac{dt^{\frac{1-n}{n}}}{n\{1+\delta(T-T)\}T^{1/n}}, t_1 \leq t \leq T \tag{3.2}$$

From (1) we get,

$$I(t) = \frac{d}{T^{1/n}}(t_1^{1/n} - t^{1/n}) \tag{3.3}$$

From (2) we get,

$$I(t) = -\frac{d}{T^{1/n}}[(t^{1/n} - t_1^{1/n})(1-\delta T) + \frac{\delta}{1+n}\{t^{(1+n)/n} - t_1^{(1+n)/n}\}] \tag{3.4}$$

So the maximum inventory level is,

$$I_m(0) = \frac{d}{T^{1/n}} t_1^{1/n} \tag{3.5}$$

And the maximum amount of demand backlogging is,
$$D_m(T) = -I(T)$$
$$= \frac{d}{T^{1/n}}[(T^{1/n} - t_1^{1/n})(1-\delta t) + \frac{\delta}{1+n}\{T^{(1+n)/n} - t_1^{(1+n)/n}\}] \tag{3.6}$$

Hench the order quantity per cycle is;
$$Q = I_m(0) + D_m(T)$$
$$= \frac{d}{T^{1/n}}[t_1^{1/n} + (T^{1/n} - t_1^{1/n})(1-\delta T) + \frac{\delta}{1+n}\{T^{(1+n)/n} - t_1^{(1+n)/n}\}] \tag{3.7}$$

The fixed cost per cycle is,
$$FC = c_1 \int_0^T e^{-(r-i)t} dt,$$
$$= c_1\{T + \frac{(i-r)T^2}{2}\}.$$

Shortage cost per cycle is,
$$Sc = -c_2 \int_{t1}^T e^{-(r-i)t} I(t) dt,$$
$$= -c_2 \frac{d}{T^{1/n}}[(1-\delta t)(T-t_1)t_1^{1/n} - \frac{n(1-\delta T)}{n+1}\{T^{(n+1)/n} - t_1^{(n+1)/n}\} + \frac{\delta}{1+n}t_1^{(n+1)/n}(T-T_1) - \frac{n\delta}{(n+1)(2n+1)}\{T^{(2n+1)/n} - t_1^{(2n+1)} - \frac{(1-\delta T)(1-r)}{2}t_1^{1/n}(T^2 - T_1^2) - \frac{(1-\delta T)(1-r)n}{2n+1}\{T^{(2n+1)/n} - t_1^{(2n+1)/n}\} + \frac{\delta(i-r)}{2n+2}t_1^{(n+1)/n}(T^2 - t_1^2) - \frac{\delta(i-r)n}{(n+1)(3n+1)}\{T^{(3n+1)/n} - t_1^{(3n+1)}\}].$$

Opportunity cost due to lost sales is,
$$OC = c_3 \int_{t1}^T R(t)[1 - \frac{1}{1+\delta(T-t)}]e^{-(r-i)t} dt$$
$$= c_3 \frac{\delta d}{T^{1/n}}[T(T^{1/n} - t_1^{1/n}) - \frac{1}{n+1}\{T^{(n+1)/n} - t_1^{(n+1)/n}\} + \frac{(i-r)T}{n+1}\{T^{(n+1)/n} - t_1^{(n+1)/n}\} - \frac{(i-r)}{2n+1}\{T^{(2n+1)/n} - t_1^{(2n+1)/n}\}].$$

Holding cost per cycle is,

As θ, r, and i is too small, so neglecting higher power and product of θ, r, and i.

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$HC = c_4 \frac{d}{T^{1/n}} \int_0^{t_1} e^{\theta t} \cdot e^{-(r-i)t} I(t) dt$

$= c_4 \frac{d}{T^{1/n}} [\frac{1}{1+n} t_1^{(n+1)/n} + \frac{(\theta+i-r)}{2+2n} t_1^{(2n+1)/n}]$

