γ < 2 and 0 < μ < 2; and also E_0 is a non-hyperbolic if r = 1 or γ = 2 or μ = 2.

Proposition: 2

The equilibrium point E_1 is a

(a) Sink if 1< r < 3; $r < \frac{\delta k}{\delta k-\gamma}$ and 0 < μ < 2;

(b) Source if r > 3; $r > \frac{\delta k}{\delta k-\gamma}$ and μ > 2;

(c) Saddle if r > 3; $r < \frac{\delta k}{\delta k-\gamma}$ and μ < 2;

Proof:

In order to prove this result, we determine the eigenvalues of the Jacobian matrix J at E_1. The Jacobian matrix evaluated at the equilibrium point E_1 has the form

$$J(E_1) = \begin{bmatrix} 2-r & \beta k\left(\frac{1}{r}-1\right) & \frac{\sigma\alpha k(r-1)}{r\omega} \\ 0 & 1-\gamma+\delta k\left(1-\frac{1}{r}\right) & 0 \\ 0 & 0 & 1-\mu \end{bmatrix}$$

Hence the eigen values of the matrix $J(E_1)$ are $\lambda_1 = 2 - r$; $\lambda_2 = 1-\gamma+\delta k\left(1-\frac{1}{r}\right)$; $\lambda_3 = 1-\mu$. By using theorem, it is easy to see that, E_1 is a sink if 1< r < 3; $r < \frac{\delta k}{\delta k-\gamma}$ and 0 < μ < 2; E_1 is a source if r > 3; $r > \frac{\delta k}{\delta k-\gamma}$ and μ > 2; and finally E_1 is a saddle if r > 3; $r < \frac{\delta k}{\delta k-\gamma}$ and μ < 2.

Proposition: 3

The equilibrium point E_2 is a

(a) Sink if $\frac{\delta k}{\delta k-\gamma} < r < \frac{\delta k(\beta\mu+\rho)}{\rho(\delta k-\gamma)}$

(b) Source if $\frac{\delta k(\beta\mu+\rho)}{\rho(\delta k-\gamma)} < r < \frac{\delta k}{\delta k-\gamma}$;

(c) Saddle if $r > \frac{\delta k(\beta\mu+\rho)}{\rho(\delta k-\gamma)}$ and $r > \frac{\delta k}{\delta k-\gamma}$.

Proof:

In order to prove this result, we determine the eigenvalues of the Jacobian matrix J at E_2. The Jacobian matrix evaluated at the equilibrium point E_2 has the form

$$J(E_2) = \begin{bmatrix} 1-\frac{r\gamma}{k\delta} & \frac{-\beta\gamma}{\delta} & \frac{\sigma\alpha\gamma}{\omega\delta} \\ \frac{r(\delta k-\gamma)-\delta k}{\beta k} & 1 & \frac{\lambda}{\beta}-\frac{\lambda r(\delta k-\gamma)}{\beta\delta k} \\ 0 & 0 & 1-\mu+\frac{\rho r(\delta k-\gamma)}{\beta\delta k}-\frac{\rho}{\beta} \end{bmatrix}$$

Hence the eigen values of the Jacobian matrix $J(E_2)$ are $\lambda_1 = \left[1-\mu+\frac{\rho r(\delta k-\gamma)}{\beta\delta k}-\frac{\rho}{\beta}\right]$;

$\lambda_{2,3} = \left(1-\frac{r\gamma}{2\delta k}\right) \pm \frac{1}{2}\sqrt{\frac{r^2\gamma^2}{\delta^2 k^2}-\frac{4\gamma\delta^2 r}{\beta\lambda}+\frac{4\gamma^2\delta}{\beta\lambda k}+\frac{4\gamma\delta^2 k}{\beta\lambda}}$. By using theorem, it is easy to see that, E_2 is a sink if $\frac{\delta k}{\delta k-\gamma} < r < \frac{\delta k(\beta\mu+\rho)}{\rho(\delta k-\gamma)}$; E_1 is a source if $\frac{\delta k(\beta\mu+\rho)}{\rho(\delta k-\gamma)} < r < \frac{\delta k}{\delta k-\gamma}$; and finally E_1 is a saddle if $r > \frac{\delta k(\beta\mu+\rho)}{\rho(\delta k-\gamma)}$ and $r > \frac{\delta k}{\delta k-\gamma}$.

