A New Quasi-Newton Update of the Newton's Iterative Method for Optimizing Non-Linear Multi-Variable Optimization Problems

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Abstract: ***We modified the Newton's method of optimization. The new modification is of the quasi-Newton principle thereby presenting a rank 3 matrix update of the principle. A number of standard test functions were used to implement the Newton's method, one of the existing modifications and the new modification. The results obtained compared favourably with the existing ones. Maple computer application was used to ease computations.***

Keywords: Test Functions, Newton's method, Quasi-Newton Method, DavidonFlecther Powell's Method Method (DFP)

1. Introduction

In recent years, work have been carried out on the use and convergence of the classical Newton's method [3], the method have been modified by several researchers such as Weerakoon (2000) and Fernando (2000)[10] who re-derived the classical Newton's method by using the rectangular rule to compute the integral. Homerier (2003)[11] developed on the work of Weerakoon and Fernando (2000) and presented a modification of the method for the solution of multi variable case and thus having a convergence of order 3. Polyak (2005)[6] stated in his publication the two main methods of modification of the Newton's formula as the damped and the MarquardartLeverberg methods of modifying. Others are Sathya (2005)[9], Nestrov Yu (2003)[9]; Baghmisheh (2013)[1]

2. Derivation of the Method

Existing modification was based on the choice of the rank of the update matrix, the computation of $[B_i]$, to be 1 and 2, this work adopted the rank 3 principle and is as follows:

The approximate inverse of the Hessian matrix, $[B_i] \equiv [A_i]^{-1} \equiv [J_i]^{-1}$, is to be computed. For this, we first expand the gradient of f about an arbitrary reference point $X_\circ$, using Taylor's series as:

$$\nabla f(X) \equiv \nabla f(X_\circ) + [J_\circ](X - X_\circ) \qquad (1)$$

If we pick **THREE POINTS:** X_i, X_{i+1} *and* X_{i+2} and $[A_i] \equiv [J_i]$

Then (1) can be rewritten as:

$$\nabla f_{i+2} = \nabla f(X_\circ) + [A_i](X_{i+2} - X_\circ) \qquad (2)$$

$$\nabla f_{i+1} = \nabla f(X_\circ) + [A_i](X_{i+1} - X_\circ) \qquad (3)$$

$$\nabla f_i = \nabla f(X_\circ) + [A_i](X_i - X_\circ) \qquad (4)$$

So, (2)-(3)-(4) gives

$$\nabla f_{i+2} - \nabla f_{i+1} - \nabla f_i = -\nabla f(X_\circ) + [A_i](X_{i+2} - X_{i+1} - X_i) + [A_i]X_\circ \qquad (5)$$

Let

$$g_{i+2} = \nabla f_{i+2} - \nabla f_{i+1} - \nabla f_i$$

$$d_{i+2} = X_{i+2} - X_{i+1} - X_i$$

$$m_{i+2} = -\nabla f(X_\circ) + [A_i]X_\circ$$

Let

$$[A_i] \equiv [B_{i+2}]^{-1}$$

$\Rightarrow$

$$m_{i+2} = -\nabla f(X_\circ) + [B_{i+2}]^{-1}X_\circ$$

$$= [B_{i+2}]^{-1}X_\circ - \nabla f(X_\circ)$$

(5) now become:

$$g_{i+2} = m_{i+2} + [A_i]d_{i+2}$$

$\Rightarrow$

$$d_{i+2} = [A_i]^{-1}(g_{i+2} - m_{i+2})$$

Thus;

$$d_{i+2} = [B_{i+2}](g_{i+2} - m_{i+2}) \qquad (6)$$

It can be seen that (6) represents a system of n-equations in n^3 -unknown elements of matrix $[B_{i+2}]$ The update $[B_{i+3}]$ is of

