Optimal Portfolio Construction (A Case Study of LQ45 Index in Indonesia Stock Exchange)

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Abstract: *The purpose of this paper is to test whether single index model or constant correlation model of portfolio selection offers better investment alternatives to investors in IDX. Sample taken for this research is 22 stocks consistently joined in LQ45 index for the time period of February 2010 to January 2015. After obtaining the optimal portfolios, the performance of each portfolio is evaluated and analyzed in terms of their expected return and risk. The measurement of that performance uses the risk adjusted methods: Sharpe, Treynor, and Jensen Index. By using the single index model, it is observed that only six out of 22 sample stocks are allowed to be included in the optimal portfolio. Meanwhile the optimal portfolio constructed by using the constant correlation model consists of eight stocks. The final results then show that the optimal portfolio constructed by using the single index model has a better performance. The three indexes – Sharpe, Treynor, and Jensen – give the same performance ranking for both portfolios, it means that both portfolios have been well-diversified.*

Keywords: optimal portfolio, Single Index Model, Constant Correlation Model, Indonesia Stock Exchange.

1. Introduction

The theory of investment says that the decision-making in investment is not only based on the expected rate of return but also the risk level. "Either return or risk are two inseparable things since the consideration of an investment is both's trade off. The return and the risk have a positive relationship: the bigger the risk which has to be borne, the bigger the return which has to be compensated" [1]. The risk-return trade off becomes a dilemma in decision-making of investment for the investors.

In most cases, the risk might be able to be reduced by combining some single securities into a portfolio. This effect of reducing risks by including a large number of investments or securities in a portfolio is called diversification [2]. This diversification is important, since "studies indicate that diversification can help insulate one's investment against market and management risks without much sacrificing the desired level of return" [3]. The part of risk which can be eliminated by constructing the well-diversified portfolio is the unsystematic risk. This risk is unique for a company, so the bad thing occurs in a company can be offset by the good thing occurs in another company [1].

It is investors' job to find the optimal combination of stocks for their portfolios that can minimize the risks and maximize the returns. However, an investor is faced with a choice from among an enormous number of stocks. When one considers the number of possible stocks and the various possible proportions in which each can be held, the decision process seems overwhelming.

In a series of publications and books, Elton, Gruber, and Padberg [4]-[5] propose simple techniques used to select optimal portfolio. These models allow the development of a system for computing the composition of optimum portfolios that is so simple it can often be performed without the use of computer. That model can be used if the assumption of the single index model or the constant correlation model regarding the covariance structure return among its securities

is accepted (both assumptions will be discussed in the next part).

Generally, both models will be able to determine whether a stock can be included in the optimal portfolio or not by using a unique ranking criteria. The stock will be sorted based on the performance measured by using a ratio of excess return against the risk, so if a stock is included into an optimal portfolio, any higher ranked stock must also be included into the optimal portfolio.

After obtaining the optimal portfolios, it is important to measure the performance of each portfolio. The Sharpe, Treynor, and Jensen index are the measures of portfolio performance which have inserted the return or the risk factor. The Sharpe and Treynor index are ratios of compensation against a risk. The difference is that the indicator of the risk of Sharpe index is a total risk, meanwhile the Treynor index is a systematic risk. The Jensen index is an index which shows the difference between the level of actual return gained by the portfolio and the level of expected return if the portfolio is in line with the capital market [6].

2. Objective of the Study

The main objective of this research is to construct the optimal portfolio which contains the stock of index LQ45 using the single index model and constant correlation model as well as its performance analysis by using the risk adjusted performance approach: Sharpe, Treynor, and Jensen. The LQ45 Index is a capitalization-weighted index of the 45 most heavily traded stocks on the Indonesia Stock Exchange.

3. Research Method

The type of this research is descriptive quantitative research. The sample of research is 22 stocks part of LQ45 index selected by using the purposive sampling technique. The criterion of stock selection is the stock which is consistently joined in LQ45 index during February 2010 – January 2015. The data are from secondary data, such as the adjusted

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weekly closing stock price, IHSG as the market index, and SBI as the proxy of risk free rate for the observation period during February 2010-January 2015.

4. Theoretical Foundation and Methodology

4.1 Single Index Model

Single index model has an assumption that one reason security returns might be correlated is because of a common response to market changes. This case comes up from an observation that when the market goes up (as measured by any of the widely available stock market indexes), most stocks tend to increase in price, and when the market goes down, most stocks tend to decrease in price [4]. Thus, the return on a stock and the return on stock market index can be commonly stated as a relationship:

$$R_i = \alpha_i + \beta_i R_m + e_i \qquad (1)$$

Explanation:

R_i = return on a stock or security i

β_i = beta is the coefficient which measures the change of R_i caused by the change of R_m

R_m = rate of return on the market index

α_i = the expected value of the component of security i's return that is independent of the market's performance.

e_i = residual error, which is the random variable with zero expected value or E(e_i) = 0.