4. Numerical Analysis

In this section, we illustrate the analytical findings with the help of numerical simulations performed with MATLAB programming. We present time series plots for system (1) to confirm the theoretical results with hypothetical set of data. They show interesting complex dynamical behaviours.

Case: 1

We shall consider r =0.15, k = 1, β = 0.15, σ =0.5, α = 0.5, ω = 0.1, γ =0.1, δ =0.15, λ =0.04,
μ =0.95 and ρ =0.94. At equilibrium point E_0 = (0, 0, 0), the eigenvalues are λ_1 = 0.15, λ_2 = 0.9, λ_3 = 0.05 so that $|\lambda_{1,2,3}|$ < 1. Hence the trivial equilibrium point is stable (see Figure: 1).

Case: 2

We shall consider r =1.1, k = 1, β = 0.65, σ =0.5, α = 0.5, ω = 0.1, γ =0.1, δ =0.5, λ =1.14, μ =0.5 and ρ =0.4. The equilibrium point E_1 = (0.0909, 0, 0) and the eigenvalues are λ_1 = 0.9, λ_2 = 0.945, λ_3 = 0.5 so that $|\lambda_{1,2,3}|$ < 1. Hence system (1) is stable (see Figure: 2).

Case: 3

We shall consider r =1.5, k = 1, β = 0.65, σ =0.5, α = 0.5, ω = 0.1, γ =0.1, δ =0.5, λ =1.14, μ =0.5 and ρ =0.4. The equilibrium point E_1 = (0.2, 0.3076, 0) and the eigenvalues are λ_1 = 0.62, λ_2 = 0.95, λ_3 = 0.74 so that $|\lambda_{1,2,3}|$ < 1. Thus system (1) is stable (see Figure: 3).

5. Discussion

We developed a plant-larva-insect model with Allee effect on the plant that considers two interaction types, pollination and herbivory. In particular case, interaction types depend on the stage of the insect's life cycle as inspired by the interaction between Manduca sexta and Datura wrightii or the interaction between Manduca sexta and Nicotiana attenuata. The parameters in this analysis can change due to external factors. One of the most important is temperature. Flowering, pollination, herbivory, growth and mortality rates are temperature-dependent and they can increase with warming depending on the thermal impacts on insect and plant metabolisms. However, we get the general picture that warming or cooling can change the balance between costs and benefits impacting the stability of the plant-insect association.

Some pesticides are hormone retarding agents. This means that their release can reduce maturation rates, altering the balance of the interaction towards more herbivory and less pollination and finally endangering pollination service. Another factor that can increase or decrease larvae maturation rates, is the level of nutrients present in the plant's vegetative tissue.An important factor that can affect the balance between mutualism and herbivory is the diet breadth of pollinators. From the lack of alternative pollinators could also lead to increase herbivory and loss of stability.

6. Conclusion

Many insect pollinators are herbivores during their larval phases. If pollination and herbivory targets the same plant (e.g. as between tobacco plants and hawk moths), the overall outcome of the association depends on the balance between costs and benefits for the plant. We proved that system (1) has a unique positive equilibrium point, which is locally asymptotically stable. The main objective of dynamical system is to predict the global behaviour of a system based on the knowledge of its present state. An approach to this problem consists of determining the possible global behaviours of the system and determining which initial conditions lead to these long-term behaviours.

Due to the simplicity of our PLA model with Allee effect, we have carried out a systematic, local and global stability analysis of it. Particularly, the condition for local asymptotic stability in population biology is a very interesting mathematical problem. Usually the biologists believe that a unique, positive, locally asymptotically stable equilibrium point in an ecological system is very important in biological point of view. Therefore, it is very important to find conditions which may guarantee the local asymptotic stability of the unique positive equilibrium point of the given system. In this paper, we proved the necessary and sufficient condition for the local asymptotic stability of the unique positive equilibrium point of system (1). Some numerical examples were provided to support theoretical results.

