$P(\widetilde{A})=\frac{\int_0^{W_A}\left(\frac{L^{-1}(h)+R^{-1}(h)}{2}\right)dh}{\int_0^{W_A} h\,dh}$,($0\leq h\leq w_A$ and $0\leq w_A\leq 1$)

$P(\widetilde{A})=\frac{\int_0^1 [\frac{a+(b-a)h+c-(c-b)h]dh}{2}]}{\int_0^1 h\,dh}$

$=\frac{a+4b+c}{6}$

Suppose $\tilde{a}=(a_1,a_2,a_3)$ and $\tilde{b}=(b_1,b_2,b_3)$ are two fuzzy triangular number then

(1) $\tilde{a}+\tilde{b}=(a_1,a_2,a_3)+(b_1,b_2,b_3)=(a_1+b_1, a_2+b_2, a_3+b_3)$.

(2) $\tilde{a}-\tilde{b}=(a_1,a_2,a_3)-(b_1,b_2,b_3)=(a_1-b_1, a_2-b_2, a_3-b_3)$.

(3) $\tilde{a}\times\tilde{b}=(a_1,a_2,a_3)\times(b_1,b_2,b_3)=(a_1b_1,a_2b_2,a_3b_3)$.

(4) $\frac{1}{\tilde{b}}=(\frac{1}{b_3},\frac{1}{b_2},\frac{1}{b_1})$ where b_1,b_1,b_1 are all non zero positive real number, then $\frac{\tilde{a}}{\tilde{b}}=(\frac{a_1}{b_3},\frac{a_2}{b_2},\frac{a_3}{b_1})$.

(5) Let $k\in R$, then $k\tilde{a}=k(a_1,a_2,a_3)=(ka_1,ka_2,ka_3)$ for $k\geq 0$.

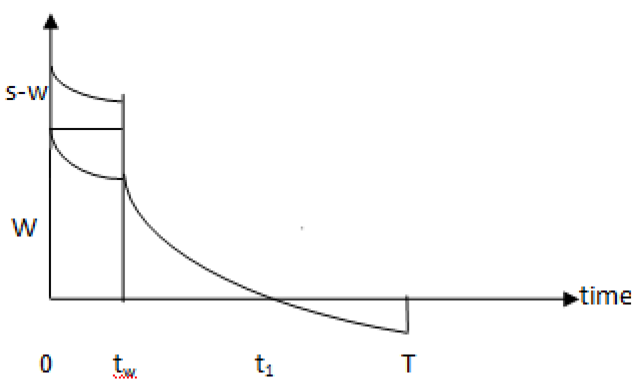
2.1.Notation

$I_r(t)$:Inventory level at time t in RW, $t\geq 0$.
$I_0(t)$:Inventory level at time t in OW, $t\geq 0$.
T:Cycle of length.
t_w: Time point when stock level of RW reaches to zero.
t_1: Time point when stock level of OW reaches to zero.
c_1:Fixed cost.
c_2:Shortages cost per unit.
c_3:Opportunity cost due to lost sales.
C_4 :Holding cost per unit per unit time.
S:Highest stock level at the beginning of the cycle.
W:Stoarge capacity of OW.
R:Highest shortages level.
Q:Total order quantity per cycle.
$TAC(t_w,t_1)$:Total average cost per unit.
$\tilde{c}_1$=Fuzzy fixed cost.
$\tilde{c}_2$=Fuzzy shortage cost per unit.
$\tilde{c}_3$=Fuzzy opportunity cost due to lost sales.
$\tilde{c}_4$ =Fuzzy holding cost per unit.
$\widetilde{TAC}(t_w,t_1)$=Fuzzy total cost per unit.

2.2 Assumption

a: The inventory system involves only one item.
b: The replenishment occur instantaneously at infinite rate.
c: The lead time is negligible.
d: Demand rate is constant, assume it D.
e: Shortages with fully backlogged.

2.3. Model Devolopment (CRISP SET)



Initially w units are store in OW and the rest (S-W) in RW. During $0\leq t\leq t_w$, the inventory (S-W) units in RW decrease due to customer demand, deterioration and it vanishes at $t=t_w$. during the time interval $0\leq t\leq t_w$, the inventory level decrease due to deterioration in OW and during the interval $t_w\leq t\leq t_1$ it decrease due to customer demand, deterioration and reaches to zero at $t=t_1$. During the time interval $t_1<t\leq T$, the fully backlogged shortages are allowed.

So the differential equation describing as follows

$\frac{dI_r(t)}{dt}+\alpha\beta t^{\beta-1}$=-D , $0\leq t\leq t_w$ (3.1)

Subject to boundary condition $I_r(t_w)=0$, and $I_r(0)=S-W$.