Total average cost per cycle is,

$TAC = \frac{1}{T}[FC + SC + OC + HC]$

$= \frac{d}{T^{(n+1)/n}} [[c_1 T^{(n+1)/n} + c_1(i-r)\frac{T^{(2n+1)/n}}{2} - c_2 [(1-\delta t)(T-t_1)t_1^{1/n} - \frac{n(1-\delta T)}{n+1}\{T^{(n+1)/n} - t_1^{(n+1)/n}\} + \frac{\delta}{1+n}t_1^{(n+1)/n}(T-T_1) - \frac{n\delta}{(n+1)(2n+1)}\{T^{(2n+1)/n} - t_1^{(2n+1)} - \frac{(1-\delta T)(1-r)}{2}t_1^{1/n}(T^2 - T_1^2) - \frac{(1-\delta T)(1-r)n}{2n+1}\{T^{(2n+1)/n} - t_1^{(2n+1)/n}\} + \frac{\delta(i-r)}{2n+2}t_1^{(n+1)/n}(T^2 - t_1^2) - \frac{\delta(i-r)n}{(n+1)(3n+1)}\{T^{(3n+1)/n} - t_1^{(3n+1)}\}] + c_3\delta [T(T^{1/n} - t_1^{1/n}) - \frac{1}{n+1}\{T^{(n+1)/n} - t_1^{(n+1)/n}\} + \frac{(i-r)T}{n+1}\{T^{(n+1)/n} - t_1^{(n+1)/n}\} - \frac{(i-r)}{2n+1}\{T^{(2n+1)/n} - t_1^{(2n+1)/n}\}] + c_4 [\frac{1}{1+n}t_1^{(n+1)/n} + \frac{(\theta+i-r)}{2+4n}t_1^{(2n+1)/n}]].$

For minimum cost it should be,

$\frac{dTAC(t_1)}{dt_1} = 0$ and $\frac{d^2TAC(t_1)}{dt^2_1} > 0$

3.3 FUZZY Model

Due to uncertainly lets us assume that,

$\tilde{c_1} = (c_1^1, c_1^2, c_1^3, c_1^4, c_1^5)$,

$\tilde{c_2} = (c_2^1, c_2^2, c_2^3, c_2^4, c_2^5)$, $\tilde{c_3} = (c_3^1, c_3^2, c_3^3, c_3^4, c_3^5)$, $\tilde{c_4} = (c_4^1, c_4^2, c_4^3, c_4^4, c_4^5)$, be pentagonal fuzzy number then the total average cost is given by,

Then

$\widetilde{TAC}(t_1) = \frac{d}{T^{(n+1)/n}} [[\hat{c}_1 T^{(n+1)/n} + c_1(i-r)\frac{T^{(2n+1)/n}}{2} - \hat{c}_2 [(1-\delta t)(T-t_1)t_1^{1/n} - \frac{n(1-\delta T)}{n+1}\{T^{(n+1)/n} - t_1^{(n+1)/n}\} + \frac{\delta}{1+n}t_1^{(n+1)/n}(T-T_1) - \frac{n\delta}{(n+1)(2n+1)}\{T^{(2n+1)/n} - t_1^{(2n+1)} - \frac{(1-\delta T)(1-r)}{2}t_1^{1/n}(T^2 - T_1^2) - \frac{(1-\delta T)(1-r)n}{2n+1}\{T^{(2n+1)/n} - t_1^{(2n+1)/n}\} + \frac{\delta(i-r)}{2n+2}t_1^{(n+1)/n}(T^2 - t_1^2) - \frac{\delta(i-r)n}{(n+1)(3n+1)}\{T^{(3n+1)/n} - t_1^{(3n+1)}\}] + \hat{c}_3\delta [T(T^{1/n} - t_1^{1/n}) - \frac{1}{n+1}\{T^{(n+1)/n} - t_1^{(n+1)/n}\} + \frac{(i-r)T}{n+1}\{T^{(n+1)/n} - t_1^{(n+1)/n}\} - \frac{(i-r)}{2n+1}\{T^{(2n+1)/n} - t_1^{(2n+1)/n}\}] + \hat{c}_4 [\frac{1}{1+n}t_1^{(n+1)/n} + \frac{(\theta+i-r)}{2+4n}t_1^{(2n+1)/n}]].$

We defuzzifi the fuzzy total cost $\widetilde{TAC}(t_1)$ by graded mean representation method as follows,

$\widetilde{TAC}(t_1) = \frac{1}{12}[\widetilde{TAC}^1(t_1), \widetilde{TAC}^2(t_1), \widetilde{TAC}^3(t_1), \widetilde{TAC}^4(t_1), \widetilde{TAC}^5(t_1)]$