References

[1] Adam kent, Patrick Doncaster, C., Tim Sluckin, 2003. Consequences for predators of rescue and Allee effects on prey, Ecological Modelling, 162, 233-245.
[2] Allee, W.C., 1931. Animal Aggregations: A study in General Sociology. University of Chicago Press, Chicago.
[3] Courchamp, F., Berec, L., Gascoigne, J., 2008. Allee effects in Ecology and Conservation, Oxford University Press, Oxford.
[4] Elaydi, S.N., 1996. An Introduction to Difference Equations, Springer - Verlag Publishers.
[5] Encinas-Viso, F., Revilla, T.A., Etienne, R.S., 2014. Shifts in pollinator population structure may jeopardize pollination service. Journal of Theoretical Biology, 352: 24-30.
[6] George Maria Selvam, A., Dhineshbabu, R., Sathish, V., 2014. Dynamical behaviour in a discrete three species prey-predator system, International journal of innovative science, engineering and technology, volume 1, issue 8.
[7] Irwin, R.E., 2010. Evolutionary ecology: when pollinators are also herbivores. Current Biology 20: 100-101.
[8] Kang, Y., 2013. Scramble competitions can rescue endangered species to strong Allee effects, Mathematical Biosciences, 241, 75-87.
[9] Kang, Y., Lanchier, N., 2011. Expansion or extinction: Deterministic and stochastic two-patch models with Allee effects, Journal of Mathematical Biology, 67, 1227-1259.
[10] Kang, Y., Yakubu, A., 2011. Weak Allee effects and species coexistence, Nonlinear Analysis: Real world applications, 12, 3329-3345.
[11] Kearns, C., Inouye, D., Waser, N., 1998. Endangered mutualisms: The conservation of plant-pollinator interactions. Annual Review of Ecology and Systematic, 29: 83-112.
[12] Leah Edelstein-Keshet, 1986. Mathematical theory for plant-herbivore systems. J. Math. Biol. 24: 25-58.
[13] Lotka, A.J., 1925. Elements of Mathematical Biology, Williams and Wilkins, Baltimore,USA.
[14] Madhusudanan, V., Vijaya, S., Gunasekaran, M., 2014. Impact of harvesting in three species food web model with two distinct Functional responses, IJIRSET, Vol. 3, Issue 2, 9505-9513.
[15] Murray, J.D., 1998. Mathematical Biology. Springer-Verlag, Berlin, Germany.
[16] Revilla, T.A., Francisco Encinas-Viso, 2015. Dynamical Transitions in Pollination- Herbivory Interaction: A Conflict between Mutualism and Antagonism, PLOS ONE.
[17] Sujatha, K., Gunasekaran, M., 2015. Qualitative and Quantitative Approaches in dynamics of two different prey-predator systems. IJSCE, volume-5, issue-1, pp 76-80.
[18] Tapan Saha, Malay Bandyopadhyay, 2005. Dynamical analysis of a plant-herbivore model: bifurcation and global stability, Journal of Applied Mathematics and computing, volume 19, issue 1-2, pp 327-344.
[19] Vijayalakshmi, S., Sahayajothi, S., Gunasekaran, M., 2015. Stability analysis in dynamics of plant-herbivore system with mating Induced Allee effect, IOSR Journal of Mathematics, volume 11, issue 2 Ver. III , pp 33-37.
[20] Volterra, V., 1931. Le conssen La Theirie Mathematique De LaLeitte Pou Lavie. Gauthier-Villars, Paris.
[21] Wang, J., Shi, J., Wei, J., 2011. Predator-prey system with strong Allee effect in prey, Journal of Mathematical Biology, 62, 291-331.
[22] Yun Kang, Sourav Kumar Sasmal, Amiya Ranjan Bhowmick, Joydev Chattopadhyay, 2014. Dynamics of a predator-prey system with Prey subject to Allee effects and disease, Mathematical Biosciences and Engineering, volume 11, number 4.