RANK 3(because of the three point selected).Thus:

$$[B_{i+3}] = [B_{i+2}] + [\Delta B_{i+2}]$$

Let

$$[\Delta B_{i+2}] = C_1 Z_1 Z_1^T + C_2 Z_2 Z_2^T + C_3 Z_3 Z_3^T \qquad (7)$$

Forcing(7) to satisfy quasi-Newton condition,

$$d_{i+2} = [B_{i+3}](g_{i+2} - m_{i+2})$$

Thus:

$$\left.\begin{array}{c} d_{i+2}=[B_{i+2}](g_{i+2}-m_{i+2})+[\Delta[B_{i+2}](g_{i+2}-m_{i+2})] \\ =[B_{i+2}](g_{i+2}-m_{i+2})+C_1Z_1Z_1^T(g_{i+2}-m_{i+2})+C_2Z_2Z_2^T(g_{i+2}-m_{i+2})+ \\ C_3Z_3Z_3^T(g_{i+2}-m_{i+2}) \\ or \\ d_{i+2}-[B_{i+2}](g_{i+2})+[B_{i+2}](m_{i+2})= \\ C_1Z_1Z_1^T(g_{i+2}-m_{i+2})+C_2Z_2Z_2^T(g_{i+2}-m_{i+2})+C_3Z_3Z_3^T(g_{i+2}-m_{i+2}) \end{array}\right\} \quad (8)$$

Since $Z_r^T(g_{i+2}-m_{i+2})$; $r=1(1)3$ is a scalar, we rewrite (8) as:

$$\frac{d_{i+2}-[B_{i+2}](g_{i+2})+[B_{i+2}](m_{i+2})}{Z_1^T(g_{i+2}-m_{i+2})Z_2^T(g_{i+2}-m_{i+2})Z_3^T(g_{i+2}-m_{i+2})}$$
$$=\frac{C_1Z_1}{Z_2^T(g_{i+2}-m_{i+2})Z_3^T(g_{i+2}-m_{i+2})}+\frac{C_2Z_2}{Z_1^T(g_{i+2}-m_{i+2})Z_3^T(g_{i+2}-m_{i+2})}+$$
$$\frac{C_3Z_3}{Z_1^T(g_{i+2}-m_{i+2})Z_2^T(g_{i+2}-m_{i+2})}$$

Equating the equation term by term, $\Rightarrow$

$$C_1Z_1=\frac{d_{i+2}}{Z_1^T(g_{i+2}-m_{i+2})}$$
$$\Rightarrow$$
$$Z_1=d_{i+2} \quad and \quad C_1=\frac{1}{d_{i+2}^T(g_{i+2}-m_{i+2})} \quad (9)$$

Also

$$C_2Z_2=\frac{-[B_{i+2}]g_{i+2}}{Z_2^T(g_{i+2}-m_{i+2})}$$
$$\Rightarrow$$
$$Z_2=[B_{i+2}]g_{i+2} \quad and \quad C_2=\frac{-1}{[B_{i+2}]g_{i+2}^T(g_{i+2}-m_{i+2})} \quad (10)$$

and also:

$$C_3Z_3=\frac{[B_{i+2}]m_{i+2}}{Z_3^T(g_{i+2}-m_{i+2})}$$
$$\Rightarrow$$
$$Z_3=[B_{i+2}]m_{i+2} \quad and \quad C_3=\frac{1}{([B_{i+2}]m_{i+2})^T(g_{i+2}-m_{i+2})}$$

(11)

Now the new update matrix $[B_{i+3}]$ is

$$[B_{i+3}]=[B_{i+2}]+[\Delta B_{i+2}]$$

$$=[B_{i+2}]+\frac{d_{i+2}d_{i+2}^T}{d_{i+2}^T(g_{i+2}-m_{i+2})}-\frac{[B_{i+2}]g_{i+2}([B_{i+2}]g_{i+2})^T}{([B_{i+2}]g_{i+2})^T(g_{i+2}-m_{i+2})}$$
$$+\frac{[B_{i+2}]m_{i+2}([B_{i+2}]m_{i+2})^T}{([B_{i+2}]m_{i+2})^T(g_{i+2}-m_{i+2})} \quad (12)$$

where

$$d_{i+2}=X_{i+2}-X_{i+1}-X_i$$

$$g_{i+2}=\nabla f_{i+2}-\nabla f_{i+1}-\nabla f_i$$

$$m_{i+2}=[B_{i+2}]^{-1}X_\circ-\nabla f_\circ$$

Suppose:

$$[L_{i+2}]=\frac{d_{i+2}d_{i+2}^T}{d_{i+2}^T(g_{i+2}-m_{i+2})},$$

$$[M_{i+2}]=-\frac{[B_{i+2}]g_{i+2}([B_{i+2}]g_{i+2})^T}{([B_{i+2}]g_{i+2})^T(g_{i+2}-m_{i+2})}$$

and

$$[N_{i+2}]=\frac{[B_{i+2}]m_{i+2}([B_{i+2}]m_{i+2})^T}{([B_{i+2}]m_{i+2})^T(g_{i+2}-m_{i+2})}$$

