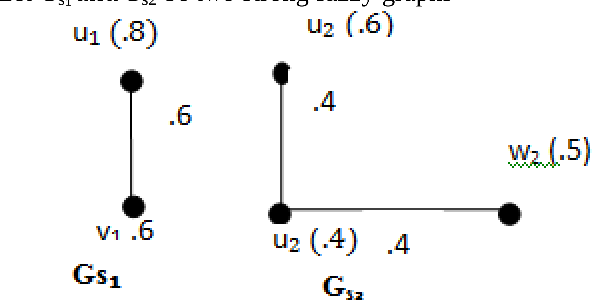
Example 3.3

Let G_{s_1} and G_{s2} be two strong fuzzy graphs



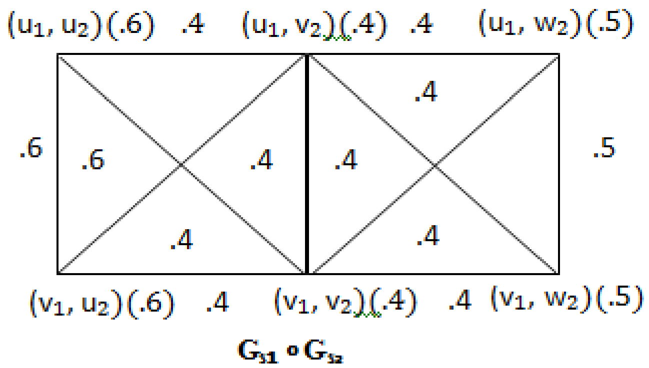


Figure 2

From fig 2 implies $G_{s1} \circ G_{s2}$ is the normal product of two strong fuzzy graphs is also a strong fuzzy graph.

From the above figs
$\mu((u_1,u_2)(v_1,u_2)) = \sigma(u_1,u_2) \wedge \sigma(v_1, u_2) = .6 \wedge .6 = .6$
$\mu((u_1,v_2)(u_1,w_2)) = \sigma(u_1,v_2) \wedge \sigma(u_1,w_2) = .4 \wedge .5 =.4$

Similarly finding the membership value of all the edges, we get a strong fuzzy graph. Hence the normal product $G_{s1} \circ G_{s2}$ of two strong fuzzy graphs G_{s1} and G_{s2} is also a strong fuzzy graph.

Theorem 3.4 If G_{s1} :(σ_1, μ_1) and G_{s2}: (σ_2, μ_2) be two strong fuzzy graphs then $\overline{Gs \phi Gs_2} = \overline{Gs_1} \circ \overline{Gs_2}$

Proof: Let G_{s1} :(σ_1, μ_1) and G_{s2}: (σ_2, μ_2) are strong fuzzy graphs. $\bar{G}$:$(\sigma,\bar{\mu})= \overline{Gs_1 \circ Gs_2}$ $\bar{\mu}=\overline{\mu_1 \circ \mu_2}$, $\bar{G}$: $(V,\bar{E})$ $\overline{Gs_1}$ $(\sigma_1,\overline{\mu_1})= \overline{G_1}$ $(V_1 ,\overline{E_1})$ $\overline{Gs_2}(\sigma_2, \overline{\mu_2})= \overline{G_2}$ $(V_2 ,\overline{E_2})$ $\overline{Gs_1} \circ \overline{Gs_2}$: ($\sigma_1 \circ \sigma_2$, $\overline{\mu_1} \circ \overline{\mu_2}$) Now , the various types of edges say e , joining the vertices of V are the following and it suffices to prove that $\overline{\mu \phi \mu_2} = \overline{\mu_1} \circ \overline{\mu_2}$ in each case. Case(i)

$e=(u,u_2)(u,v_2)$ $u_2v_2 \in E_2$ Then $e \in E$ and G being strong hence $\bar{\mu}$ (e)=0 Also ($\overline{\mu_1} \circ \overline{\mu_2}$)(e)=0 $u_2v_2 \notin E_2 = \bar{\mu}$ (e). Case (ii)

$e=(u,u_2)(u,v_2)$ $u_2 \neq v_2$ and $u_2v_2 \notin E_2$ Then $e \notin E$, so $\mu(e)=0$ Now $\bar{\mu}$ (e) = $\sigma(u, u_2) \wedge \sigma(u, v_2)$
$= [\sigma_1(u) \wedge (\sigma_2(u_2)] \wedge [(\sigma_1(u) \wedge \sigma_2(v_2)]$
$= \sigma_1(u) \wedge [\sigma_2(u_2) \wedge \sigma_2(v_2)]$ $u_2v_2 \notin E_2$ ($\overline{\mu_1} \circ \overline{\mu_2}$)(e) = $\sigma_1(u) \wedge \overline{\mu_2}(u_2v_2)$
$= \sigma_1(u) \wedge [\sigma_2(u_2) \wedge \sigma_2(v_2)]$
$= \overline{\mu_1 \circ \mu_2}$ (e)