If the assumption of single index model is met as the description of the covariance structure among securities, the method of optimal portfolio selection which will be explained can be used. Note that all procedures used to calculate the optimal portfolio will only consider the short sales disallowed situation.

4.2 Constructing the Optimal Portfolio Based on the Single Index Model

The calculation to determine the optimal portfolio should be eased if it is only based on a number which can determine whether securities can be included into the optimal portfolio or not. The number is the excess return to beta ratio [1]. The excess return to beta ratio (ERB) shows the relationship between return and risk. This ERB measures the relative excess against a unit of risk which can be diversified measured by beta.

$$ERB_i = \frac{E(R_i) - R_f}{\beta_i} \qquad (2)$$

Explanation:

R_f = risk-free asset return

The optimal portfolio will contain securities with high ERB values. Thus, the cut off point is needed to determine the limitation of ERB values which are considered as high. This cut off point can be determined by using these following steps:

1. Sorting the securities based on the highest ERB values to the lowest ERB values.

2. Calculating the value of A_i and B_i for each securities

$$A_i = \frac{[E(R_i) - R_f].\beta_i}{\sigma_{ei}^2} \quad \text{and} \quad B_i = \frac{\beta_i^2}{\sigma_{ei}^2} \qquad (3)$$

Where σ_{ei} is the variance from the residual error of securities i which is also a unique risk or unsystematic risk.

3. Calculating the value of C_i that is the value of C for securities i, calculated based on the following formula:

$$C_i = \frac{\sigma_m^2 \sum_{j=1}^{i} A_j}{1 + \sigma_m^2 \sum_{j=1}^{i} B_j} \qquad (4)$$

where:

a. The cut-off point (C*) is the value of C_i where the last ERB value is still greater than the value of C_i.

b. The securities included into the optimal portfolio are those which posses the bigger ERB value or same with the ERB value in C* point.

To determine the amount of the proportion of each selected securities in the optimal portfolio, the formula is:

$$w_i = \frac{Z_i}{\sum_{j=1}^{N} Z_j} \qquad (5)$$

With the value of Z_i:

$$Z_i = \frac{\beta_i}{\sigma_{ei}^2}(ERB_i - C^*)$$

Explanation:

w_i = proportion of security i

σ_{ei}^2 = variance from residual error of security i

Expected return based on the single index model can be calculated by using this formula:

$$E(R_p) = \alpha_p + \beta_p.E(R_m) \qquad (6)$$

Meanwhile, the variance or the risk of portfolio can be calculated by using this formula:

$$\sigma_p^2 = \beta_p^2.\sigma_m^2 + \left(\sum_{i=1}^{N} w_i.\sigma_{ei}\right)^2 \qquad (7)$$

4.3 Constant Correlation Model

The constant correlation model assumes the correlation between all pairs of securities is the same. The procedures assuming a constant correlation coefficient exactly parallel those presented for the case of single index model [1]. Thus the correlation coefficient value is the average of correlation coefficient value among those securities.

If the assumption of constant correlation model is met as the description of the covariance structure among securities, the procedures of optimal portfolio determination which will be explained can be used. Note that all procedures used to calculate the optimal portfolio will only consider the short sales disallowed situation.

4.4 Constructing the Optimal Portfolio Based on the Constant Correlation Model

The procedures of determining the optimal portfolio by using this model are the same with the procedures by using the single index model, but the selections of securities which are included into the portfolio are based on the excess return to standard deviation value (ERS). ERS can be calculated by using this formula:

$$ERS_i = \frac{(E(R_i) - R_f)}{\sigma_i} \qquad (8)$$

Either single index model or constant correlation model divide the excess return value with the risk. However, in the constant correlation model, the standard deviation (σ_i) substitutes the beta as its risk indicator.

Furthermore, the value of C_i is used to determine the cut-off point calculated based on following formula:

$$C_i = \frac{\rho}{1 - \rho + i\rho} \sum_{j=1}^{i} \frac{E(R_j) - R_f}{\sigma_j} \qquad (9)$$

where ρ is a correlation coefficient assumed constant for all securities. To meet its assumption that the correlation coefficients among stocks are constant, the used value is the average value of correlation coefficient value (ρ_{ij}) among stocks, as following:

$$\rho = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{ij}}{N}$$

where the number of ρ_{ij} which are calculated as following:

$$N = \frac{n(n-1)}{2}$$

Next, all stocks or securities which have a higher excess return to standard deviation than C* are included into the optimal portfolio.

