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Soliton Solutions of Double Sine–Gordon Equation

Anil Kumar Ram[1], Gautam Johri[2], Nand Kishore Pandey[3]

Department of Postgraduate Studies and Research in Physics and Electronics, Rani Durgavati University Jabalpur-482001, India

Abstract: ***Double Sine –Gordon Equation is a type of a nonlinear equation which appears in the problem of the nonlinear interacting potential between the nearest neighbours of the the quartic type. The soliton solutions are obtained for the different range of the parameter values.***

Keywords: Double Sine –Gordon Equation, critical points, linear stability analysis, saddle point, solitons.

1. Introduction

Nonlinear theories which admit of soliton – like solution have found increasing application in physics in recent years .In particular the Double Sine –Gordon Equation appears in the problem of diatomic lattice with quartic nonlinear interatomic potential. There are some questions about the validity of the solutions with regard to the parameters of Double Sine –Gordon Equation [1-7] .

2. Double Sine –Gordon Equation

The Double Sine –Gordon Equation is given by

$$\frac{d^2\theta}{dx^2} - 4\Delta\sigma\sin\theta + 2\sigma^2\sin 2\theta = 0 \quad (1)$$

The soliton- like solution of the equation (1) corresponds to the path of the separatrix connecting the unstable saddle points[8].

3. Solution of Equation

The equivalent autonomous system for equation(1) are

$$\frac{d\theta}{dx} = \eta \qquad -(2)$$

$$\frac{d\eta}{dx} = 4\sigma\Delta\sin\theta - 2\sigma^2\sin 2\theta \qquad -(3)$$

For the above system, thecritical points are $(0,\pm n\pi)$ (where n = 0,1,2,3,..) and (0, $\pm cos^{-1}\frac{\Delta}{\sigma}$) . By the linear stability analysis, it can be shown that $(0,\pm n\pi)$ (n = 0,1,2,3,.) are centres and (0, $\pm cos^{-1}\frac{\Delta}{\sigma}$) and (0, $2n\pi \pm cos^{-1}\frac{\Delta}{\sigma}$)are the saddle points[9] . The soliton like solutions lie on the separatix which connects the saddle points i.e.(0, $cos^{-1}\frac{\Delta}{\sigma}$) and (0, $2n\pi \pm cos^{-1}\frac{\Delta}{\sigma}$)

From equation (2) and (3), We obtain

$$\int \eta d\eta = \int (4\sigma\Delta\sin\theta - 2\sigma^2\sin 2\theta)d\theta$$

$$\frac{1}{2}\eta^2 + C = -4\sigma\Delta cos\theta + \sigma^2 cos2\theta\text{-} \qquad (4)$$

C can be obtained from the condition that the separatrix should asymptotically approach the saddle points for x→ $\pm\infty$

$$2c = -\Delta^2 - \frac{\sigma^2}{2} \qquad (5)$$

Substituting the value from equation (5) in equation (4), we obtain

$$\eta = \pm 2(\Delta - \sigma cos\theta) \qquad (6)$$

Integrating equations(6), We obtain

$$X = \pm\frac{1}{2}\int\frac{d\theta}{(\Delta - \sigma\cos\theta)} + X_0 \qquad (7)$$

When $\Delta^2 < \sigma^2$,The integration of the equation (7) can be written as [10]

$$X - X_0 = \pm\frac{1}{2}\frac{1}{\sqrt{\sigma^2-\Delta^2}}\ln\frac{\sqrt{\sigma^2-\Delta^2}\tan\frac{\theta}{2} - (\sigma-\Delta)}{\sqrt{\sigma^2-\Delta^2}\tan\frac{\theta}{2} + (\sigma-\Delta)} \qquad -(8)$$

After simplification equations (8) can be written as

$$\theta = \pm 2\tan^{-1}\left(\sqrt{\frac{\sigma-\Delta}{\sigma+\Delta}}\coth\sqrt{\sigma^2-\Delta^2}\,(x-x_0)\right) \qquad -(9)$$

From equations (9),we obtain two solutions for phase

$$\theta_I^{\pm} = m\pi \pm \tan^{-1}\left(\sqrt{\frac{\sigma-\Delta}{\sigma+\Delta}}\coth\sqrt{\sigma^2-\Delta^2}\,(x-x_0)\right) \qquad -(10)$$

Where ,m= 0,1,2,3……. , (+) and (-) correspond to the kink and antkink solution respectively.

When $\Delta^2 > \sigma^2$,The integration of the equation (7) can be written as [10]

$$X - X_0 = \pm\left(\frac{1}{2}\right)\frac{2}{\sqrt{\Delta^2-\sigma^2}}tan^{-1}\left(\frac{(\Delta+\sigma)tan\frac{\theta}{2}}{\sqrt{\Delta^2-\sigma^2}}\right) \qquad -(11)$$

After simplification equation (11) can be written as

$$\theta = \pm 2\tan^{-1}\left(\sqrt{\frac{\sigma-\Delta}{\sigma+\Delta}}\tanh\sqrt{\sigma^2-\Delta^2}\,(x-x_0)\right) \qquad -(12)$$

From equation (12), we obtain two solutions for phase

$$\theta^{\pm} = m\pi \pm 2\tan^{-1}\left(\sqrt{\frac{\sigma-\Delta}{\sigma+\Delta}}\tanh\sqrt{\sigma^2-\Delta^2}\,(x-x_0)\right) \qquad -(13)$$

Where ,m= 0,1,2,3……. , (+) and (-) correspond to the kink and antkink solution respectively.

4. Discussion and Conclusion

In the present work, we obtain two solution for the Double Sine –Gordon equation (1) for the range of parameter values $\Delta^2 < \sigma^2\ and\ \Delta^2 > \sigma^2$. For the problem of the diatomic lattice with quartic nearest neighbours interatomic potential[6], the solutions (10) corresponds to the soliton solutions in the gap region of the linear spectrum whereas the solutions (13)corresponds to the soliton solutions for the frequencies below and above the gap. However in literature [7,11]. The solution (13) is quoted as the gap soliton solutions . The two solutions to reduce the same solutions for the frequency corresponding to $\Delta = 0$ (at which the maximum linear damping occurs)

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