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A Numerical Solution of First Order Simultaneous Fuzzy Differential Equation by Fifth Order Runge-Kutta Merson Method

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Abstract: ***In this paper we introduce a new technique for getting the solution of "First order simultaneous Fuzzy differential equation by Fifth Order Runge-Kutta Merson method" based on Seikala derivative of fuzzy process [11]. A numerical method based on the Runge-Kutta Merson method of order five is discussed in detail, followed by a complete error analysis.***

Keywords: Fuzzy Cauchy Problem – FDE - Fifth Order Runge-Kutta Merson method

1. I

2. ntroduction

The topics of fuzzy differential equations which attracted a growing interest for some time, in particular, in relation to the fuzzy control, have been rapidly developed recent years. The fuzzy derivative was first introduced by S.L. Chang, L.A. Zadeh in [4], followed up by D. Dubois, H. Prade in [5], who defined and used the extension principle. Other methods have been discussed by M.L. Puri, D.A. Ralescu in [10] and R. Goetschel, W. Voxman in [6]. Fuzzy differential equations and initial value problems were regularly treated by O. Kaleva in [7] and [8], S. Seikkala in [11]. A numerical method for solving fuzzy differential equations has been introduced by M. Ma, M. Friedman, A. Kandel in [9] via the standard Euler method.

The structure of this paper is organized as follows. In section 2, some basic results on fuzzy numbers and definition of fuzzy derivative which have been discussed by S. Seikkala in [11] are given. In section 3 we define the problem, a fuzzy Cauchy problem whose numerical solution is the main interest of this paper. We find a numerical solution of the first order simultaneous fuzzy differential equation by fifth order Runge-Kutta Merson method in section 4. Sample problem is illustrated and the complete error analysis is also included in section 5.

3. Preliminaries

Consider the first order simultaneous differential equation

$\frac{dy}{dt} = f(t,y,z)$ & $\frac{dz}{dt} = g(t,y,z)$, $t_0 \le t \le b$ with initial conditions $y(t_0) = y_0$, $z(t_0) = z_0$ . . . (2.1)

Since, $y(t_{n+1}) = y(t_n) + w_1k_1 + w_2k_2 + w_3k_3 + w_4k_4$

where, $k_1 = hf(t_n, y(t_n))$

$$\left.\begin{aligned} k_2 &= hf(t_n+c_2h,\ y(t_n)+a_{21}k_1) \\ k_3 &= hf(t_n+c_3h,\ y(t_n)+a_{31}k_1+a_{32}k_2) \\ k_4 &= hf(t_n+c_4h,\ y(t_n)+a_{41}k_1+a_{42}k_2+a_{43}k_3) \\ k_5 &= hf(t_n+c_5h,\ y(t_n)+a_{51}k_1+a_{52}k_2+a_{53}k_3+a_{54}k_4) \end{aligned}\right\} \ldots (2.2)$$

Utilizing the Taylor's series expansion techniques, Runge-Kutta Merson method of fifth order is given by,

$$y_{n+1} = y_n + \frac{k_1 + 4k_4 + k_5}{6}$$

$$z_{n+1} = z_n + \frac{l_1 + 4l_4 + l_5}{6}$$

where $k_1 = hf(t_n, y(t_n), z(t_n))$

$$k_2 = hf(t_n+\frac{h}{3},\ y(t_n)+\frac{1}{3}k_1,\ z(t_n)+\frac{1}{3}l_1)$$

$$k_3=hf(t_n+\frac{h}{3},y(t_n)+\frac{1}{6}(k_1+k_2),z(t_n)+\frac{1}{6}(l_1+l_2)),$$

$$k_4=hf(t_n+\frac{h}{2},y(t_n)+\frac{1}{8}(k_1+3k_3),z(t_n)+\frac{1}{8}(l_1+3l_3)),$$

$$k_5 = hf(t_n+h,\ y(t_n)+\frac{1}{2}(k_1 - 3k_3 + 4k_4),\ z(t_n)+\frac{1}{2}(l_1 - 3l_3 + 4l_4))$$