The amount of the optimum investment for each securities is calculated as following:

$$w_i = \frac{Z}{\sum_{j=1}^{N} Z_j} \qquad (10)$$

Where,

$$Z_i = \frac{1}{(1-\rho)\sigma_i} \left[\frac{E(R_i) - R_f}{\sigma_i} \right] - C*$$

The expected return of portfolio is the weighted average of expected returns from each single securities in the portfolio [1]. The expected return of portfolio is calculated as following:

$$E(R_P) = \sum_{i=1}^{n} w_i . E(R_i) \qquad (11)$$

Then, the risk of this portfolio can be calculated by using the following equation:

$$\sigma_p{}^2 = \sum_{i=1}^{n} w_i{}^2 \sigma_i{}^2 + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}}^{n} w_i . w_j . \sigma_{ij} \qquad (12)$$

Explanation:

$\sigma_i{}^2$ = variance return of security i

σ_{ij} = covariance between securities i and j return

5. Results and Discussion

5.1 Single Index Model

The calculation results are presented to determine the optimal portfolio by using single index model as following:

Table 1: The Construction of Optimal Portfolio (Single Index Model) with R_f = 0.117353% and $E(R_m)$ = 0.3207%

No	Issuers Code	E(R)	β_i	α_i	$\sigma_{ei}{}^2$	ERB	A	B	$\sum_{j=1}^{i} A_j$	$\sum_{j=1}^{i} B_j$	C_i
1	JSMR	0.668	0.761	0.424	9.3645	0.7235	0.0447	0.0618	0.0447	0.0618	0.1917
2	TLKM	0.5927	0.676	0.376	13.7406	0.7032	0.0234	0.0333	0.0681	0.0951	0.2554
3	KLBF	0.9583	1.285	0.546	12.7419	0.6544	0.0848	0.1296	0.1529	0.2247	0.3859
4	UNVR	0.5915	0.777	0.342	13.485	0.6103	0.0273	0.0448	0.1803	0.2695	0.4087
5	ASII	0.7855	1.476	0.312	14.7843	0.4527	0.0667	0.1474	0.247	0.4168	0.4197
6	GGRM	0.4812	0.857	0.206	19.4037	0.4246	0.0161	0.0379	0.263	0.4547	0.42
7	BBNI	0.6609	1.403	0.211	11.8684	0.3874	0.0643	0.1659	0.3273	0.6205	0.4132
8	BBCA	0.5104	1.066	0.169	5.6818	0.3687	0.0737	0.2	0.401	0.8205	0.4042
9	BBRI	0.6018	1.375	0.161	8.4879	0.3523	0.0785	0.2227	0.4795	1.0433	0.3947
10	LPKR	0.553	1.322	0.129	28.6897	0.3296	0.0201	0.0609	0.4996	1.1042	0.3916
11	INDF	0.4381	1.057	0.099	11.9199	0.3035	0.0284	0.0937	0.528	1.1979	0.3856
12	BMRI	0.5007	1.483	0.025	7.334	0.2585	0.0775	0.2999	0.6056	1.4978	0.3627
13	SMGR	0.3905	1.133	0.027	10.899	0.2411	0.0284	0.1178	0.634	1.6156	0.3547
14	LSIP	0.2879	0.796	0.032	28.2111	0.2142	0.0048	0.0225	0.6388	1.638	0.353
15	INTP	0.3608	1.154	-0.009	10.7226	0.2109	0.0262	0.1242	0.665	1.7622	0.3439
16	PGAS	0.2943	1.004	-0.028	12.0647	0.1762	0.0147	0.0836	0.6797	1.8458	0.3369
17	AALI	0.2015	0.691	-0.02	25.2593	0.1218	0.0023	0.0189	0.682	1.8647	0.3349
18	UNTR	0.2237	1.192	-0.159	13.9899	0.0892	0.0091	0.1016	0.6911	1.9662	0.3232
19	BDMN	0.1574	0.867	-0.121	21.6294	0.0462	0.0016	0.0348	0.6927	2.001	0.3188
20	PTBA	0.0764	1	-0.244	22.0071	-0.041	-				
21	ITMG	0.0553	0.878	-0.226	24.9296	-0.0707	-				
22	ADRO	-0.0528	0.974	-0.365	26.8194	-0.1747	-				

Table 1 is a stock list sorted based on the ERB value. From the table, it can be seen that there are three stocks which have a negative ERB value so those three stocks are removed from the optimal portfolio candidate. It is due to those stocks have the smaller return compared to the SBI return (risk free rate). However, those stocks are risky assets. The rational investors certainly will not decide to invest on the risky assets if they cannot get the compensation from the risk.

