A Fuzzy Two-Warehouse Inventory Model for Weibull Deteriorating Items with Constant Demand, Shortages and Fully Backlogging

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Abstract: In this paper deals with fuzzy inventory model for deteriorating item, constant demand pattern, shortage under fully backlogged with two warehouse, is formulated and solved. After illustrate the model it test validity of the same, one numerical example have been solved then test sensitivity analyses. Fuzziness is applying by allowing the cost components (holding cost, shortage cost, deterioration cost etc). In fuzzy environment it considered all required parameter to be triangular fuzzy numbers. The purpose of the model is to minimize total cost function.

Keywords: Inventory, Two Warehouse, Constant demand, Fuzzy number, Shortages, Triangular fuzzy number

1. Introduction

An inventory deal with decision that minimum the total average cost or maximize The total average profit. For this purpose the task is to construct a mathematical model of the real life Inventory system, such a mathematical model is based on various assumption and approximation

In a inventory model deterioration and shortages is very important condition. There are many deterioration products in an inventory like drugs, foods, pharmaceuticals etc. During inventory there are several type of customer. At shortage period some customers are waiting for actual product and others do not it.

When a retailer purchases a large quantity of goods at a time, then it hired one or more warehouse. In this paper it considered two warehouse OW and RW.


But in real life situation it will have some little fluctuations. So consideration of fuzzy variables is more realistic.

In this paper we first consider crisp inventory model with power demand where shortage are allowed and partially backlogged. Thereafter we developed fuzzy inventory model with fuzzy power demand rate under partially backlogged.

2. Preliminaries

For graded representation method to defuzzyfy, we need the following definitions,

Definition2.1: A fuzzy set \( \tilde{A} \) on the given universal set \( X \) is a set of order pairs, \( \tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X \} \) where \( \mu_{\tilde{A}}(x) \in [0,1] \) is called a membership function.

Definition2.2: The \( \alpha \)-cut of \( \tilde{A} \), is defined by \( A_\alpha = \{ x : \mu_{\tilde{A}}(x) = \alpha, \alpha \geq 0 \} \)

Definition2.3: \( \tilde{A} \) is normal if there exists \( x \in X \) such that \( \mu_{\tilde{A}}(x) = 1 \)

Definition2.4: A triangular fuzzy number \( \tilde{A} = (a,b,c) \) is represented with membership function \( \tilde{A} \).

\( \tilde{A} \) is defined as,

\[ L(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c \\ 0, & \text{otherwise} \end{cases} \]

when \( a = b = c \), we have fuzzy point \((a,a,a)\). The family of all triangular fuzzy number on \( R \), denoted as \( F_T = \{(a,b,c), a,b,c \in R \} \), the \( \alpha \)-cut of \( \tilde{A} = (a,b,c) \in F_T, 0 \leq \alpha \leq 1 \) is \( A(\alpha) = [A_l(\alpha), A_R(\alpha)] \) where \( A_l(\alpha) = a + (\alpha-a) \) and \( A_R(\alpha) = c - (\alpha-c) \) are the left and right end Point of \( A(\alpha) \).

Definition2.5: If \( \tilde{A} = (a,b,c) \) is a triangular fuzzy number then the graded mean integration of \( \tilde{A} \) is defined as,
Initially w units are store in OW and the rest (S-W) in RW. During 0≤t≤t, the inventory (S-W) units in RW decrease due to customer demand, deterioration and it vanishes at t=t, during the time interval 0≤t≤t, the inventory level decreases due to deterioration in OW and during the interval t=t≤t, it decrease due to customer demand, deterioration and reaches to zero at t=t. During the time interval t<t≤T, the fully backlogged shortages are allowed.

So the differential equation describing as follows
\[
\frac{dI(t)}{dt} + \alpha \beta t^{\beta-1} = D, \quad 0 \leq t \leq t
\]
Subject to boundary condition I(t)=0, and I(0)=S-W.
\[
\frac{dI(t)}{dt} + \alpha \beta t^{\beta-1} = 0, \quad 0 \leq t \leq t
\]
Subject to boundary condition I(t)=0.
\[
\frac{dI(t)}{dt} = -D, \quad t \leq t \leq T
\]
Subject to boundary condition I(t)=0. And I(T)=R