Case(iii)
$e = (u_1,w)(v_1,w)$ $u_1v_1 \in E_1$ Here $e \in E$ and $\bar{\mu}$ (e)=0 Also $u_1v_1 \in \overline{E_1}$ Hence ($\overline{\mu_1} \circ \overline{\mu_2}$)(e) = 0

Case(iv)
$e = (u_1,w)(v_1,w)$ $u_1v_1 \notin E_1$ Here $e \notin E$, so $\mu(e)=0$ and
$\bar{\mu}$ (e) = $\sigma(u_1, w_2) \wedge \sigma (v_1,w)$
$= [\sigma_1(u_1) \wedge (\sigma_2(w)] \wedge [(\sigma_1(v_1) \wedge \sigma_2(w)]$
$= [\sigma_1(u_1) \wedge \sigma_1(v_1)] \wedge \sigma_2(w)$
Also $u_1v_1 \notin \overline{E_1}$
($\overline{\mu_1} \circ \overline{\mu_2}$)(e) = $\overline{\mu_1}$ $(u_2v_2) \wedge \sigma_2(w)$
$= [\sigma_1(u_1) \wedge \sigma_1(v_1)] \wedge \sigma_2(w)$
$= \bar{\mu}$ (e)

Case(v)
$e=(u_1,u_2)(v_1,v_2)$ $u_1v_1 \in E_1$, and $u_2v_2 \in E_2$ Here $e \in E$ and $\bar{\mu}$ (e)=0 Also since $u_1v_1 \notin \overline{E_1}$ and $u_2v_2 \notin \overline{E_2}$ We have ($\overline{\mu_1} \circ \overline{\mu_2}$)(e) = 0

Case(vi)
$e=(u_1,u_2)(v_1,v_2)$ $u_1v_1 \in E_1$, and $u_2v_2 \notin \overline{E_2}$ Then $e \notin E$, so $\mu(e)=0$ Also $\bar{\mu}$ (e) = $\overline{\mu_1 \circ \mu_2}$ (e) = 0 $u_1v_1 \notin \overline{E_1}$ and $u_2v_2 \in E_2$ Then ($\overline{\mu_1} \circ \overline{\mu_2}$)(e) = 0

Case(vii)
$e=(u_1,u_2)(v_1,v_2)$ $u_1v_1 \notin E_1$, and $u_2v_2 \in E_2$ Then $e \notin E$, so $\mu(e)=0$ Also $\bar{\mu}$ (e)=0 Then ($\overline{\mu_1} \circ \overline{\mu_2}$)(e) = 0

Case(viii)
$e=(u_1,u_2)(v_1,v_2)$ $u_1v_1 \notin E_1$, and $u_2v_2 \notin \overline{E_2}$ Then $e \notin E$, so $\mu(e)=0$
$\bar{\mu}$ (e) = $\sigma(u_1, u_2) \wedge \sigma (v_1,v_2)$
$= [\sigma_1(u_1) \wedge \sigma_2(u_2)] \wedge [\sigma_1(v_1) \wedge \sigma_2(v_2)]$
$= [\sigma_1(u_1) \wedge \sigma_1(v_1)] \wedge [\sigma_2(u_2) \wedge \sigma_2(v_2)]$
Since $u_1v_1 \notin E_1 \Rightarrow u_1v_1 \in \overline{E_1}$
$u_2v_2 \notin E_2 \Rightarrow u_2v_2 \in \overline{E_2}$
Hence ($\overline{\mu_1} \circ \overline{\mu_2}$)(e) = $\overline{\mu_1}$ $(u_1v_1) \wedge$ $\overline{\mu_2}$ (u_2v_2)
$=[\sigma_1(u_1) \wedge \sigma_1(v_1)] \wedge [\sigma_2(u_2) \wedge \sigma_2(v_2)]$
$= \bar{\mu}$ (e)

Thus from case (i) to (viii) it follows that $\overline{Gs \phi Gs_2} = \overline{Gs_1} \circ \overline{Gs_2}$.

4. Conclusion

In this paper we have proposed, complement of strong fuzzy graphs, normal products of strong fuzzy graphs and the complement properties for tensor product of strong fuzzy graphs. In the fuzzy environment it is reasonable to discuss complement of strong fuzzy graphs and its properties.

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