and $l_1 = hg(t_n, y(t_n), z(t_n))$

$$l_2 = hg(t_n+\frac{h}{3},\ y(t_n)+\frac{1}{3}k_1,\ z(t_n)+\frac{1}{3}l_1)$$

$$l_3 = hg(t_n+\frac{h}{3},y(t_n)+\frac{1}{6}(k_1+k_2),z(t_n)+\frac{1}{6}(l_1+l_2))$$

$$l_4=hg(t_n+\frac{h}{2},y(t_n)+\frac{1}{8}(k_1+3k_3),z(t_n)+\frac{1}{8}(l_1+3l_3))$$

$$l_5 = hg(t_n+h,\ y(t_n)+\frac{1}{2}(k_1 - 3k_3 + 4k_4),\ z(t_n)+\frac{1}{2}(l_1 - 3l_3 + 4l_4)) \ldots (2.3)$$

Definition – 2.1

A fuzzy number u is a fuzzy subset of R ie) $u: R \to [0, 1]$ satisfying the following conditions

1) u is normal, ie $\exists\ x_0 \in R \ni u(x_0) = 1$
2) u is a convex fuzzy set ie) $u(tx+(1-t)y) \ge \min\{u(x),u(y)\}$, $\forall t \in [0, 1]$ & $x, y \in R$

3) u is upper semi continuous on R
4) $\{x \in R, u(x) > 0\}$ is compact

The set E is the family of fuzzy numbers and arbitrary fuzzy number is represented by an ordered pair of functions $(\underline{u}(r), \overline{u}(r))$, $0 \le r \le 1$, satisfying the following requirements

1. $\underline{u}(r)$ is a bounded left continuous non-decreasing function over [0, 1] w.r.to any 'r'.
2. $\overline{u}(r)$ is a bounded right continuous non-increasing function over [0, 1] w.r.to any 'r'.
3. $\underline{u}(r) \le \overline{u}(r)$, $0 \le r \le 1$, r-level cut is $[u]_r = \{x/u(x) \ge r\}$, $0 \le r \le 1$ as a closed & bounded interval denoted by $[u]_r = [\underline{u}(r), \overline{u}(r)]$ and $[u]_0 = \{x|u(x) > 0\}$ is compact.

Definition – 2.2
A triangular fuzzy number u is a fuzzy set in E that is characterized by an ordered triple $(u_l, u_c, u_r) \in R^3$ with $u_l < u_c < u_r$ such that $[u]_0 = [u_l : u_r]$ and $[u]_1 = [u_c]$. The membership function of the triangular fuzzy number u is given by

$$u(x) = \begin{cases} \dfrac{x - u_l}{u_c - u_l}, & u_l \le x \le u_c \\ 1 & x = u_c \\ \dfrac{u_r - x}{u_r - u_c}, & u_c \le x \le u_r \end{cases} \quad ..(2.4)$$

and we will have i) $u > 0$ if $u_l > 0$ ii) $u \ge 0$ if $u_l \ge 0$ iii) $u < 0$ if $u_c < 0$ iv) $u \le 0$ if $u_c \le 0$
Let I be a real interval. A mapping $y : I \to E$ is called a fuzzy process and its α - level set is denoted by $[y(t)]_\alpha = [\underline{y}(t, y, z), \overline{y}(t, y, z)]$, $t \in I$, $0 < \alpha \le I$. Seikkala derivative y(t) of a fuzzy process is defined by $[y^1(t)]_\alpha = [\underline{y}^1(t, y, z), \overline{y}^{-1}(t, y, z)]$, $t \in I$, $0 < \alpha \le I$ provided an equation defines fuzzy number as in [11]. Similarly, let I be a real interval. A mapping $z: I \to E$ is called a fuzzy process and its α - level set is denoted by $[z(t)]_\alpha = [\underline{z}(t, y, z), \overline{z}(t, y, z)]$, $t \in I$, $0 < \alpha \le I$. Seikkala derivative z(t) of a fuzzy process is defined by, $[z^1(t)]_\alpha = [\underline{z}^1(t, y, z), \overline{z}^{-1}(t, y, z)]$, $t \in I$, $0 < \alpha \le I$ provided the equation defines fuzzy number as in [11]. For $u, v \in E$ and $\lambda \in R$, $u + v$ and the product λu can be defined by (i) $[u + v]_\alpha = [u]_\alpha + [v]_\alpha$ (ii) $[\lambda u]_\alpha = \lambda [u]_\alpha$ where $\alpha \in [0, 1]$ and (iii) $[u]_\alpha + [v]_\alpha$ means the addition of two intervals of R and $[\lambda u]_\alpha$ means the product between a scalar and an interval of R.