Where $\widetilde{TAC}^r(t_1) = \frac{d}{T^{(n+1)/n}} [[\tilde{c}^r_1 T^{(n+1)/n} + c_1(i-r)\frac{T^{(2n+1)/n}}{2} - \tilde{c}^r_2 [(1-\delta t)(T-t_1)t_1^{1/n} - \frac{n(1-\delta T)}{n+1}\{T^{(n+1)/n} - t_1^{(n+1)/n}\} + \frac{\delta}{1+n}t_1^{(n+1)/n}(T-T_1) - \frac{n\delta}{(n+1)(2n+1)}\{T^{(2n+1)/n} - t_1^{(2n+1)} - \frac{(1-\delta T)(1-r)}{2}t_1^{1/n}(T^2 - T_1^2) - \frac{(1-\delta T)(1-r)n}{2n+1}\{T^{(2n+1)/n} - t_1^{(2n+1)/n}\} + \frac{\delta(i-r)}{2n+2}t_1^{(n+1)/n}(T^2 - t_1^2) - \frac{\delta(i-r)n}{(n+1)(3n+1)}\{T^{(3n+1)/n} - t_1^{(3n+1)}\}] + \tilde{c}^r_3\delta [T(T^{1/n} - t_1^{1/n}) - \frac{1}{n+1}\{T^{(n+1)/n} - t_1^{(n+1)/n}\} + \frac{(i-r)T}{n+1}\{T^{(n+1)/n} - t_1^{(n+1)/n}\} - \frac{(i-r)}{2n+1}\{T^{(2n+1)/n} - t_1^{(2n+1)/n}\}] + \tilde{c}^r_4 [\frac{1}{1+n}t_1^{(n+1)/n} + \frac{(\theta+i-r)}{2+4n}t_1^{(2n+1)/n}]].$ r=1,2,3,4,5

$\widetilde{TAC}(t_1) = \frac{1}{12}[\widetilde{TAC}_1(t_1) + 3\widetilde{TAC}_2(t_1) + 4\widetilde{TAC}_3(t_1) + 3\widetilde{TAC}_4(t_1) + \widetilde{TAC}_5(t_1)]$ For minimum cost it should be,

$\frac{\widetilde{TAC}(t_1)}{dt_1} = 0$ and $\frac{d^2\widetilde{TAC}(t_1)}{dt^2_1} > 0$

4. Numerical Solution

For crisp model: Let us take the in-put value:

C_1	C_2	C_3	C_4	C_5	i	r	δ	θ	m	n	d	T
100	5	6	5	2	0.1	0.2	0.1	0.2	0.1	3	20	1

And the out-put value:

Q	t_1	$TAC(t_1)$
19.532	0.216	1845.760

For fuzzy model:

$\tilde{c_1} = (90,95,100,105,110)$, $\tilde{c_2} = (3,4,5,6,7)$, $\tilde{c_3} = (4,5,6,7,8)$, $\tilde{c_4} = (3,4,5,6,7)$

The solution of fuzzy model by graded mean representation is,

1) When $\tilde{c_1}, \tilde{c_2}, \tilde{c_3}, \tilde{c_4}$ are all pentagonal fuzzy numbers then, $TAC(t_1) = 1853.668$, $t_1 = 0.215$
2) When $\tilde{c_1}, \tilde{c_2}, \tilde{c_3}$ are all pentagonal fuzzy numbers then $TAC(t_1) = 1853.667$, $t_1 = 0.215$
3) When $\tilde{c_1}, \tilde{c_2}$ are pentagonal fuzzy numbers then, $TAC(t_1) = 1853.667$, $t_1 = 0.215$
4) When $\tilde{c_1}$ are pentagonal fuzzy numbers then, $TAC(t_1) = 1853.677$, $t_1 = 0.216$

5.1 Sensitivity Analysis

We now examine to sensitivity analysis of the optimal solution of the model for change in I, keeping the other parameters unchanged. The initial data from the above numerical example.

i	% change	t_1	$TAC(t_1)$
0.05	-50	0.216	1795.760
0.06	-40	0.216	1805.760
0.07	-30	0.216	1815.760
0.08	-20	0.217	1825.760
0.09	-10	0.217	1835.760
0.10	0	0.217	1845.760
0.11	10	0.217	1855.760
0.12	20	0.217	1865.760
0.13	30	0.217	1875.760
0.14	40	0.218	1885.760
0.15	50	0.218	1895.760

5.2 Effect, for increment parameter i,

(a) The inventory time t_1 increase very slow.
(b) Total average cost $TAC(t_1)$ increase.

5. Conclusion

In this paper, we have proposed a real life inventory problem in a fuzzy environment and presented solution along with

sensitivity analysis approach. The inventory model developed with power pattern demand with shortages. Shortages have been allow partially backlogged in this model. In case where large portion of demand occurs at the beginning of the period the author, use $n>1$ and when it is large at end we use $0<n<1$. This model has been developed for single item.

In this paper, we have considered pentagonal fuzzy number and solved by graded mean integration method. In future, the other type of membership functions such as piecewise linear hyperbolic, L-R fuzzy number etc can be considered to construct the membership function and then model can be easily solved.

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