$\frac{dI_0(t)}{dt}+\alpha\beta t^{\beta-1}=0$, $0\leq t\leq t_w$ (3.2)

Subject to boundary condition $I_0(t_0)=W$.

$\frac{dI_o(t)}{dt}+\alpha\beta t^{\beta-1}=-D$, $t_w\leq t\leq t_1$ (3.3)

Subject to boundary condition $I_0(t_1)=0$.

$\frac{dI_0(t)}{dt}=-D$, $t_1\leq t\leq T$ (3.4)

Subject to boundary condition $I_0(t_1)=0$. And $I_0(T)=-R$

Here α is too small so it neglecting the higher power of α.

From (3.1) we get,

$I_r(t)=D(t_w-t)(1-\alpha t^{\beta})+\frac{\alpha}{\beta+1}(t_w^{\beta+1}-t^{\beta+1})$ (3.5)

So $S-W=Dt_w+\frac{\alpha}{\beta+1}t_w^{\beta+1}$

And $S=W+Dt_w+\frac{\alpha}{\beta+1}t_w^{\beta+1}$.

From (3.2) we get,

$I_0(t)=W(1-\alpha t^{\beta})$ (3.6)

From (3.3) we get,

$I_0(t)=D(t_1-t)(1-\alpha t^{\beta})+\frac{\alpha}{\beta+1}(t_1^{\beta+1}-t^{\beta+1})$ (3.7)

From (3.4) we get,

$I_0(t)=D(t_1-t)$ (3.8)

So the maximum stock level at the beginning of the cycle is,

$S=W+Dt_w+\frac{\alpha}{\beta+1}t_w^{\beta+1}$ (3.9)

And the maximum amount of demand backlogging is,

$R=-I_0(T)$
$=D(T-t_1)$ (3.10)

Hench the order quantity per cycle is;

$Q=S+R$
$=W+Dtw+\frac{\alpha}{\beta+1}t_w^{\beta+1}+D(T-t_1)$ (3.11)

The fixed cost per cycle is,

$FC=c_1$

Shortages cost per cycle is,

$SC=-c_2\int_{t_1}^{T} I_0(t)dt$,

$=-c_2D(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2})$

Deterioration cost per cycle is,

$DC=c_3\int_0^{t_w}\alpha t^{\beta-1}I_r(t)+c_3\int_0^{t_w}\alpha t^{\beta-1}I_0(t)+c_3\int_{t_w}^{t_1}\alpha t^{\beta-1}I_0(t)$

$=c_3D[\frac{\alpha}{\beta+1}t_w^{\beta+1}+w\alpha t_w^{\beta}+\alpha t_w t_1^{\beta}-\frac{\alpha\beta}{\beta+1}t_1^{\beta+1}-\frac{\alpha}{\beta+1}t_w^{\beta+1}]$.

Holding cost per cycle is,

$HC=c_4\int_0^{t_w}I_r(t)dt+c_4\int_0^{t_w}I_0(t)dt+c_4\int_{t_w}^{t_1}I_0(t)dt$

$=c_4[\frac{t_w^2}{2}-\frac{\alpha t_w^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha t_w^{\beta+2}}{(\beta+2)}+\frac{w}{D}(t_w-\frac{\alpha t_w^{\beta}}{(\beta+1)})+\frac{t_1^2}{2}-\frac{\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha t_1^{\beta+2}}{(\beta+2)}-(t_1t_w-\frac{t_w^2}{2}-\frac{\alpha t_1 t_w^{\beta+1}}{(\beta+1)}+\frac{\alpha t_w^{\beta+2}}{(\beta+2)})-\frac{\alpha t_1^{\beta+2}}{(\beta+2)}-\frac{\alpha}{\beta+1}(t_1^{\beta+1}t_w-\frac{t_w^{\beta+2}}{\beta+2})$.

So total average cost per cycle is,

$TAC(t_w,t_1)=\frac{1}{T}[FC+SC+DC+HC]$

$$=\frac{D}{T}[\frac{c_1}{D} - c_2(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2})+ c_3(\frac{\alpha}{\beta+1}t_w^{\beta+1}+w\alpha t_w^{\beta}+\alpha t_w t_1^{\beta}-\frac{\alpha\beta}{\beta+1}t_1^{\beta+1}-\frac{\alpha}{\beta+1}t_w^{\beta+1}+ c_4\{\frac{t_w^2}{2}-\frac{\alpha t_w^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha t_w^{\beta+2}}{(\beta+2)}+\frac{w}{D}(t_w-\frac{\alpha t_w^{\beta}}{(\beta+1)})+\frac{t_1^2}{2}-\frac{\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha t_1^{\beta+2}}{(\beta+2)} - (t_1t_w - \frac{t_w^2}{2} - \frac{\alpha t_1 t_w^{\beta+1}}{(\beta+1)}+\frac{\alpha t_w^{\beta+2}}{(\beta+2)}) - \frac{\alpha t_1^{\beta+2}}{(\beta+2)} - \frac{\alpha}{\beta+1}(t_1^{\beta+1}t_w - \frac{t_w^{\beta+2}}{\beta+2})\}].$$