Then

$$[B_{i+3}]=[B_{i+2}]+[L_{i+2}]+[M_{i+2}]+[N_{i+2}] \quad (13)$$

A major concern is that in addition to satisfying (13), the symmetry and positive definiteness of the matrix $[B_{i+3}]$ is to be maintained; i.e if $[B_{i+3}]$ is symmetric and positive definite (to obtain minimum), $[B_{i+4}]$ must remain symmetric and positive definite. This property is retained for all i.

3. Numerical Results

In this section we present some numerical results to demonstrate the performance of the new scheme with the implemented Maple computer application and the results are as follow (we take n = 10 to be fixed and stopping criterion 0.00001):

Example 1: The CUTE Quartic Function

$$f(X):=\sum_{r=1}^{n}(x_r-1)^4, \quad X_\circ=(2,2,2,\cdots,2)^T$$

For $n=10$

$$f(X):=\sum_{r=1}^{10}(x_r-1)^4, \quad X_\circ=(2,2,2,2,2,2,2,2,2,2)^T$$

$$f(X):=(x_1-1)^4+(x_2-1)^4+(x_3-1)^4+(x_4-1)^4+(x_5-1)^4+(x_6-1)^4+(x_7-1)^4+$$
$$(x_8-1)^4+(x_9-1)^4+(x_{10}-1)^4$$

Numerical Results of the Newton's Method

Table 1: Table of Numerical Results of the Newton's Method

$x\|_r^i$	0	1	2	3	4	5	6
x_1	2.000000000	1.666666667	1.444444445	1.296296297	1.197530865	1.131687243	1.087791495
x_2	2.000000000	1.666666667	1.444444445	1.296296297	1.197530865	1.131687243	1.087791495
x_3	2.000000000	1.666666667	1.444444445	1.296296297	1.197530865	1.131687243	1.087791495
x_4	2.000000000	1.666666667	1.444444445	1.296296297	1.197530865	1.131687243	1.087791495
x_5	2.000000000	1.666666667	1.444444445	1.296296297	1.197530865	1.131687243	1.087791495
x_6	2.000000000	1.666666667	1.444444445	1.296296297	1.197530865	1.131687243	1.087791495
x_7	2.000000000	1.666666667	1.444444445	1.296296297	1.197530865	1.131687243	1.087791495
x_8	2.000000000	1.666666667	1.444444445	1.296296297	1.197530865	1.131687243	1.087791495
x_9	2.000000000	1.666666667	1.444444445	1.296296297	1.197530865	1.131687243	1.087791495
x_{10}	2.000000000	1.666666667	1.444444445	1.296296297	1.197530865	1.131687243	1.087791495
f_i	10.00000000	1.975308648	0.390184425	0.0770734670	0.015224388	0.0030072866	0.0005940319

Numerical Results of the DFP

Table 2: Table of Numerical Results of the DFP

$x\|_r^i$	0	1
x_1	2.000000000	1.000000000
x_2	2.000000000	1.000000000
x_3	2.000000000	1.000000000
x_4	2.000000000	1.000000000
x_5	2.000000000	1.000000000
x_6	2.000000000	1.000000000
x_7	2.000000000	1.000000000
x_8	2.000000000	1.000000000
x_9	2.000000000	1.000000000
x_{10}	2.000000000	1.000000000
f_i	10.000000000	0.000000000

Numerical Results of the New Modification

Table 3: Table of Numerical Results of the New Modification

$x\|_r^i$	0	1	2	3
x_1	2.000000000	1.666666667	1.444444445	1.000000000
x_2	2.000000000	1.666666667	1.444444445	1.000000000
x_3	2.000000000	1.666666667	1.444444445	1.000000000
x_4	2.000000000	1.666666667	1.444444445	1.000000000
x_5	2.000000000	1.666666667	1.444444445	1.000000000
x_6	2.000000000	1.666666667	1.444444445	1.000000000
x_7	2.000000000	1.666666667	1.444444445	1.000000000

x_8	2.000000000	1.666666667	1.444444445	1.000000000
x_9	2.000000000	1.666666667	1.444444445	1.000000000
x_{10}	2.000000000	1.666666667	1.444444445	1.000000000
f_i	10.000000000	1.9753086480	0.3901844250	0.000000000