Arithmetic operation of arbitrary fuzzy numbers $u = (\underline{u}(r), \overline{u}(r))$, $v = (\underline{v}(r), \overline{v}(r))$ and $\lambda \in R$ can be defined as

$$\left.\begin{array}{l} \text{i)}\ u = v \text{ if } \underline{u}(r) = \underline{v}(r) \text{ and } \overline{u}(r) = \overline{v}(r) \\ \text{ii)}\ u + v = \left(\underline{u}(r) + \underline{v}(r), \overline{u}(r) + \overline{v}(r)\right) \\ \text{iii)}\ u - v = \left(\underline{u}(r) - \overline{v}(r), \overline{u}(r) - \underline{v}(r)\right) \\ \text{iv)}\ \lambda u = \left(\lambda \underline{u}(r), \lambda \overline{u}(r)\right) \text{ if } \lambda \ge 0 \end{array}\right\} \quad ...(2.5)$$

$$= \left(\lambda \overline{u}(r), \lambda \underline{u}(r)\right) \text{ if } \lambda < 0$$

4. Fuzzy Cauchy Problem

Consider the first order simultaneous differential equation $\frac{dy}{dt} = f(t, y, z)$ & $\frac{dz}{dt} = g(t, y, z)$, $t_0 \le t \le b$ with initial conditions $y(t_0) = y_0$, $z(t_0) = z_0$
Let the function f be a continuous mapping from $R \times R \to R$ and $y_0 \in E$ with r-level sets $[y_0]_r = [\underline{y}(0 : r), \overline{y}(0 : r)]$, $r \in [0, 1]$ and the function g is a continuous mapping from $R \times R \to R$ and $z_0 \in E$ with r-level sets $[z_0]_r = [\underline{z}(0 : r), \overline{z}(0 : r)]$, $r \in [0, 1]$. The extension principle of Zadeh [4] leads to the definition of f(t, y, z), y = y(t) and z = z(t) are the fuzzy numbers. $[f(t, y, z)]_r = [\underline{f}(t, y, z : r), \overline{f}(t, y, z : r)]$, $r \in [0, 1]$, It follows that

$$\underline{f}(t, y, z : r) = \min\left\{f(t, u, v) \,\middle|\, u \in [\underline{y}(r), \overline{y}(r)], v \in [\underline{z}(r), \overline{z}(r)]\right\}$$

and $$\overline{f}(t, y, z : r) = \max\left\{f(t, u, v) \,\middle|\, u \in [\underline{y}(r), \overline{y}(r)], v \in [\underline{z}(r), \overline{z}(r)]\right\}$$

5. Fifth Order Runge–Kutta Merson Method

Let the exact solution of the given equation $[Y(t)]_r = [\underline{Y}(t : r), \overline{Y}(t : r)]$ is approximated by some solution $[y(t)]_r = [\underline{y}(t : r), \overline{y}(t : r)]$, $[Z(t)]_r = [\underline{Z}(t : r), \overline{Z}(t : r)]$ is approximated by some solution $[z(t)]_r = [\underline{z}(t : r), \overline{z}(t : r)]$ also we define

$$\underline{y}(t_{n+1} : r) - \underline{y}(t_n : r) = \sum_{i=1}^{4} w_i \underline{k}_i$$

$$\overline{y}(t_{n+1} : r) - \overline{y}(t_n : r) = \sum_{i=1}^{4} w_i \overline{k}_i$$ where w_i's are constant