For minimum cost it should be,

$$\frac{\partial TAC(t_w,t_1)}{\partial t_w}=0,\ \frac{\partial TAC(t_w,t_1)}{\partial t_1}=0$$

Provided it satisfies equation,

$$\frac{\partial^2 TAC(t_w,t_1)}{\partial t_w^2}>0,\ \frac{\partial^2 TAC(t_w,t_1)}{\partial t_1^2}>0$$

And $\left[\frac{\partial^2 TAC(t_w,t_1)}{\partial t_w^2}\right]\left[\frac{\partial^2 TAC(t_w,t_1)}{\partial t_w^2}\right]-\left[\frac{\partial^2 TAC(t_w,t_1)}{\partial t_w \partial t_1}\right]>0.$

2.4 FUZZY MODEL

Due to uncertainly lets us assume that, $\tilde{c_1}=(c_1^1,c_1^2,c_1^3)$, $\tilde{c_2}=(c_2^1,c_2^2,c_2^3)$, $\tilde{c_3}=(c_3^1,c_3^2,c_3^3)$, $\tilde{c_4}=(c_4^1,c_4^2,c_4^3)$, be triangular fuzzy number then the total average cost is given by,

$$\widetilde{TAC(t_w,t_1)}=\frac{1}{T}[FC+SC+DC+HC]$$

$$=\frac{D}{T}[\frac{\tilde{c_1}}{D} - \tilde{c_2}(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2})+ \tilde{c_3}(\frac{\alpha}{\beta+1}t_w^{\beta+1}+w\alpha t_w^{\beta}+\alpha t_w t_1^{\beta}-\frac{\alpha\beta}{\beta+1}t_1^{\beta+1}-\frac{\alpha}{\beta+1}t_w^{\beta+1}+ \tilde{c_4}\{\frac{t_w^2}{2}-\frac{\alpha t_w^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha t_w^{\beta+2}}{(\beta+2)}+\frac{w}{D}(t_w-\frac{\alpha t_w^{\beta}}{(\beta+1)})+\frac{t_1^2}{2}-\frac{\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha t_1^{\beta+2}}{(\beta+2)} - (t_1t_w - \frac{t_w^2}{2} - \frac{\alpha t_1 t_w^{\beta+1}}{(\beta+1)}+\frac{\alpha t_w^{\beta+2}}{(\beta+2)}) - \frac{\alpha t_1^{\beta+2}}{(\beta+2)} - \frac{\alpha}{\beta+1}(t_1^{\beta+1}t_w - \frac{t_w^{\beta+2}}{\beta+2})\}].$$

We defuzzyfi the fuzzy total cost $\widetilde{TAC(t_w,t_1)}$ by graded mean representation method as follows,

$$\widetilde{TAC(t_w,t_1)}=\frac{1}{6}[\widetilde{TAC^1}(t_w,t_1), \widetilde{TAC^2}(t_w,t_1), \widetilde{TAC^3}(t_w,t_1)]$$

Where

$$\widetilde{TAC^r}(t_w,t_1)=\frac{1}{T}[FC+SC+DC+HC]$$

$$=\frac{D}{T}[\frac{\widetilde{c^r}_1}{D} - \widetilde{c^r}_2(t_1T-\frac{T^2}{2}-\frac{t_1^2}{2})+ \widetilde{c^r}_3(\frac{\alpha}{\beta+1}t_w^{\beta+1}+w\alpha t_w^{\beta}+\alpha t_w t_1^{\beta}-\frac{\alpha\beta}{\beta+1}t_1^{\beta+1}-\frac{\alpha}{\beta+1}t_w^{\beta+1}+ \widetilde{c^r}_4\{\frac{t_w^2}{2}-\frac{\alpha t_w^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha t_w^{\beta+2}}{(\beta+2)}+\frac{w}{D}(t_w-\frac{\alpha t_w^{\beta}}{(\beta+1)})+\frac{t_1^2}{2}-\frac{\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha t_1^{\beta+2}}{(\beta+2)} - (t_1t_w - \frac{t_w^2}{2} - \frac{\alpha t_1 t_w^{\beta+1}}{(\beta+1)}+\frac{\alpha t_w^{\beta+2}}{(\beta+2)}) - \frac{\alpha t_1^{\beta+2}}{(\beta+2)} - \frac{\alpha}{\beta+1}(t_1^{\beta+1}t_w - \frac{t_w^{\beta+2}}{\beta+2})\}].$$ Where r=1,2,3.