Here a is too small so it neglecting the higher power of a.
From (3.1) we get,
\[
I(t)=D(t_w(1-\alpha^\beta t^{\beta-1}) + \frac{\alpha}{\beta+1} (t_w^{\beta+1} - t^{\beta+1}) (3.5)
\]
So S=W=Dt_w + \frac{\alpha}{\beta+1} t_w^{\beta+1},
\[
S=W+Dt_w + \frac{\alpha}{\beta+1} t_w^{\beta+1}.
\]
From (3.2) we get,
\[
I(t)=W(1-\alpha^\beta t^{\beta-1}) (3.6)
\]
From (3.3) we get,
\[
I(t)=D(t_w(1-\alpha^\beta t^{\beta-1}) + \frac{\alpha}{\beta+1} (t_w^{\beta+1} - t^{\beta+1}) (3.7)
\]
From (3.4) we get,
\[
I(t)=D(t_w-1) (3.8)
\]
So the maximum stock level at the beginning of the cycle is,
\[
S=W + Dt_w + \frac{\alpha}{\beta+1} t_w^{\beta+1} (3.9)
\]
And the maximum amount of demand backlogging is,
\[
R=I(t)(T)
\]
Hence the order quantity per cycle is;
\[
Q=\frac{S+R}{D}(T-t)
\]
The fixed cost per cycle is,
\[
F_C=c_1
\]
Shortage cost per cycle is,
\[
S_C=c_2 \int_{t_1}^{T} I(t) dt
\]
\[
= c_2 D(t_w - \frac{t_1^2}{2})
\]
Deterioration cost per cycle is,
\[
DC=c_1 \int_{t_w}^{t} \alpha t^{\beta-1} I(t) dt + c_2 \int_{t_w}^{t} \alpha t^{\beta-1} I(t) dt + c_3 \int_{t_w}^{t} \alpha t^{\beta-1} I(t) dt
\]
\[
= D I(t_w - \frac{t_1^2}{2} - \frac{t_1^2}{2} - \frac{t_1^2}{2} - \frac{t_1^2}{2}) + \frac{\alpha}{\beta+1} t_w^{\beta+1} + \frac{\alpha}{\beta+1} t_w^{\beta+1} + \frac{\alpha}{\beta+1} t_w^{\beta+1} - \frac{\alpha}{\beta+1} t_w^{\beta+1}.
\]
Holding cost per cycle is,
\[
HC=c_1 \int_{t_w}^{t} \alpha t^{\beta-1} I(t) dt + c_2 \int_{t_w}^{t} \alpha t^{\beta-1} I(t) dt + c_3 \int_{t_w}^{t} \alpha t^{\beta-1} I(t) dt
\]
\[
= D I(t_w^2 - \frac{t_1^2}{2} - \frac{t_1^2}{2} - \frac{t_1^2}{2} - \frac{t_1^2}{2}) + \frac{\alpha}{\beta+1} t_w^{\beta+1} + \frac{\alpha}{\beta+1} t_w^{\beta+1} + \frac{\alpha}{\beta+1} t_w^{\beta+1} - \frac{\alpha}{\beta+1} t_w^{\beta+1}.
\]
So total average cost per cycle is,
\[
TAC(t_w, t_1) = \frac{1}{T}[F_C + S_C + D_C + HC]
\]
We defuzzify the fuzzy total cost $TAC$ by graded mean representation method as follows,

$$TAC(T_w,t_1) = \frac{1}{6} [TAC^1(t_1),TAC^2(t_1),TAC^3(t_1)]$$

Where

$$TAC^r(T_w,t_1) = \frac{1}{7} [FC+SC+DC+HC]$$

For minimum cost it should be,

$$\frac{\partial TAC(T_w,t_1)}{\partial t_1} = 0, \quad \frac{\partial TAC(T_w,t_1)}{\partial t_w} = 0$$

Provided it satisfies equation,

$$\frac{\partial^2 TAC(T_w,t_1)}{\partial t_1^2} > 0, \quad \frac{\partial^2 TAC(T_w,t_1)}{\partial t_w^2} > 0$$

### 3.1 Sensitivity Analysis (Crisp Model)

We now examine to sensitivity analysis of the optimal solution of the model for change in $I$, keeping the other parameters unchanged. The initial data from the above numerical example,

<table>
<thead>
<tr>
<th>Parameter % of change</th>
<th>$TAC(t_1,t_w)$ $t_w$ $t_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1=50.0</td>
<td>-50 358.433 1.162 3.379</td>
</tr>
<tr>
<td>C1=75.0</td>
<td>-25 360.933 1.162 3.379</td>
</tr>
<tr>
<td>C1=100</td>
<td>0   363.433 1.162 3.379</td>
</tr>
<tr>
<td>C1=125</td>
<td>25  365.933 1.162 3.379</td>
</tr>
<tr>
<td>C1=150</td>
<td>50  368.433 1.162 3.379</td>
</tr>
<tr>
<td>C2=100</td>
<td>-50 362.240 1.000 1.630</td>
</tr>
<tr>
<td>C3=0</td>
<td>0   363.433 1.162 3.379</td>
</tr>
<tr>
<td>C3=25</td>
<td>25  365.933 1.162 3.379</td>
</tr>
<tr>
<td>C3=50</td>
<td>50  368.433 1.162 3.379</td>
</tr>
<tr>
<td>C4=50</td>
<td>-50 362.240 1.000 1.630</td>
</tr>
<tr>
<td>C4=75</td>
<td>0   363.433 1.162 3.379</td>
</tr>
<tr>
<td>C4=100</td>
<td>25  365.933 1.162 3.379</td>
</tr>
<tr>
<td>C4=125</td>
<td>50  368.433 1.162 3.379</td>
</tr>
<tr>
<td>C4=150</td>
<td>75  368.433 1.162 3.379</td>
</tr>
</tbody>
</table>

For crisp model:

Let us take the in-put values:

<table>
<thead>
<tr>
<th>$t_w$</th>
<th>$t_1$</th>
<th>S</th>
<th>Q</th>
<th>TAC($t_1,t_w$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.162</td>
<td>3.379</td>
<td>9.910</td>
<td>43.015</td>
<td>363.433</td>
</tr>
</tbody>
</table>

### 3.2 Effect for increment parameters

1. $(TAC(t_1,t_w))$ increase and $t_w$, $t_1$ remain same,
2. $(TAC(t_1,t_w))$ decrease, for increase of $c_1$.
3. $(TAC(t_1,t_w))$ increase and $t_w$ decrease, for increase of $c_2$.
4. $(TAC(t_1,t_w))$ decrease and $t_w$, $t_1$ decrease, for increase of $c_3$. 

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4. Conclusion

In this paper, we have proposed a real life two warehouse inventory problem in a fuzzy environment and presented solution along with sensitivity analysis approach. The inventory model developed with constant demand with shortages. Shortages have been allow fully backlogged in this model. Here deterioration also be used, and which is weibull deterioration. In this paper, we have considered triangular fuzzy number and solved by graded mean integration method. In future, the other type of membership functions such as piecewise linear hyperbolic, L-R fuzzy number, fuzzy pentagonal number, etc can be considered to construct the membership function and then model can be easily solved.

References