$[k_i(t, y(t, r))]_r = [\underline{k}_i(t, y(t, r)), \overline{k}_i(t, y(t, r))]$, i = 1, 2, 3, 4 and 5 also

$$\underline{z}(t_{n+1}:r)-\underline{z}(t_n:r)=\sum_{i=1}^{4} w_i \underline{l}_i$$

$$\overline{z}(t_{n+1}:r)-\overline{z}(t_n:r)=\sum_{i=1}^{4} w_i \overline{l}_i$$ where l_i's are constant

$[l_i(t,z(t,r))]_r=[\underline{l}_i(t,z(t,r)),\overline{l}_i(t,z(t,r))]$, i = 1, 2, 3, 4 and 5

$$\underline{k}_1(t,y(t:r))=hf(t_n,\underline{y}(t_n:r))$$

$$\underline{l}_1(t,z(t:r))=hf(t_n,\underline{z}(t_n:r))$$

$$\overline{k}_1(t,y(t:r))=hf(t_n,\overline{y}(t_n:r))$$

$$\overline{l}_1(t,z(t:r))=hf(t_n,\overline{z}(t_n:r))$$

$$\underline{k}_2(t,y(t:r))=hf\left(t_n+\frac{h}{3},\underline{y}(t_n:r)+\frac{1}{3}\underline{k}_1\right)$$

$$\underline{l}_2(t,z(t:r))=hf\left(t_n+\frac{h}{3},\underline{z}(t_n:r)+\frac{1}{3}\underline{l}_1\right)$$

$$\overline{k}_2(t,y(t:r))=hf\left(t_n+\frac{h}{3},\overline{y}(t_n:r)+\frac{1}{3}\overline{k}_2\right)$$

$$\overline{l}_2(t,z(t:r))=hf\left(t_n+\frac{h}{3},\overline{z}(t_n:r)+\frac{1}{3}\overline{l}_2\right)$$

$$\underline{k}_3(t,y(t:r))=hf\left(t_n+\frac{h}{3},\underline{y}(t_n:r)+\frac{1}{6}(\underline{k}_1+\underline{k}_2)\right)$$

$$\underline{l}_3(t,z(t:r))=hf\left(t_n+\frac{h}{3},\underline{z}(t_n:r)+\frac{1}{6}(\underline{l}_1+\underline{l}_2)\right)$$

$$\overline{k}_3(t,y(t:r))=hf\left(t_n+\frac{h}{3},\overline{y}(t_n:r)+\frac{1}{6}(\overline{k}_1+\overline{k}_2)\right)$$

$$\overline{l}_3(t,z(t:r))=hf\left(t_n+\frac{h}{3},\overline{z}(t_n:r)+\frac{1}{6}(\overline{l}_1+\overline{l}_2)\right)$$

$$\underline{k}_4(t,y(t:r))=hf\left(t_n+\frac{h}{2},\underline{y}(t_n:r)+\frac{1}{8}(\underline{k}_1+3\underline{k}_3)\right)$$

$$\underline{l}_4(t,z(t:r))=hf\left(t_n+\frac{h}{2},\underline{z}(t_n:r)+\frac{1}{8}(\underline{l}_1+3\underline{l}_3)\right)$$

$$\overline{k}_4(t,y(t:r))=hf\left(t_n+\frac{h}{2},\overline{y}(t_n:r)+\frac{1}{8}(\overline{k}_1+3\overline{k}_3)\right)$$

$$\overline{l}_4(t,z(t:r))=hf\left(t_n+\frac{h}{2},\overline{z}(t_n:r)+\frac{1}{8}(\overline{l}_1+3\overline{l}_3)\right)$$

$$\underline{k}_5(t,y(t:r))=hf\left(t_n+h,\underline{y}(t_n:r)+\frac{1}{2}(\underline{k}_1-3\underline{k}_3+4\underline{k}_4)\right)$$

$$\underline{l}_5(t,z(t:r))=hf\left(t_n+h,\underline{z}(t_n:r)+\frac{1}{2}(\underline{l}_1-3\underline{l}_3+4\underline{l}_4)\right)$$

$$\overline{k}_5(t,y(t:r))=hf\left(t_n+h,\overline{y}(t_n:r)+\frac{1}{2}(\overline{k}_1-3\overline{k}_3+4\overline{k}_4)\right)$$

