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Solving Fully Fuzzy Linear Programming Problem Via Similarity Measure

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Abstract: ***In this paper, we have focused on fully fuzzy linear programming (FFLP) problem in which all the coefficients and decision variables are triangular fuzzy numbers and all the constraints are fuzzy equality or inequality. A new method for solving FFLP problem is proposed using similarity measure and ranking function, the FFLP problem is transformed into crisp nonlinear programming (CNLP) problem. In the end, to illustrate our method, we provide a numerical example.***

Keywords: Similarity measure, Ranking function, optimal solution.

1. Introduction

In last few years, many authors have shifted their focus to fully fuzzy linear programming (FFLP) problem as it is easily correlated with the real world problems. Several methods have been proposed in the literature to solve FFLP problem. Lotfi et al. [1] have solved FFLP problem by approximated the fuzzy parameters to the nearest symmetric triangular fuzzy numbers and find the fuzzy optimal approximate solution. Amit et al. [2] have used the ranking function method to transform fuzzy objective function into crisp one and obtain the fuzzy optimal solution of FFLP problem. Khan et al. [3] have given a modified version of simplex method for FFLP problem. Ezzati et al. [5] have introduced a new lexicographical ordering of triangular fuzzy numbers and transform the FFLP problem into crisp multi-objective linear programming problem and find the exact optimal solution of FFLP problem. Recently, Kaur and Kumar [4] have applied Mehar's method on FFLP problem in which parameters are L-R fuzzy number.

In this paper, we introduce fully fuzzy linear programming problem, in which resources, decision parameters and decision variables are triangular fuzzy numbers. The paper is structured as follows. Some basic definitions and arithmetic operations of triangular fuzzy numbers are presented in Section 2. In Section 3, FFLP problem is discussed and a new algorithm is given for solving FFLP problem. Section 4 presents a numerical illustration and finally the paper concludes in Section 5.

2. Preliminaries

In this section, some basic definitions and arithmetic operations related to triangular fuzzy numbers, which will be used in the rest of the paper, are given.

Definition 2.1[2]. A fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2, \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \le x \le a_3, \\ 0, & otherwise. \end{cases}$$

Let TF(R) denotes the set of all triangular fuzzy numbers.

Definition 2.2 [2]. A triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is said to be non-negative fuzzy number if and only if $a_1 \ge 0$.
Let $TF(R)^+$ denotes the set of all non-negative triangular fuzzy numbers.

Definition 2.3 [2]. A ranking function is a function $\Re : F(R) \to R$, where F(R) is the set of fuzzy numbers defined on the set of real numbers, which maps each fuzzy number into the real line, where a natural order exists. Let $\tilde{A} = (a_1, a_2, a_3)$ be a triangular fuzzy number then $\Re(\tilde{A}) = \dfrac{a_1 + 2a_2 + a_3}{4}$.

Definition 2.4 [6]. Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers then the similarity between two fuzzy numbers is defined as

$$d(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{3}\left[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2\right]}$$

Definition 2.5[2]. The arithmetic operations on two triangular fuzzy numbers $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are given by:

1) $\tilde{A} \oplus \tilde{B} = (a_1, a_2, a_3) \oplus (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
2) $-\tilde{A} = -(a_1, a_2, a_3) = (-a_3, -a_2, -a_1)$
3) $\tilde{A} \Theta \tilde{B} = (a_1, a_2, a_3) \Theta (b_1, b_2, b_3) = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$

4) Let $\tilde{A} = (a_1, a_2, a_3)$ be any triangular fuzzy number and $\tilde{X} = (x_1, x_2, x_3)$ be a non-negative triangular fuzzy numbers then

$$\tilde{A} \otimes \tilde{X} \cong \begin{cases} (a_1x_1,\ a_2x_2,\ a_3x_3), & \mathrm{a}_1 \ge 0, \\ (a_1x_3,\ a_2x_2,\ a_3x_3), & \mathrm{a}_1 < 0,\ \mathrm{a}_3 \ge 0, \\ (a_1x_3,\ a_2x_2,\ a_3x_1), & \mathrm{a}_3 < 0. \end{cases}$$