Example 2: The Extended Penalty Function

$$f(X) := \sum_{r=1}^{n-1}(x_r - 1)^2 + (\sum_{r=1}^{n} x_r^2 - 0.25)^2, \quad X_\circ := (1,2,\ldots,n)^T$$

For n = 10

$$f(X) := \sum_{r=1}^{9}(x_r - 1)^2 + (\sum_{r=1}^{n} x_r^2 - 0.25)^2, \quad X_\circ := (1,2,3,4,5,6,7,8,9,10)^T$$

$$f(X) := (x_1-1)^2 + (x_2-1)^2 + (x_3-1)^2 + (x_4-1)^2 + (x_5-1)^2 + (x_6-1)^2 + (x_7-1)^2 +$$
$$(x_8-1)^2 + (x_9-1)^2 + (x_1{}^2 + x_2{}^2 + x_3{}^2 + x_4{}^2 + x_5{}^2 + x_6{}^2 + x_7{}^2 + x_8{}^2 + x_9{}^2 + x_{10}{}^2 - 0.25)^2$$

Numerical Results of the Newton's Method

Table 4(A): Table of Numerical Results of the Newton's Method

$x\|_r^i$	0	1	2	3	4	5	6
x_1	1.000000000	0.667569248	0.447294544	0.303569664	0.214869397	0.171499055	0.175338526
x_2	2.000000000	1.333840638	0.890807174	0.598074835	0.409086502	0.297043345	0.251796370
x_3	3.000000000	2.000112026	1.334319800	0.892580003	0.603303604	0.422587633	0.328254214
x_4	4.000000000	2.666383416	1.777832428	1.187085171	0.797520708	0.548131922	0.404712059
x_5	5.000000000	3.332654807	2.221345057	1.481590343	0.991737812	0.673676211	0.481169903
x_6	6.000000000	3.998926196	2.664857686	1.776095512	1.185954916	0.799220500	0.557627748
x_7	7.000000000	4.665197586	3.108370318	2.070600685	1.380172020	0.924764789	0.634085593
x_8	8.000000000	5.331468973	3.551882943	2.365105851	1.574389123	1.050309077	0.710543437
x_9	9.000000000	5.997740364	3.995395571	2.659611021	1.768606226	1.175853365	0.787001280
x_{10}	10.00000000	6.671372395	4.453886274	2.977017847	1.992490609	1.331379531	0.872693160
f_i	148236.5625	29265.86075	5770.666467	1134.412339	222.3449378	44.84338980	11.45268999

Table 4(B): Table of Numerical Results of the Newton's Method

$x\|_r^i$	7	8	9	10	11
x_1	0.234275248	0.322951101	0.356999334	0.357381948	0.357340138
x_2	0.273163849	0.334599565	0.358092776	0.357402797	0.357340149
x_3	0.312052449	0.346248028	0.359186218	0.357423645	0.357340162
x_4	0.350941050	0.357896492	0.360279660	0.357444494	0.357340174
x_5	0.389829651	0.369544955	0.361373102	0.357465343	0.357340186
x_6	0.428718253	0.381193419	0.362466544	0.357486192	0.357340198
x_7	0.467606853	0.392841882	0.363559986	0.357507041	0.357340210
x_8	0.506495454	0.404490345	0.364653428	0.357527889	0.357340222
x_9	0.545384055	0.416138809	0.365746870	0.357548738	0.357340234
x_{10}	0.519210003	0.208132322	0.029016674	0.0008519305	7.636906E-7
f_i	5.626061593	4.647398508	4.528569570	4.525718286	4.525718286

