$x^{(k+1)} = b - Lx^{(k+1)} - Ux^k$ 3.4

Where $x^{(k+1)}$ and $x^k$ are the new and old approximations respectively. At this point our approximation $a_{jj}$=1 is no longer valid since it is taking care of automatically by the factor $1/a_{jj}$.

The algorithm computes solution for x of the system Ax=b given initial approximation $x^{(0)}$, where $A = [a_{jk}]$ is an $n \times n$ matrix with $a_{jj} \neq 0, j = 1, \ldots, n$

INPUT: A, b initial approximation $x^{(0)}$, tolerance $\epsilon > 0$, maximum number of iterations N.

$For\ k = 0, \ldots, N-1, do:$

$For\ j = 1, \ldots n, do:$

$$x_j^{(k+1)} = \frac{1}{a_{jj}}\left(b_j - \sum_{i=1}^{j-1} a_{ji}\, x_i^{(k+1)} - \sum_{i=j+1}^{n} a_{ji}\, x_i^{(k)}\right)$$

End

$if\ max\left|x_j^{(k+1)} - x_j^k\right| < \epsilon$

$then\ OUTPUT\ x^{(k+1)}$ Stop

End

*OUTPUT:* no satisfying solution. The tolerance condition failed.

### 3.3 Program Design and Structure

The application is a two page application that allows the user to enter all the coefficients of the system of linear equation in form of matrix A.

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_2 \end{pmatrix}$$

Where $x_1$, $x_2$ and $x_3$ are the expected values that solves the system. We have represented a system of three linear equations to depict most of the cases obtained while solving physical problems. The designed view of the program is shown in fig1. below.

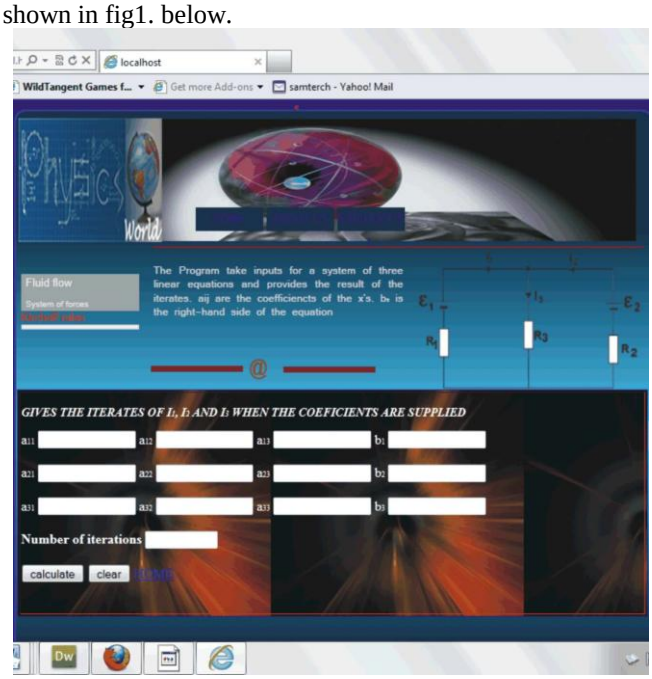


**Figure 1:**Designed web-based application for Gauss-Sidel iteration

After the coefficients of the system $a_{11}$, $a_{12}$, $a_{13}$, $b_1$…,$a_{jj}$ is inputted and the calculate button clicked. The result is displayed on a new page.

## 4. Results and Discussion

Here, examples on systems of linear equations were taken and tested in the application to see the results of the iterates.

(a)Suppose the system below

$5x_1 + 2x_2 - x_3 = 6$

$x_1 + 6x_2 - 3x_3 = 4$

$2x_1 + x_2 + 4x_3 = 7$

Inputting the values of the coefficients in the program we have

**Table 1:** Results of iterates for example 1

| $i_1$ | $i_2$ | $i_3$ |
|---|---|---|
| 0.0000 | 0.0000 | 0.0000 |
| 1.2000 | 0.4667 | 1.0333 |
| 1.2200 | 0.9800 | 0.8950 |
| 0.9870 | 0.9497 | 1.0191 |
| 1.0239 | 1.0056 | 0.9866 |
| 0.9951 | 0.9941 | 1.0039 |
| 1.0031 | 1.0014 | 0.9981 |
| 0.9991 | 0.9992 | 1.0007 |
| 1.0005 | 1.0003 | 0.9997 |
| 0.9998 | 0.9999 | 1.0001 |
| 1.0001 | 1.0000 | 1.0000 |

The results as displayed on the browser page is shown in fig2.

