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(Here the constant a is the slope containing the constants B, G etc. ρ_0 is the density at the centre of the planet)

Considering the above approximation, we solve the differential equation for pressure with suitable boundary conditions obtaining the following:

$$P = \frac{\pi G}{3}[2\rho_0{}^2(A^2 - R^2) - \frac{7}{3}\rho_0 a(A^3 - R^3) + \frac{3}{4}a^2(A^4 - R^4)]$$

Where A is the radius of the planet.

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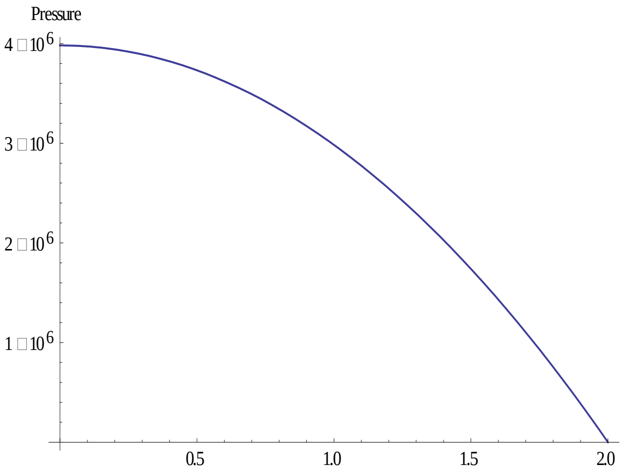
This is the graph showing pressure variation with the planetary radius for some appropriate parameters. It is as it should be, with the pressure being the highest at the centre (where R = 0) and falling to zero at the surface (where R=A).



The graph has been plotted again, on Wolfram Mathematica. The above approximate relation helps us to approximate the variation of volcanic activity with the radius of the planet.

3. Deductions and Results

3.1 The Critical Arguments:

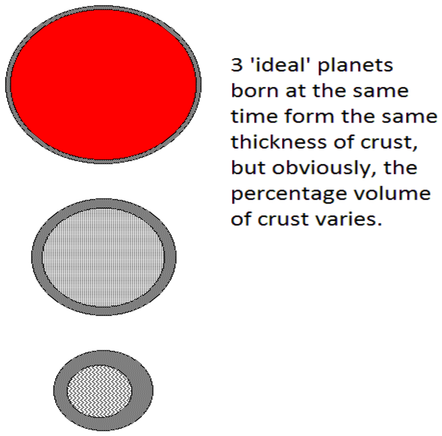
Nearing the end of my analysis, I only have a few critical arguments to present, which shall lead to my conclusion:

- Volcanism in a planet depends on the pressure under the crust.
- For two planets born at the same time, their crusts have cooled to the same thickness, say d, in fixed time T. Since $(d \propto \sqrt{t})$

We want the pressure just below the crust, i.e., the pressure at radial distance:

$$R = R_p - d = A - d$$

A is the radius of the planet in question.



- The density at the centre ρ_0, is greater for larger planets, due to greater depth from the surface (hence greater pressure)

When $R = A - d$:

$A^2 - R^2 = (2A - d)d = X$

$A^3 - R^3 = d(A^2 + d^2 + Ad) = Y$

$A^4 - R^4 = d(2A - d)(2A^2 + d^2 + 2Ad) = Z$

So with all these arguments and using the following approximate expression for pressure:

$$P = \frac{\pi G}{3}[2\rho_0{}^2(A^2 - R^2) - \frac{7}{3}\rho_0 a(A^3 - R^3) + \frac{3}{4}a^2(A^4 - R^4)]$$

When $R = A - d$, we write the pressure just under the crust P_c as:

$$P_c = \frac{\pi G}{3}[2\rho_0{}^2 X - \frac{7}{3}\rho_0 Y + \frac{3}{4}a^2 Z]$$

This quantity is obviously larger for larger planets, since all terms and variables contained within at are positive, and even ρ_0 is larger for larger planets. As A increases, ρ_0 increases and so does the pressure under the crust P_c.

3.2 Conclusion

Our analysis reveals that for planets of same homogeneous chemical composition, which have lived equal time t, the crust thickness is same, i.e., d. So the pressure just below the crust is proportional to A, the radius of the planet. Hence bigger planets shall have greater pressure below the crust and more chance for magma to break through weak spots and form a volcano. Thus a larger planet is more volcanic.

3.3 A note:

There might be a question that if density varies with radius, how come a constant ρ was used when I was talking about crust formation.

We have $\rho = \rho_0 - aR$.

When $R = A - d$,

$\rho = \rho_0 - a(A - d)$

Or $\rho = \rho_0 - aA\left(1 - \frac{d}{A}\right) = \rho_0 - aA \quad (because\ d \ll A)$

a and A being constants, density at around the surface is almost constant, and carrying the analysis forward, the same can be said of the temperature, so that we can assume a constant ΔT. The key point to note is that the thickness of the crust is very small as compared to the radius of the entire planet.

4. Achievements and Limitations

This paper deals with the comparative study of the degree of volcanism between two different planets formed at the same time with the assumption that the planet is 'ideal', i.e., has the properties as discussed above. However real planets are far from this, as there are different kinds of magmatic flows within the molten interior and so heat loss occurs by convection as well, which has been completely ignored in this paper for the sake of simplicity.

However, even a more rigorous study of the interior reveals a very similar pressure gradient. The density profile we obtained was a monotonically decreasing function of radius. Exact studies in the past have shown flat regions in the curve of the density profile, but on the whole, the density indeed decreases with increase in radius. Herein lies the success of our model – a lot was inferred from it, with hardly any complicated mathematics.

5. Acknowledgements

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References

[1] Notes on Geology and the Adam-Williamson Equation from Cornell University, sourced from the website http://www.geo.cornell.edu/geology/classes/geol388/pdf_files/density2.pdf

[2] www.wikipedia.org for additional data on the Adam-Williamson equation

Author Profile

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