From the table, it is also known that there are 12 stocks which have more than one beta value so it is called aggressive stocks, which are a stock possessing a high risk. This stock will experience the higher increase from the market increase and the sharper decrease if the market decreases. Then, nine stocks have less than one beta value. This stock is called defensive stock. This stock has smaller sensitivity against the market. It is due to the stock price might move to the same direction with the market, however, the amount is not same (smaller). PTBA has one beta value, as big as the beta of market so this stock is neutral.

Furthermore, the results of cut-off point (C*) simulation require that only stocks with $ERB \geq C_i$ value which will be included into the optimal portfolio. From 19 samples, there are only six stocks which have bigger ERB values than C_i. Thus, the six stocks become the last candidate included into the optimal portfolio. The value of C* is known as 0.42. Then, the amount of investment allocation percentage on the selected six stocks is as following:
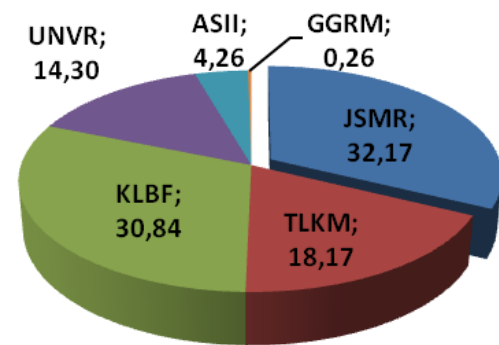

Figure 1: Allocation of Investment Funds (%)

Table 2: Return, Risk, and Portfolio Beta

E(Rp) %	Risk %		β_p
	(σ_p)	(σ_p^2)	
0.7374%	2.8437	8.087	0.9401

From the figure 1 above, it can be seen that JSMR, KLBF and TLKM get the biggest allocation of investment funds whilst GGRM gets the smallest portion which is 0.26%. Meanwhile, table 2 shows the calculation results of the risk and the return of portfolio. Based on the table 2, this optimal portfolio can give the level of portfolio return by 0.73736% weekly calculated, with the risk stated in the standard deviation by 2.8437%. The amount of the portfolio risk is smaller than the individual stock risk (the individual risk is in table 3). It proves that the diversification of portfolio can decrease the portfolio risk, by remaining to maintain the rate of portfolio return.

5.2 Constant Correlation Model

The calculation results are presented to determine the optimal portfolio by using the constant correlation model (table 3) as following:

Table 3: The Establishment of Optimal Portfolio (Constant Correlation Model) with $\rho = 0.3226$

No	Issuers Code	E(R)	Excess Return	σ_i	ERS	$\dfrac{\rho}{1-\rho+\iota\rho}$	$\sum_{j=1}^{i}\dfrac{E(R_j)-R_f}{\sigma_j}$	C_i
1	KLBF	0.9583	0.8409	4.7284	0.1779	0.3226	0.1779	0.0574
2	JSMR	0.668	0.5506	3.5695	0.1543	0.2439	0.3321	0.081
3	ASII	0.7855	0.6681	5.2427	0.1274	0.1961	0.4596	0.0901
4	TLKM	0.5927	0.4754	4.0505	0.1174	0.1639	0.5769	0.0946
5	UNVR	0.5915	0.4742	4.1237	0.115	0.1408	0.6919	0.0975
6	BBNI	0.6609	0.5435	4.8308	0.1125	0.1235	0.8044	0.0993
7	BBCA	0.5104	0.3931	3.5072	0.1121	0.1099	0.9165	0.1007
8	BBRI	0.6018	0.4845	4.4158	0.1097	0.099	1.0262	0.1016
9	BMRI	0.5007	0.3834	4.4888	0.0854	0.0901	1.1116	0.1001
10	GGRM	0.4812	0.3638	4.8669	0.0748	0.0826	1.1864	0.098
11	INDF	0.4381	0.3208	4.2935	0.0747	0.0763	1.2611	0.0963
12	LPKR	0.553	0.4357	6.2352	0.0699	0.0709	1.331	0.0944
13	SMGR	0.3905	0.2732	4.2867	0.0637	0.0662	1.3947	0.0924
14	INTP	0.3608	0.2434	4.2994	0.0566	0.0621	1.4513	0.0901
15	PGAS	0.2943	0.177	4.2358	0.0418	0.0585	1.4931	0.0873
16	LSIP	0.2879	0.1705	5.6487	0.0302	0.0552	1.5233	0.0842
17	UNTR	0.2237	0.1064	4.7189	0.0225	0.0524	1.5458	0.0809
18	AALI	0.2015	0.0842	5.2954	0.0159	0.0498	1.5617	0.0777
19	BDMN	0.1574	0.0401	5.1001	0.0079	0.0474	1.5696	0.0744
20	PTBA	0.0764	-0.041	5.2755	-0.008		-	
21	ITMG	0.0553	-0.0621	5.4239	-0.011		-	
22	ADRO	-0.053	-0.1701	5.687	-0.03		-	