$$\overline{l}_5(t,z(t:r))=hf\left(t_n+h,\overline{z}(t_n:r)+\frac{1}{2}(\overline{l}_1-3\overline{l}_3+4\overline{l}_4)\right)$$

$$F(t,y(t:r))=\underline{k}_1(t,y(t:r))+4\underline{k}_4(t,y(t:r))+\underline{k}_5(t,y(t:r))$$

$$G(t,y(t:r))=\overline{k}_1(t,y(t:r))+4\overline{k}_4(t,y(t:r))+\overline{k}_5(t,y(t:r))$$

and

$$P(t,z(t:r))=\underline{l}_1(t,z(t:r))+4\underline{l}_4(t,z(t:r))+\underline{l}_5(t,z(t:r))$$

$$Q(t,z(t:r))=\overline{l}_1(t,z(t:r))+4\overline{l}_4(t,z(t:r))+\overline{l}_5(t,z(t:r))$$

The exact and approximate solution at t_n, $0 \le n \le N$ are denoted by

$[Y(t_n)]_r = [\underline{Y}(t_n:r),\overline{Y}(t_n:r)]$, $[y(t_n)]_r = [\underline{y}(t_n:r),\overline{y}(t_n:r)]$ and

$[Z(t_n)]_r = [\underline{Z}(t_n:r),\overline{Z}(t_n:r)]$, $[z(t_n)]_r = [\underline{z}(t_n:r),\overline{z}(t_n:r)]$ respectively.

The solution calculated by the grid points at $a = t_0 \le t_1 \le t_2 \le \ldots \le t_N = b$, $h = \frac{b-a}{N} = t_{n+1} - t_n$. Therefore we have

$$\underline{Y}(t_{n+1}:r)=\underline{Y}(t_n:r)+\frac{1}{6}F[t_n,\underline{Y}(t_n:r)]$$

$$\overline{Y}(t_{n+1}:r)=\overline{Y}(t_n:r)+\frac{1}{6}G[t_n,\overline{Y}(t_n:r)]$$

$$\underline{Z}(t_{n+1}:r)=\underline{Z}(t_n:r)+\frac{1}{6}P[t_n,\underline{Z}(t_n:r)]$$

$$\overline{Z}(t_{n+1}:r)=\overline{Z}(t_n:r)+\frac{1}{6}Q[t_n,\overline{Z}(t_n:r)]$$

$$\underline{y}(t_{n+1}:r)=\underline{y}(t_n:r)+\frac{1}{6}F[t_n,\underline{y}(t_n:r)]$$

$$\overline{y}(t_{n+1}:r)=\overline{y}(t_n:r)+\frac{1}{6}G[t_n,\overline{y}(t_n:r)]$$

$$\underline{z}(t_{n+1}:r)=\underline{z}(t_n:r)+\frac{1}{6}P[t_n,\underline{z}(t_n:r)]$$

$$\overline{z}(t_{n+1}:r)=\overline{z}(t_n:r)+\frac{1}{6}Q[t_n,\overline{z}(t_n:r)]$$

To show the convergence of these approximation

$$\lim_{h\to 0}\underline{y}(t:r)=\underline{Y}(t:r),\quad \lim_{h\to 0}\overline{y}(t:r)=\overline{Y}(t:r),$$

$$\lim_{h\to 0}\underline{z}(t:r)=\underline{Z}(t:r),\quad \lim_{h\to 0}\overline{z}(t:r)=\overline{Z}(t:r)$$

Lemma 4.1:

Let a sequence of numbers $\{W\}_{n=0}^{N}$ satisfies $|W_{n+1}| \le A\,|W_n| + B$, $0 \le n \le N-1$ for some given positive constants A and B, then $|W_n| \le A^n |W_0| + B\dfrac{A^n - 1}{A-1}$, $0 \le n \le N-1$. [11]

Lemma 4.2:

Let a sequence of numbers $\{W\}_{n=0}^{N}$ and $\{V\}_{n=0}^{N}$ satisfies the conditions

$$|W_{n+1}| \le |W_n| + A\max\{|W_n|, |V_n|\} + B$$

and $$|V_{n+1}| \le |V_n| + A\max\{|W_n|, |V_n|\} + B$$

for some given positive constants A and B and denote $U_n = |W_n| + |V_n|$, $0 \le n \le N$ then

$$U_n \le \overline{A}^n U_0 + \overline{B}\,\frac{\overline{A}^n - 1}{\overline{A} - 1},\ 1 \le n \le N$$

where $\overline{A} = 1 + 2A$ and $\overline{B} = 2B$

Proof: Consider,

$$|W_{n+1}| + |V_{n+1}| \le |W_n| + |V_n| + 2A\,\{|W_n| + |V_n|\} + 2B$$

$$= (1 + 2A)\,\{|W_n| + |V_n|\} + 2B$$

By lemma 4.1, for W_n, $0 \le n \le N$ hence it is valid.

Theorem - 4.1

Let F(t, u, v) & G(t, u, v) belong to $C^1(K)$ and let the partial derivatives of F & G be bounded over K, then for arbitrary fixed value r, $0 \le r \le 1$ are approximate solutions converge to the exact solutions of $\underline{Y}(t_n : r)$ and $\overline{Y}(t_n : r)$ uniformly in t. Similarly, P(t, u, v) & Q(t, u, v) belong to $C^1(K)$ and let the partial derivatives of P and Q be bounded over K, then for arbitrary fixed value r, $0 \le r \le 1$ are approximate solutions converge to the exact solutions of $\underline{Z}(t_n : r)$ and $\overline{Z}(t_n : r)$ uniformly in t.

6. Numerical Example

Consider $\dfrac{dy}{dt} = -4y + 5z$ and $\dfrac{dz}{dt} = 8y - 6z$ with an initial conditions y(0) = (7.6 + 0.4r, 8.2 - 0.2r) and z(0) = (3.8 + 0.2r, 4.3-0.3r); $0 \le r \le 1$.

Solution:

The exact solution is $\underline{Y}(t:r) = 3\underline{y_1}(t:r)e^{t} + 5\underline{y_2}(t:r)e^{-3t}$,

$$\underline{Z}(t:r) = 3\underline{z_1}(t:r)e^{t} + \underline{z_2}(t:r)e^{-3t}$$

when t = 1 then the exact solution is given by,

$$\underline{Y}(1:r) = (2.75 + 0.25r)e + 5(0.8 + 0.2r)e^{-3}$$

and $$\overline{Y}(1:r) = (3.25 - 0.25r)e + (1.2 - 0.2r)e^{-3}$$

$$\underline{Z}(1:r) = (2.75 + 0.25r)e + (0.8 + 0.2r)e^{-3}$$

and $$\overline{Z}(1:r) = (3.25 - 0.25r)e + (1.2 - 0.2r)e^{-3}$$

The exact and approximate solutions obtained by the fifth order Runge-Kutta Merson method with initial condition for taking k = 0.1

Table 5.1

Exact Solution with h = 0.1				
r	$[\underline{Y}$	$\overline{Y}]$	$[\underline{Z}$	$\overline{Z}]$
0.0	7.6744233017	9.1331383527	7.5151046830	8.8941604245
0.1	7.7473590543	9.0602026002	7.5840574700	8.8252076375
0.2	7.8202948068	8.9872668476	7.6530102571	8.7562548504
0.3	7.8932305594	8.9143310951	7.7219630442	8.6873020633
0.4	7.9661663119	8.8413953425	7.7909158313	8.6183492762
0.5	8.0391020645	8.7684595900	7.8598686184	8.5493964891
0.6	8.1120378170	8.6955238374	7.9288214054	8.4804437021
0.7	8.1849735696	8.6225880849	7.9977741925	8.4114909150
0.8	8.2579093221	8.5496523323	8.0667269796	8.3425381279
0.9	8.3308450747	8.4767165798	8.1356797667	8.2735853408
1.0	8.4037808272	8.4037808272	8.2046325537	8.2046325537