$$\widetilde{TAC(t_w,t_1)}=\frac{1}{6}[\widetilde{TAC_1}(t_w,t_1)+4\widetilde{TAC_2}(t_w,t_1)+\widetilde{TAC_3}(t_w,t_1)]$$

For minimum cost it should be,

$$\frac{\partial \widetilde{TAC(t_w,t_1)}}{\partial t_w}=0,\ \frac{\partial \widetilde{TAC(t_w,t_1)}}{\partial t_1}=0$$

Provided it satisfies equation,

$$\frac{\partial^2 \widetilde{TAC(t_w,t_1)}}{\partial t_w}>0,\ \frac{\partial^2 \widetilde{TAC(t_w,t_1)}}{\partial t_w}>0$$

And $\left[\frac{\partial^2 \widetilde{TAC(t_w,t_1)}}{\partial t_w}\right]\left[\frac{\partial^2 \widetilde{TAC(t_w,t_1)}}{\partial t_w}\right]-\left[\frac{\partial^2 \widetilde{TAC(t_w,t_1)}}{\partial t_w \partial t_1}\right]>0.$

3. Numerical Solution

For crisp model:

Let us take the in-put value:

C_1	C_2	C_3	C_4	α	β	D	T	W
100	20	70	50	0.1	0.2	5	10	4

And the out-put value is:

t_w	t_1	S	Q	$TAC(t_w,t_1)$
1.162	3.379	9.910	43.015	363.433

For fuzzy model:

$\tilde{c_1}$=(95,100,105,), $\tilde{c_2}$ =(18,20,22), $\tilde{c_3}$=(65,70,75), $\tilde{c_4}$=(47,50,53)

The solution of fuzzy model by graded mean representation is,

1) When $\tilde{c_1},\tilde{c_2},\tilde{c_3}, ,\tilde{c_4}$ are all triangular fuzzy numbers then, $TAC(t_w,t_1)$=363.389, t_w=1.161 t_1=3.377
2) When $\tilde{c_1},\tilde{c_2},\tilde{c_3}$, are all triangular fuzzy numbers then $TAC(t_w,t_1)$=363.171, t_w=1.160 t_1=3.376
3) When $\tilde{c_1},\tilde{c_2}$, are triangular fuzzy numbers then, $TAC(t_w,t_1)$=363.154, t_w=1.160 t_1=3.375
4) When $\tilde{c_1}$, is triangular fuzzy numbers then, $TAC(t_w,t_1)$=363.433, t_w=1.60 t_1=3.379

3.1 Sensitivity Analysis (Crisp Model)

We now examine to sensitivity analysis of the optimal solution of the model for change in I, keeping the other parameters unchanged. The initial data from the above numerical example.

Parameter	% of change	$TAC(t_w,t_1)$	t_w	t_1
C_1=50.0	-50	358.433	1.162	3.379
C_1=75.0	-25	360.933	1.162	3.379
C_1=100	0	363.433	1.162	3.379
C_1=125	25	365.933	1.162	3.379
C_1=150	50	368.433	1.162	3.379
C_2=10	-50	221.020	0.000	1.530
C_2=15	-25	302.483	0.802	2.676
C_2=20	0	363.433	1.162	3.379
C_2=25	25	413.477	1.451	3.952
C_2=30	50	455.574	1.691	4.431
C_3=35	-50	354.724	1.258	3.434
C_3=52.5	-25	359.117	1.210	3.407
C_3=70	0	363.433	1.162	3.379
C_3=87.5	25	367.667	1.112	3.351
C_3=105	50	371.817	1.062	3.322
C_4=25	-50	277.048	1.901	5.117
C_4=37.5	-25	328.471	1.476	4.840
C_4=50	0	363.433	1.162	3.379
C_4=62.5	25	388.571	0.924	2.856
C_4=75	50	407.359	0.739	2.455

3.2Effect, for increment parameters-

(1) $TAC(t_w,t_1)$ increase and t_w, t_1 remain same, for increase of c_1.
(2) $TAC(t_w,t_1)$, t_w, t_1, increase, for increase of c_2.
(3) $TAC(t_w,t_1)$ increase and t_w, t_1 decrease , for increase of c_3.
(4) $TAC(t_w,t_1)$ increase and t_w, t_1 decrease, for increase of c_4.

4. Conclusion

In this paper, we have proposed a real life two warehouse inventory problem in a fuzzy environment and presented solution along with sensitivity analysis approach. The inventory model developed with constant demand with shortages. Shortages have been allow fully backlogged in this model. Here deterioration also be used, and which is weibull deterioration. In this paper, we have considered triangular fuzzy number and solved by graded mean integration method. In future, the other type of membership functions such as piecewise linear hyperbolic, L-R fuzzy number, fuzzy pentagonal number, etc can be considered to construct the membership function and then model can be easily solved.

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