5) $$\lambda \tilde{A} = \begin{cases} (\lambda a_1,\ \lambda a_2,\ \lambda a_3), & \lambda \ge 0, \\ (\lambda a_3,\ \lambda a_2,\ \lambda a_1), & \lambda < 0. \end{cases}$$

3. Fully Fuzzy Linear Programming Problem

FFLP problem is formulated as follows:

(P1) $$Max\ \mathrm{Z}(\tilde{X}) = \sum_{j=1}^{n} \tilde{c}_j \otimes \tilde{x}_j$$

Subject to . $\sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j \preceq \tilde{b}_i$ $i = 1, 2\ldots, m_1$,

$\sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j \succeq \tilde{b}_i$ $i = m_1+1, m_1+2\ldots, m_2$,

$\sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j \square \tilde{b}_i$ $i = m_2+1, m_1+2\ldots, m$,

$\tilde{x}_j$ is non -negative triangular fuzzy, $j = 1, 2\ldots, n$,.

where $\tilde{X} = [\tilde{x}_j]_{n\times 1}$, $\tilde{c}_j$, $\tilde{a}_{ij}$, $\tilde{b}_i, \tilde{x}_j \in$ TF(R) , j = 1, 2…, n, p = 1, 2…, k_i and i = 1, 2…., m

Now, we are going to introduce a new algorithm to find a optimal solution of FFLP problem. The steps of the proposed algorithm are as follows:

Step 1 Regarding to Definition 2.4, adding and subtracting fuzzy slack and fuzzy surplus variable respectively, then the fuzzy constraints of the FFLP problem (P2), can be converted into the crisp nonlinear constraints and written as:

(P2) $$Max\ \mathrm{Z}(\tilde{X}) = \sum_{j=1}^{n} \tilde{c}_j \otimes \tilde{x}_j$$

Subject to $d_i\left[\sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j \oplus \tilde{s}_i, \tilde{b}_i\right] \le \delta_i$, $i = 1, 2\ldots, m_1$,

$d_i\left[\sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j \Theta \tilde{s}_i, \tilde{b}_i\right] \le \delta_i$, $i = m_1+1, m_1+2\ldots, m_2$,

$d_i\left[\sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j, \tilde{b}_i\right] \le \delta_i$, $i = m_2+1, m_1+2\ldots, m$,

$x_j, (y_j - x_j), (z_j - y_j) \ge 0$, $j = 1, 2\ldots, n$,

$s_{i1}, (s_{i2} - s_{i1}), (s_{i3} - s_{i2}) \ge 0$, $i = 1, 2\ldots, m_2$.

It's up to the DM how much similarity DM wants between L.H.S and R.H.S of the fuzzy constraints and δ_i is the degree of similarity DM wants between L.H.S and R.H.S of the i[th] constraint.

Step 2 Using Definition 2.3, the FFLP problem, obtained in Step 2, can be converted into following crisp nonlinear programming problem:

(P3) $$Max\ R(\mathrm{Z}(\tilde{X})) = R\left(\sum_{j=1}^{n} \tilde{c}_j \otimes \tilde{x}_j\right)$$

Subject to $d_i\left[\sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j \oplus \tilde{s}_i, \tilde{b}_i\right] \le \delta_i$, $i = 1, 2\ldots, m_1$,

$d_i\left[\sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j \Theta \tilde{s}_i, \tilde{b}_i\right] \le \delta_i$, $i = m_1+1, m_1+2\ldots, m_2$,

$d_i\left[\sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j, \tilde{b}_i\right] \le \delta_i$, $i = m_2+1, m_1+2\ldots, m$,

$x_j, (y_j - x_j), (z_j - y_j) \ge 0$, $j = 1, 2\ldots, n$,

$s_{i1}, (s_{i2} - s_{i1}), (s_{i3} - s_{i2}) \ge 0$, $i = 1, 2\ldots, m_2$.

Step 3 Solve the crisp non-linear programming problem by LINGO 14.0, obtained in Step 2, to find the optimal solution of x_j, y_j and z_j.

