$$\frac{(y+z)^2}{Z^2}R_{(n-2)(n-2)}=\frac{(y+z)^2}{Z^2}R_{(n-1)(n-1)}=\frac{(y+z)^2}{2}R_{nn}=\frac{(y+z)^2}{Z^2}R_{(n-2)(n-1)}$$
$$=\frac{-(y+z)^2}{Z\sqrt{2}}R_{(n-2)n}=\frac{-(y+z)^2}{Z\sqrt{2}}R_{(n-1)n} \quad (8.10)$$

Here we observed that the equation (8.9) is incompatible with the equation (8.10) which is obtained in the case of purely gravitational field. Hence we have

***Conclusion*** $[t\sqrt{2}/(y+z)]$-type plane *wave solutions of Einstein's field equation in general theory of relativity doesn't exist in the case where massive scalar field is coupled with the gravitational field.*

***Remark*** If $m^2=0$ then the equation (8.9) is compatible to (8.10) and we have a result as in the case where the zero mass scalar field is coupled with the gravitational field.

## 9. Massive scalar field coupled with gravitational & electromagnetic field in $V_n$

In this case the energy momentum tensor is given by

$$T_{ij}=\frac{1}{4\pi}[V_iV_j-\frac{1}{2}g_{ij}(V_kV^k-m^2V^2)]+E_{ij} \quad (9.1)$$

where $E_{ij}$ denotes electromagnetic energy momentum tensor.

But as in the previous case for massive scalar field

$$V_kV^k=0 \text{ and } T=T_i^i=g^{ij}T_{ij}=\frac{1}{4\pi}[\frac{n}{2}m^2V^2]. \quad (9.2)$$

Therefore equation (9.1) becomes

$$T_{ij}=\frac{1}{4\pi}[V_iV_j+\frac{1}{2}g_{ij}m^2V^2]+E_{ij}. \quad (9.3)$$

Hence Einstein's field equation $(1.11)$ becomes

$$R_{ij}=-2[V_iV_j+(\frac{2-n}{4})g_{ij}m^2V^2+4\pi E_{ij}]. \quad (9.4)$$

And from equation (9.4) we have

$$R_{(n-2)(n-2)}=-2[\frac{\bar{V}^2Z^2}{(y+z)^2}+(\frac{n-2}{4})Z^2Bm^2V^2+8\pi E_{(n-2)(n-2)}],$$
$$R_{(n-1)(n-1)}=-2[\frac{\bar{V}^2Z^2}{(y+z)^2}+(\frac{n-2}{4})Z^2Bm^2V^2+8\pi E_{(n-1)(n-1)}]$$
$$R_{nn}=-2[\frac{2\bar{V}^2}{(y+z)^2}-(\frac{n-2}{4})Bm^2V^2+8\pi E_{nn}],$$
$$R_{(n-2)(n-1)}=-2[\frac{\bar{V}^2Z^2}{(y+z)^2}+8\pi E_{(n-2)(n-1)}],$$
$$R_{(n-2)n}=-2[\frac{-\bar{V}^2Z\sqrt{2}}{(y+z)^2}+8\pi E_{(n-2)n}],$$
$$R_{(n-1)n}=-2[-\frac{\bar{V}^2Z\sqrt{2}}{(y+z)^2}+8\pi E_{(n-1)n}]. \quad (9.5)$$

But line element (1.8) gives the relation of non-vanishing components of Ricci tensor as

$$\frac{(y+z)^2}{Z^2}R_{(n-2)(n-2)}=\frac{(y+z)^2}{Z^2}R_{(n-1)(n-1)}=\frac{(y+z)^2}{2}R_{nn}=\frac{(y+z)^2}{Z^2}R_{(n-2)(n-1)}$$
$$=\frac{-(y+z)^2}{Z\sqrt{2}}R_{(n-2)n}=\frac{-(y+z)^2}{Z\sqrt{2}}R_{(n-1)n}. \quad (9.6)$$

Here we observed that the equation (9.5) is incompatible with the equation (9.6) which is obtained in the case of purely gravitational field. Hence we have

***Conclusion*** $[t\sqrt{2}/(y+z)]$-type plane *wave solutions of Einstein's field equation doesn't exist in the case where massive scalar field is coupled with gravitational and electromagnetic field.*

***Remark*** If $m^2=0$ then the equation (9.5) is compatible to (9.6) and we have a result as in the case where the zero mass scalar field is coupled with the gravitational and electromagnetic field.

## References

[1] B. G. Ambatkar (2015) : $[\frac{1}{\sqrt{2}}(y+z)-t]$-type and $[t\sqrt{2}/(y+z)]$-type four and J K Jumale dimensional plane wave solutions of Einstein's field equations in general relativity. *International Journal of Engineering and Science Vol.*5,*Issue* 2, *Feb.* 2015, *pp*. 30-38.

[2] B. G. Ambatkar (2015) : $[\frac{1}{\sqrt{2}}(y+z)-t]$-type and $[t\sqrt{2}/(y+z)]$-type plane wave and J K Jumale solutions of Einstein's field equations in $V_6$. *Online International Interdisciplinary Research. Vol.*V.*Issue* III, *May- June.2015*

[3] Thengane K D (2003) : Solutions of field equations $R_{ij}=0$ for $[\frac{1}{\sqrt{2}}(y+z)-t]$-type and

$[t\sqrt{2}/(y+z)]$-type plane waves in N-dimensional space- time (I).

[4] H Takeno(1961) : The Mathematical theory of plane gravitational wave in general relativity". *Scientific report of the Research Institute to theoretical physics. Hiroshima, University, Takchara, Hiroshima-ken, Japan-*1961