Numerical Results of the DFP

Table 5: Table of Numerical Results of the DFP

$x\|_r^i$	0	1	2	3	4
x_1	1.000000000	0.052195689	0.348256789	0.328933520	0.310096800
x_2	2.000000000	0.10315664	0.355101161	0.341819912	0.325623036
x_3	3.000000000	0.154123638	0.361945532	0.354706304	0.341149270
x_4	4.000000000	0.205087613	0.368789905	0.367592696	0.356675499
x_5	5.000000000	0.256051587	0.375634277	0.380479089	0.372201728
x_6	6.000000000	0.307015562	0.382478649	0.393365481	0.387727957
x_7	7.000000000	0.357979536	0.389323021	0.406251873	0.403254185
x_8	8.00000000	0.408943511	0.396167393	0.419138265	0.418780413
x_9	8.000000000	0.459907485	0.403011765	0.432024658	0.434306648
x_{10}	10.000000000	0.521956890	0.248214942	0.189734381	0.199011046
f_i	148236.5625	5.727319639	4.687078259	4.671672096	4.665980990

Numerical Results of the New Modification

Table 6: Table of Numerical Results of the New Modification

$x\|_r^i$	0	1	2	3	4	5	6
x_1	1.000000000	0.667569248	0.447294544	0.056228737	0.349152353	0.324199644	0.324199644
x_2	2.000000000	1.333840638	0.890807174	0.106216629	0.355778117	0.337774361	0.337774361
x_3	3.000000000	2.000112026	1.334319800	0.156204521	0.362403882	0.351349079	0.351349079
x_4	4.000000000	2.666383416	1.777832428	0.206192412	0.369029647	0.364923796	0.364923796
x_5	5.000000000	3.332654807	2.221345057	0.256180303	0.375655412	0.378498513	0.378498513
x_6	6.000000000	3.998926196	2.664857686	0.306168195	0.382281177	0.392073231	0.392073231
x_7	7.000000000	4.665197586	3.108370318	0.356156087	0.388906942	0.405647948	0.405647948
x_8	8.000000000	5.331468973	3.551882943	0.406143978	0.395532707	0.419222665	0.419222665
x_9	9.000000000	5.997740364	3.995395571	0.456131870	0.402158472	0.432797382	0.432797382
x_{10}	10.00000000	6.671372395	4.453886274	0.527890856	0.250351739	0.196083051	0.196083051
f_i	148236.5625	29265.86075	5770.666467	5.721084504	4.688902229	4.673038603	4.673038603

Example 3: The Diagonal 3 Function

$$f(X) := \sum_{r=1}^{n}(exp(x_r) - r\,sin(x_r)), \quad X_\circ := (1,1,\ldots,1)^T = (1,1,1,\cdots,1)^T$$

For n = 10,

$$f(X) := \sum_{r=1}^{10}(exp(x_r) - r\,sin(x_r)), \quad X_\circ := (1,1,1,1,1,1,1,1,1,1)^T$$

$$f(X) := e^{x_1} - \sin(x_1) + e^{x_2} - 2\sin(x_2) + e^{x_3} - 3\sin(x_3) + e^{x_4} - 4\sin(x_4) + e^{x_5} - 5\sin(x_5)$$
$$+ e^{x_6} - 6\sin(x_6) + e^{x_7} - 7\sin(x_7) + e^{x_8} - 8\sin(x_8) + e^{x_9} - 9\sin(x_9) + e^{x_{10}} - 10\sin(x_{10})$$

Numerical Results of the Newton's Method

Table 7: Table of Numerical Results of the Newton's Method

$x\|_r^i$	0	1	2	3	4	5	6
x_1	1.000000000	0.388165517	0.092030346	0.00734632904	0.000053315397	2.68181E-9	3.1815E-10
x_2	1.000000000	0.627904126	0.544206632	0.539797326	0.539785161	0.539785161	0.539785161
x_3	1.000000000	0.790684952	0.768821594	0.768578571	0.768578541	0.768578541	0.768578541
x_4	1.000000000	0.908438950	0.904794062	0.904788218	0.904788218	0.904788218	0.904788218
x_5	1.000000000	0.997578519	0.997576220	0.997576220	0.997576220	0.997576220	0.997576220
x_6	1.000000000	1.067403727	1.065759850	1.065758888	1.065758888	1.065758888	1.065758888
x_7	1.000000000	1.123578392	1.118398070	1.118389310	1.118389310	1.118389310	1.118389310
x_8	1.000000000	1.169749014	1.160480341	1.160454403	1.160454403	1.160454403	1.160454403
x_9	1.000000000	1.208369491	1.195012688	1.194962558	1.194962557	1.194962557	1.194962557
x_{10}	1.000000000	1.241151822	1.223929771	1.223851815	1.223851813	1.223851813	1.223851813
f_i	-19.0980858	-21.0928298	-21.1997810	-21.20427858	-21.20430569	-21.20430570	-21.20430570