Table 5.2

Approximation Solution by fifth order Runge-Kutta Merson with h = 0.1				
r	$[\underline{y}$	$\overline{y}]$	$[\underline{z}$	$\overline{z}]$
0.0	7.9835987091	9.2810068130	7.7944021225	9.0868310928
0.1	8.0256175995	9.1932840347	7.8354244232	8.9986104965
0.2	8.0676355362	9.1055622101	7.8764467239	8.9103908539
0.3	8.1096553802	9.0178413391	7.9174709320	8.8221721649
0.4	8.1516742706	8.9301195145	7.9584946632	8.7339515686
0.5	8.1936931610	8.8423986435	7.9995169640	8.6457328796
0.6	8.2357130051	8.7546758652	8.0405406952	8.5575122833
0.7	8.2777309418	8.6669540405	8.0815629959	8.4692935944
0.8	8.3197507858	8.5792322159	8.1225881577	8.3810720444
0.9	8.3617696762	8.4915103912	8.1636104584	8.2928533554
1.0	8.4037885666	8.4037885666	8.2046337128	8.2046337128

Table 5.3

Complete Error Analysis with h = 0.1				
r	$[\underline{y}$	$\overline{y}]$	$[\underline{z}$	$\overline{z}]$
0.0	0.3091754074	0.1478684603	0.2792974395	0.1926706683
0.1	0.2782585452	0.1330814345	0.2513669532	0.1734028590
0.2	0.2473407294	0.1182953625	0.2234364668	0.1541360035
0.3	0.2164248208	0.1035102440	0.1955078878	0.1348701016
0.4	0.1855079587	0.0887241720	0.1675788319	0.1156022924
0.5	0.1545910965	0.0739390535	0.1396483456	0.0963363905
0.6	0.1236751881	0.0591520278	0.1117192898	0.0770685812
0.7	0.0927573722	0.0443659556	0.0837888034	0.0578026794
0.8	0.0618414637	0.0295798836	0.0558611781	0.0385339165
0.9	0.0309246015	0.0147938114	0.0279306917	0.0192680146
1.0	0.0000077394	0.0000077394	0.0000011591	0.0000011591

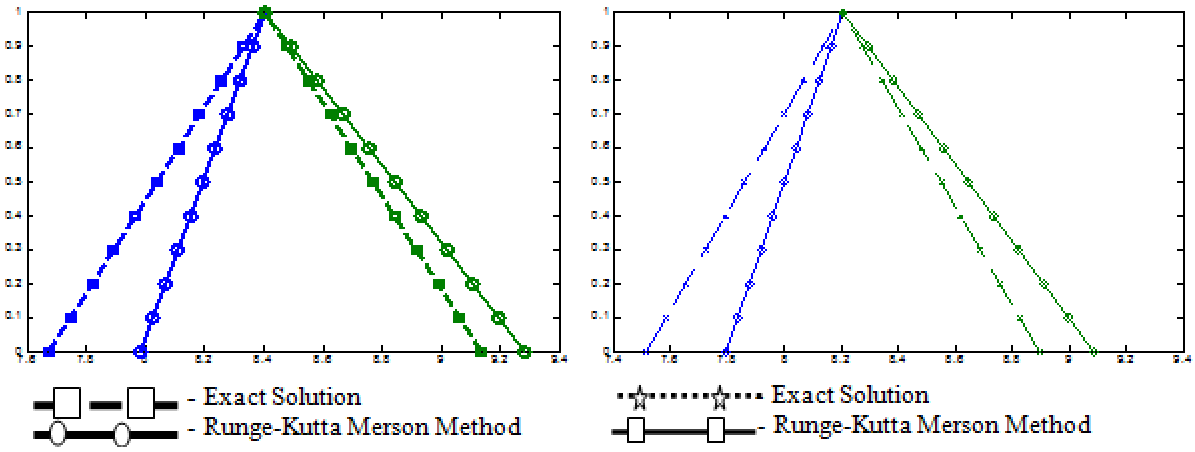

Figure 5.1

7. Conclusion

In this paper we have found the iterative solution of first order simultaneous fuzzy differential equation using fifth order Runge-Kutta Merson method.

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