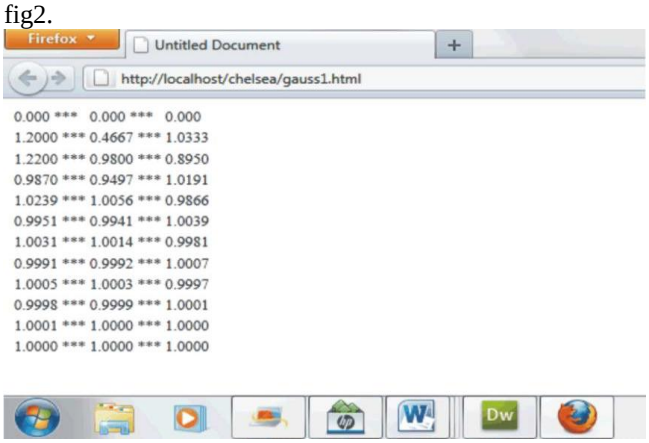


**Figure 2:** Results of example (a) on the browser window

In order to exhibit the relevance this kind of systems in Physics, we used examples in electrical network under Kirchhoff's rules; where the voltage rule states that: *In any closed loop in a network, the algebraic sum of the voltage drops (i.e. products of current and resistance) taken around the loop is equal to the resultant e.m.f. acting in that loop* [3]

**(b) Consider the electrical network below**

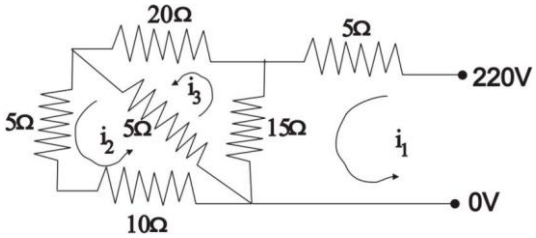


**Figure 3:** Electrical network

The system of equations developed for the three loops using Kirchhoff's rule stated above, is shown below:

$$20i_1 + 0i_2 - 15i_3 = 220$$
$$0i_1 + 20i_2 - 5i_3 = 0$$
$$-15i_1 - 5i_2 + 40i_3 = 0$$

Inputting the coefficients of the values of the currents in the web-based application we obtain the following iterates.

**Table 2:** Results of iterates for example 2

| $i_1$ | $i_2$ | $i_3$ |
|---|---|---|
| 0.0000 | 0.0000 | 0.0000 |
| 11.0000 | 0.0000 | 4.1250 |
| 14.0938 | 1.0313 | 5.4141 |
| 15.0606 | 1.3535 | 5.8169 |
| 15.3627 | 1.4542 | 5.9428 |
| 15.4571 | 1.4857 | 5.9821 |
| 15.4866 | 1.4955 | 5.9944 |
| 15.4958 | 1.4986 | 5.9982 |
| 15.4987 | 1.4995 | 5.9994 |
| 15.4995 | 1.4998 | 5.9998 |
| 15.4999 | 1.4999 | 5.9999 |
| 15.4999 | 1.5000 | 6.0000 |
| 15.5000 | 1.5000 | 6.0000 |

The result of the iterates as it appears on the browser window is shown in fig2. below.

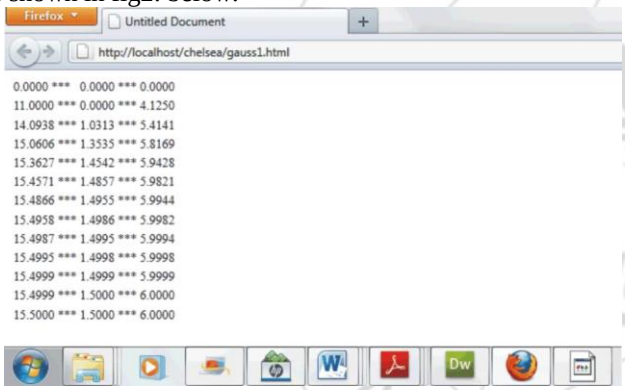


**Figure 4:** Results of example (b) on the browser window

(c)We also consider an example in electrical network using Kirchhoff's rules which is one of the practical examples in Physics below:

$$4i_1 + 2i_2 - i_3 = -6.3$$
$$0i_1 + 3i_2 + 4i_3 = 14.8$$
$$i_1 - i_2 + 5i_3 = 13.5$$

Inputting the coefficient of currents $i_1$, $i_2$, and $i_3$; the result of the computed iteration is given below:

**Table 3:** Results of iterates for example 3

| **$i_1$** | **$i_2$** | **$i_3$** |
|---|---|---|
| 0.0000 | 0.0000 | 0.0000 |
| -1.5750 | 4.9333 | 4.0017 |
| -3.0412 | -0.4023 | 3.2278 |
| -0.5669 | 0.6296 | 2.9393 |
| -1.1550 | 1.0143 | 3.1338 |
| -1.2987 | 0.7549 | 3.1107 |
| -1.1748 | 0.7857 | 3.0921 |
| -1.1948 | 0.8105 | 3.1011 |
| -1.2050 | 0.7985 | 3.1007 |
| -1.1991 | 0.7991 | 3.0996 |
| -1.1997 | 0.8005 | 3.1000 |
| -1.2003 | 0.8000 | 3.1001 |
| -1.2000 | 0.7999 | 3.1000 |
| -1.1999 | 0.8000 | 3.1000 |
| -1.2000 | 0.8000 | 3.1000 |

The result is also shown on another page of the browser in figure two below:

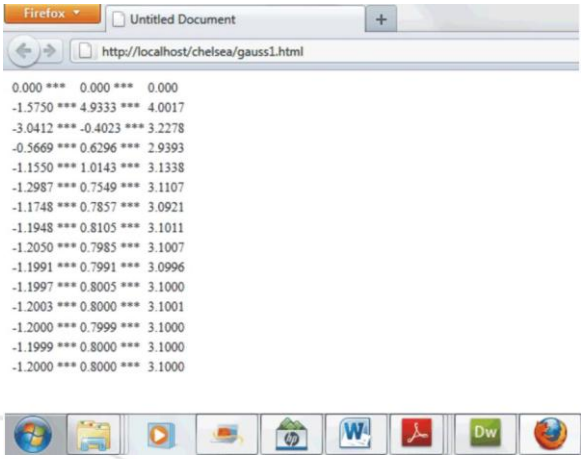


**Figure 5:** Results of example (c) on the browser window

The source code for the program is shown in the appendix.

## 5. Conclusion

In this research a web-based application for numerical solution of linear systems using Gauss-Seidel iterative method iteration has been developed and tested. The application performed effectively and is compatible with most browsers and is compliant with many platforms even with the mobile phones this will enhance accessibility even in regions of the world where computers and electricity supply is scarce.

This application will be an addition to database of existing applications in the teaching and learning of Physics. We therefore recommend that other web-based numerical applications should be designed in so many areas of physical Sciences to solve like problems.

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## Appendix

```
//JavaScript Codes
<script type="text/javascript">
//program gets data from form.
var a1 = parseFloat(document.getElementById("a1").value);
var a2 = parseFloat(document.getElementById("a2").value);
var a3 = parseFloat(document.getElementById("a3").value);
var b1 = parseFloat(document.getElementById("b1").value);
var b2 = parseFloat(document.getElementById("b2").value);
var b3 = parseFloat(document.getElementById("b3").value);
var c1 = parseFloat(document.getElementById("c1").value);
var c2 = parseFloat(document.getElementById("c2").value);
var c3 = parseFloat(document.getElementById("c3").value);
var d1 = parseFloat(document.getElementById("d1").value);
var d2 = parseFloat(document.getElementById("d2").value);
var d3 = parseFloat(document.getElementById("d3").value);
var n = parseInt(document.getElementById("n").value);
//Program performs iteration
if(a1!=0 || b2!=0 || c3!=0)
{
x1=0; x2=0; x3=0;
for (i=1; i<=n; i++)
{
 x1=(d1-(b1*x2)-(c1*x3))/a1;
 x2=(d2-(a2*x1)-(c2*x3))/b2;
 x3=(d3-(a3*x1)-(b3*x2))/c3;
 var x1=x1.toFixed(4);
 var x2=x2.toFixed(4);
 var x3=x3.toFixed(4);
//Program prints out the result of the iteration
document.write("<tr><td>");
document.siedel.value= x1;;
document.write("*** ");
document.write("<td>");
document.siedel.value= x2;
document.write(" *** ");
document.write("<td>");
document.siedel.value= x3;
document.write("<br>");
 }
}
else
{
document.write("no satisfying solution. The tolerance condition failed. ajj
        must not be zero, rearrange your equations so that no main
        diagonal element is zero");
}
</script>
```

## Author Profile

**Mr. Samuel Terungwa Temaugee** holds a Bachelor of Science degree B.Sc. (Ed) and a Master of Science degree, M.Sc. in Physics, from the University of Ilorin and University of Ibadan, Nigeria in 2006 and 2010 respectively. He is currently a lecturer with Department of Physics, Federal College of Education, Kontagora, Niger State, Nigeria. His research interest is in Radiation & Health and Computational Physics.

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