Numerical Results of the DFP

Table 8: Table of Numerical Results of the DFP

$x\|_r^i$	0	1	2	3	4	5
x_1	1.000000000	0.746169215	0.642572859	0.604459344	0.590305204	0.590305204
x_2	1.000000000	0.809138291	0.747460216	0.722653745	0.716056673	0.716056673
x_3	1.000000000	0.872107367	0.842159107	0.827577464	0.825675280	0.825675280
x_4	1.000000000	0.935076444	0.925892618	0.919157894	0.919436105	0.919436105
x_5	1.000000000	0.998045520	0.997956931	0.997760954	0.997805210	0.997805210
x_6	1.000000000	1.061014596	1.057725522	1.064099263	1.061643109	1.061643109
x_7	1.000000000	1.123983672	1.104652865	1.119140050	1.112073138	1.112073138
x_8	1.000000000	1.186952748	1.138277620	1.164020078	1.150372295	1.150372295
x_9	1.000000000	1.249921825	1.158225290	1.199970964	1.177883864	1.177883864
x_{10}	1.000000000	1.312890901	1.164210319	1.228254545	1.195951206	1.195951206
f_i	-19.09808588	-20.56393954	-20.79075548	-20.88449321	-20.88807938	-20.88807940

Numerical Results of the New Modification

Table 9: Table of Numerical Results of the New Modification

$x\|_r^i$	0	1	2	3	4	5
x_1	1.000000000	0.388165517	0.092030346	0.0022887626	0.0015163975	0.0011934025
x_2	1.000000000	0.627904126	0.544206632	0.533358761	0.537918311	0.538428333
x_3	1.000000000	0.790684952	0.768821594	0.767901999	0.768935634	0.768628531
x_4	1.000000000	0.908438950	0.904794062	0.904764791	0.905097511	0.904818571
x_5	1.000000000	0.997578519	0.997576220	0.997576220	0.997769497	0.997615189

x_6	1.000000000	1.067403727	1.065759850	1.065752857	1.065797329	1.065856073
x_7	1.000000000	1.123578392	1.118398070	1.118324978	1.118310142	1.118592472
x_8	1.000000000	1.169749014	1.160480341	1.160236828	1.160420867	1.160749409
x_9	1.000000000	1.208369491	1.195012688	1.194490718	1.195237044	1.195214851
x_{10}	1.000000000	1.241151822	1.223929771	1.223039556	1.224752045	1.223779621
f_i	-19.09808588	-21.09282982	-21.19978105	-21.20423981	-21.20429343	-21.20430136

Example 4: The Cosine Function

$$\max \ f(X)=\sum_{r=0}^{n-1} cos(-0.5x_{r+1}+x_r^2), \quad X_0=(1,1,\dots,1)^T$$

For n = 10, we have:

$$\sum_{r=0}^{9} cos(-0.5x_{r+1}+x_r^2), \quad X_0=(1,1,1,1,1,1,1,1,1,1)^T$$

$$\left.\begin{array}{l} f(X)=\cos\left(x_1^{\,2}-0.5000000000x_2\right) \\ +\cos\left(x_2^{\,2}-0.5000000000x_3\right)+\cos\left(x_3^{\,2}-0.5000000000x_4\right) \\ +\cos\left(x_4^{\,2}-0.5000000000x_5\right)+\cos\left(x_5^{\,2}-0.5000000000x_6\right) \\ +\cos\left(x_6^{\,2}-0.5000000000x_7\right)+\cos\left(x_7^{\,2}-0.5000000000x_8\right) \\ +\cos\left(x_8^{\,2}-0.5000000000x_9\right)+\cos\left(x_9^{\,2}-0.5000000000x_{10}\right) \end{array}\right\}$$