Author Profile

Dr. M. Gunasekaran received B.Sc., M.Sc., and M.Phil, in Mathematics from Bharathidasan University, Tiruchirapalli, TamilNadu in 1989, 1991 and 1992 respectively. He completed his Ph.D degree in Mathematics-Zoology from University of Madras, Chennai. He is a Professor of Mathematics with 23 years of teaching experience and now he is working in Sri Subramaniya Swami Government Arts College, Tiruttani. His areas of research are Drug designing, Mathematical models in Prey-Predator system, Plant-Herbivore system and Queuing system. He has published 16 international journals, presented 7 research papers in various National and International Conferences. He has supervised 15 M.Phil, scholars in the area of various Queuing models. He has also given invited talks in National and International conferences in his specialized areas.

ISSN (Online): 2319-7064
Index Copernicus Value (2013): 6.14 | Impact Factor (2013): 4.438

Appendix:

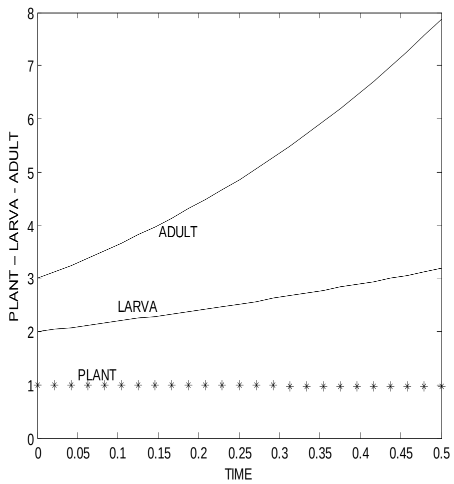

Figure: 1
Time series graph for PLA model with Allee effect
(r =0.15, k = 1, β = 0.15, σ =0.5, α = 0.5, ω = 0.1, γ=0.1, δ =0.15, λ =0.04, μ =0.95 and ρ =0.94.)

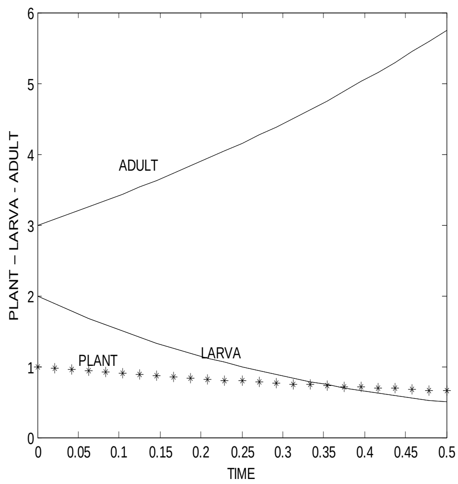

Figure: 2
Time series graph for PLA model with Allee effect
(r =1.1, k = 1, β = 0.65, σ =0.5, α = 0.5, ω = 0.1, γ =0.1, δ =0.5, λ =1.14, μ =0.5 and ρ =0.4.)

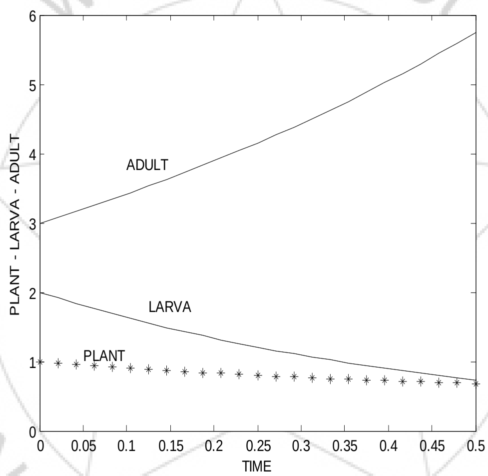

Figure: 3
Time series graph for PLA model with Allee effect
(r =1.5, k = 1, β = 0.65, σ =0.5, α = 0.5, ω = 0.1, γ =0.1, δ =0.5, λ =1.14, μ =0.5 and ρ =0.4.)