4. Numerical Example

In this section proposed algorithm is illustrated with the help of numerical example.

Example 4.1

$$MaxZ_1(\tilde{X}) = (5,7,9)\tilde{x}_1 + (4,5,6)\tilde{x}_2 + (1,2,3)\tilde{x}_3$$

Subject to

$$(2,5,7)\tilde{x}_1 + (2,3,4)\tilde{x}_2 + (1,2,3)\tilde{x}_3 \approx (8,16,24)$$
$$(1,2,3)\tilde{x}_1 + (1,2,3)\tilde{x}_2 + (1,3,4)\tilde{x}_3 \preceq (7,17,22) \quad (1)$$
$$(2,3,4)\tilde{x}_1 + (1,2,4)\tilde{x}_2 + (2,3,4)\tilde{x}_3 \succeq (12,18,25)$$

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3$ are non – negative triangular fuzzy numbers.

Now, we take $\delta = (\delta_1, \delta_2, \delta_3)$ = (0.6, 0.6, 0.6) and using step 2, the above problem can be formulated as:

$$MaxZ_1(\tilde{X}) = (5,7,9)\tilde{x}_1 + (4,5,6)\tilde{x}_2 + (1,2,3)\tilde{x}_3$$

subject to

$$d\left((2,5,7)\tilde{x}_1 + (2,3,4)\tilde{x}_2 + (1,2,3)\tilde{x}_3, (8,16,24)\right) \le 0.6$$
$$d\left((1,2,3)\tilde{x}_1 + (1,2,3)\tilde{x}_2 + (1,3,4)\tilde{x}_3 + \tilde{s}_1, (7,17,22)\right) \le 0.6 \quad (2)$$
$$d\left((2,3,4)\tilde{x}_1 + (1,2,4)\tilde{x}_2 + (2,3,4)\tilde{x}_3 - \tilde{s}_2, (12,18,25)\right) \le 0.6$$
$$x_j, y_j - x_j, z_j - y_j \ge 0 \quad j = 1, 2, 3$$
$$s_{i1}, s_{i2} - s_{i1}, s_{i3} - s_{i2} \ge 0 \quad i = 1 \text{and } 2.$$

Regarding to definition 2.4 and step 2, problem (7) is converted into crisp multi-objective nonlinear programming problem as follows:

$$MaxR\left(Z_1\left(\tilde{X}\right)\right)=\frac{1}{4}\left(5x_1+4x_2+x_3+14y_1+10y_2+4y_3+9z_1+6z_2+3z_3\right)$$

subject to

$$\left(2x_1+2x_2+x_3-8\right)^2+\left(5y_1+3y_2+2y_3-16\right)^2+\left(7z_1+4y_2+2y_3-24\right)^2\le 1.08$$

$$\left(x_1+x_2+x_3+s_{11}-7\right)^2+\left(2y_1+2y_2+3y_3+s_{12}-17\right)^2+\left(3z_1+3y_2+4y_3+s_{13}-22\right)^2\le 1.08$$

(3)

$$\left(2x_1+x_2+2x_3-s_{13}-12\right)^2+\left(3y_1+2y_2+3y_3-s_{12}-18\right)^2+\left(4\right.$$

$$x_j, y_j-x_j, z_j-y_j\ge 0 \quad j = 1, 2, 3$$

$$s_{i1}, s_{i2}-s_{i1}, s_{i3}-s_{i2}\ge 0 \quad i = 1 and 2.$$

Now solving the problem (3) by LINGO 14.0, we get the optimal solution and optimal value of objective functions of problem (1) are given in the table 1.

Table 1

$\tilde{x}_1^*$	$\tilde{x}_2^*$	$\tilde{x}_3^*$	$Z\left(\tilde{X}^*\right)$
(0.83,0.83,0.83)	(1.55,1.55,1.55)	(3.98, 3.98, 3.98)	(14.33, 21.52, 28.71)

5. Conclusions

In this paper, a novel approach for solving the FFLP problem is proposed. In this method, we apply similarity measure on the fuzzy constraints to convert it into crisps ones and ranking on objective functions.

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