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From 22 stocks, there three stocks which have negative ERS values so those three stocks are removed from the optimal portfolio candidate. It is due to those stocks have smaller return compared to SBI return (risk free rate), although those stocks are risky assets.

From the cut-off point simulation, it is known that from 19 stock candidates, there are only eight stocks which have more ERS value than Ci. Thus, those eight stocks become the last candidate which will be included into the optimal portfolio. The C* value is known as 0.1016.

After the stocks are selected, the next step is to calculate the percentage or the proportion of investment funds for each stock.
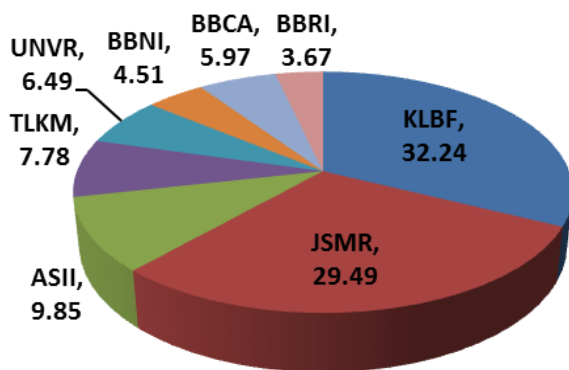

Figure 2: Allocation of Investment Funds (%)

From the figure above, it can be seen that KLBF has the biggest percentage of allocation of investment funds which is 32.24%, then it is followed by JSMR with the allocation of funds by 29.84%. While BBRI gets the smallest percentage of allocation by 3.67%.

Table 4: Return, Risk, and Portfolio Beta

E(Rp) %	Total Risk %		β_p
0.7502%	(σ_p)	(σ_p^2)	1.0645
	3.0454	9.2746	

The amount of portfolio risk is smaller than the risk of each stock. It means that the unsystematic risk of stock has been able to be reduced by establishing the well-diversified portfolio. It occurs due to the decrease of return happened in a stock or some stocks in the portfolio have been able to be covered by the increase of another stock in the portfolio.

5.3 The Comparison of Portfolio Performance

Table 5: The Comparison of Optimal Portfolio Performance

	Single Index Model	Constant Correlation Model
Sharpe (%)	0.2180	0.2078
Treynor (%)	0.6595	0.5945
Jensen (%)	0.4288	0.4163

The results of three portfolio performance indicator indexes show that the optimal portfolio constructed by single index model has a higher ranking or a higher performance value compared to the portfolio constructed by using constant correlation model. Besides, due to the three performances

indicator shows the same performance ranking for both portfolios, it means that both portfolios have been well-diversified.

Based on the Sharpe index, the optimal portfolio constructed by single index model will be able to give the compensation of portfolio return against each total risk by 0.218%. Meanwhile, the Treynor index for the compensation of portfolio return against systematic risk (beta) is 0.6595%. Seen from the Jensen index, the portfolio established by using single index model has a bigger return from the expected return if the portfolio is in the capital market line of 0.4288%.

6. Conclusion

From the research above, there are some main points of the conclusion. First, the optimal portfolio constructed by using single index model contains six combinations of stock, whilst the optimal portfolio by using constant correlation model contains eight stocks. JSMR and KLBF are the stocks with the biggest contribution in the portfolio since each of them gets the biggest portion of funds (around 29%-32%).

Second, the amount of portfolio risk is smaller compared to each individual risk. It indicates that the unsystematic risk has been successful to be reduced by establishing the well-diversified portfolio. It occurs due to the decrease of return happened in a stock or some stocks in the portfolio have been able to be covered by the increase of another stock in the portfolio.

Last, the value of three portfolio performance indicator indexes: Sharpe, Treynor, and Jensen, consistently show that the portfolio constructed by using single index model has a better performance compared to the portfolio constructed by using constant correlation model. So, the portfolio constructed by single index model automatically becomes a recommendation from the writer for the investors to invest. Besides, the three performance indicator shows the same performance ranking for both portfolios, it means that both portfolios have been well-diversified.

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