Numerical Results of the Newton's Method

Table 10: Table of Numerical Results of the Newton's Method

$x\vert_r^i$	0	1	2	3	4	5	6
x_1	1.000000000	0.736618798	0.595907502	0.526708286	0.504061097	0.501265349	0.501220390
x_2	1.000000000	0.751308556	0.609366408	0.533631247	0.506218085	0.502507524	0.502443752
x_3	1.000000000	0.754174789	0.613664471	0.537365926	0.508933804	0.504968545	0.504899430
x_4	1.000000000	0.754798184	0.615328923	0.540431716	0.513484422	0.509904134	0.509846844
x_5	1.000000000	0.755262501	0.617127152	0.545629232	0.522609270	0.519921114	0.519887586
x_6	1.000000000	0.757119769	0.622158175	0.558087481	0.542109205	0.540572623	0.540566189
x_7	1.000000000	0.766578104	0.638787312	0.589974368	0.584935772	0.584402251	0.584423604
x_8	1.000000000	0.815252984	0.692985118	0.672351105	0.683063275	0.683037229	0.683101895
x_9	1.000000000	1.065845301	0.863627067	0.893536925	0.932648704	0.933074499	0.933256390
x_{10}	1.000000000	2.355986180	1.409869685	1.595072975	1.736607746	1.741255683	1.741934913
f_i	7.898243058	8.869906895	8.983196936	8.999062171	8.999988034	8.999999998	8.999999998

Numerical Results of the DFP

Table 11: Table of Numerical Results of the DFP

$x\vert_r^i$	0	1	2	3	4	5
x_1	1.000000000	0.477069419	0.503908374	0.529776936	0.528409960	0.510216963
x_2	1.000000000	0.607802064	0.537794567	0.535290496	0.534223760	0.521609578
x_3	1.000000000	0.607802064	0.574869430	0.553364185	0.553246860	0.533470733
x_4	1.000000000	0.607802064	0.574869430	0.565474883	0.566036384	0.550955128
x_5	1.000000000	0.607802064	0.574869430	0.565474883	0.565557823	0.560413001
x_6	1.000000000	0.607802064	0.574869430	0.565474883	0.565557823	0.534086962

x_7	1.000000000	0.607802064	0.574869430	0.565474883	0.563326215	0.559814383
x_8	1.000000000	0.607802064	0.574869430	0.621335038	0.620692709	0.613298949
x_9	1.000000000	0.607802064	0.740355693	0.747653795	0.747063343	0.729238766
x_{10}	1.000000000	1.130732645	1.082600671	1.075882025	1.075939888	1.076392099
f_i	7.898243058	8.962933909	8.994444520	8.997197641	8.997202432	8.998442574

Numerical Results of the New Modification

Table 12: Table of Numerical Results of the New Modification

$x\|_r^i$	0	1	2	3	4	5
x_1	1.000000000	0.736618798	0.595907502	0.524830456	0.523137456	0.523989718
x_2	1.000000000	0.751308556	0.609366408	0.546242713	0.537582673	0.530612019
x_3	1.000000000	0.754174789	0.613664471	0.551786114	0.544610028	0.536838779
x_4	1.000000000	0.754798184	0.615328923	0.554125645	0.547257867	0.540510825
x_5	1.000000000	0.755262501	0.617127152	0.556738610	0.550275824	0.544415543
x_6	1.000000000	0.757119769	0.622158175	0.563817844	0.558281272	0.554234382
x_7	1.000000000	0.766578104	0.638787312	0.585805886	0.582104183	0.579555174
x_8	1.000000000	0.815252984	0.692985118	0.650013397	0.644528265	0.660227161
x_9	1.000000000	1.065845301	0.863627067	0.808657417	0.845708221	0.841013042
x_{10}	1.000000000	2.355986180	1.409869685	1.434070555	1.421299203	1.421697946
f_i	7.898243058	8.869906895	8.983196936	8.995931011	8.998620223	8.999032696

4. Conclusion

The new method presented in this paper resulted into a multi step scheme which at each iteration process, uses the immediate previous **three points** in the computation of the next point. It could be seen from the examples that the new method produces results that compare favourably with those existing methods. The basic modification presented is a convergent scheme and thus produces a new way of modifying the Classical Newton's method.

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