

MHD Three Dimensional Free Convection Couette Flow with Transpiration Cooling

Mamta Goyal, Namrata Naraniya

Department of Mathematics, University of Rajasthan, Jaipur-302004, India

Corresponding Author E-mail address: Vahi0714@gmail.com

Abstract: A theoretical analysis of three dimensional free convection couette flow of a viscous incompressible fluid between two vertical parallel plates with transpiration cooling in the presence of a stationary magnetic field applied perpendicular to the planes of the insulating plates is presented. The stationary plate and the plate in uniform motion are, respectively, subjected to transverse sinusoidal injection and uniform suction of the fluid. The flow becomes three dimensional due to this type of injection velocity distribution. The series expansion method is used to obtain the expressions for the velocity and temperature fields. A comprehensive study of the problem is made for various parameters.

Keywords: MHD, free convection, couette flow, transpiration cooling, three dimensional flow

1. Introduction

The magnetohydrodynamic (MHD) flows has many practical applications such as electromagnetic flow meters, electromagnetic pumps and hydromagnetic generators etc. Magnetohydrodynamics deals with dynamics of an electrically conducting fluid, which interacts with a magnetic field. The study of MHD flow in a channel has applications in many engineering problems. Chang and Lundgren [4], Hughes and Young [20], Hartmann [7], Jaffery [2], Cowling [18] discussed the exact solutions to the classical problem of hydrodynamic channel flow for different simplified solutions. The flow and heat transfer of electrically conducting fluids in channels under the effect of a transverse magnetic field has been discussed by several authors notably Attia and Kotb [6], Nigam and Singh [15] and Alpher [14]. Convective flows in channels driven by temperature differences of bounding walls have been studied and reported, extensively in literature. Free convection flows in vertical slots were discussed by Burch et al. [17], Kim et al. [16], Buhler [8], and Weidman [13]. Ferraro and Plumpton [19] and Cramer and Pai [11] are notable authors for major contribution about MHD free convection flows and their significant application in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering, electronics, and so on. An extensive contribution on heat transfer flow has been made by Gebhart [3] to highlight the insight on the phenomena. Raptis [1] investigated Hydrodynamic free convection flow through a porous medium between two parallel plates. Transpiration cooling is a very effective process to protect certain structural elements in turbojet and rocket engines, like combustion chamber walls, exhaust nozzles, or gas turbine blades, from the influence of hot gases. In view of this Eckert [5] obtained an exact solution of the plane couette flow with transpiration cooling. Singh [10] studied Three dimensional couette flow with transpiration cooling. Singh and Sharma [9] investigated MHD three dimensional couette flow with transpiration cooling. Jain and gupta [12] were analyzed Three dimensional free convection couette flow with transpiration cooling.

The aim of the present problem is to study the effects of the transverse magnetic field on three dimensional free

convection couette flow. If the effect of magnetic field is neglected ($M=0$), in the present work, the results obtained are similar to those of Jain and gupta [12].

2. Formulation of the Problem

Consider a steady laminar couette flow of viscous incompressible fluid between two vertical parallel flat plates distance 'd' apart. A co-ordinate system is introduced with its origin at the stationary plate lying vertically on the $x^* - z^*$ plane. The another plate in uniform motion U_0 along the x^* axis is subjected to a constant suction V_0 and stationary plate to a transverse sinusoidal injection velocity distribution of the form

$$v^*(z) = V_0 \left(1 + \varepsilon \frac{\cos \pi z^*}{d} \right) \quad (1)$$

where $\varepsilon (\square 1)$ is a positive constant quantity.

A magnetic field of uniform strength B_0 applied along y^* axis perpendicular to the plane of plates. The value of this uniform magnetic field is assumed to be unaltered by making the necessary assumptions that guarantee the neglect of the induced electric and magnetic fields. Hall effects, electrical and polarization effects are also neglected. The plates are assumed to be at constant temperature T_0^* and T_1^* , respectively. All the physical quantities are independent of x^* for this problem of fully developed laminar flow but the flow remains three-dimensional due to the injection velocity Eq. (1). Denoting the velocity components u^*, v^*, w^* in the x^*, y^*, z^* directions respectively and the temperature by T^* , the problem is governed by the following equations:

$$v_y^* + w_z^* = 0, \quad (2)$$

$$v^* u_y^* + w^* u_z^* = g\beta(T^* - T_e^*) + \nu(u_{yy}^* + u_{zz}^*) - \frac{\sigma B_0^2}{\rho} u^* \quad (3)$$

$$v^* v_y^* + w^* v_z^* = -\frac{P_y^*}{\rho} + \nu(v_{yy}^* + v_{zz}^*) \quad (4)$$

$$v^* w_y^* + w^* w_z^* = -\frac{p_z^*}{\rho} + \nu(w_{yy}^* + w_{zz}^*) - \frac{\sigma B_0^2}{\rho} w^* \quad (5)$$

$$v^* T_y^* + w^* T_z^* = \frac{k}{\rho C_p} (T_{yy}^* + T_{zz}^*) + \frac{Q}{\rho C_p} (T^* - T_e^*) \quad (6)$$

where ρ is the density, ν is the kinematic viscosity, σ is the electrical conductivity, C_p is the specific heat at constant pressure, k is the thermal conductivity, g is the acceleration due to gravity, β is the coefficient of volume expansion, T_e^* is the equilibrium temperature and p^* is the pressure. Here the '*' stands for the dimensional quantities. The boundary conditions of the problem are

$$y^* = 0: u^* = 0, v^* = V_0 \left(1 + \varepsilon \frac{\cos \pi z^*}{d} \right), w^* = 0, T^* = T_0^*$$

$$y^* = d: u^* = U_0 + L_1 \frac{\partial u^*}{\partial y^*}, v^* = V_0, w^* = 0, T^* = T_1^*$$

(7)

where $L_1 = \frac{2-m_1}{m_1} L$, L being mean free path and m_1 the Maxwell's reflection coefficient.

Introducing the following non-dimensional quantities:

$$y = \frac{y^*}{d}, z = \frac{z^*}{d}, u = \frac{u^*}{U_0}, v = \frac{v^*}{V_0}, w = \frac{w^*}{V_0}, p = \frac{p^*}{\rho V_0^2},$$

$$\theta = \frac{T^* - T_e^*}{T_0^* - T_e^*}, m = \frac{T_1^* - T_e^*}{T_0^* - T_e^*}, P = \frac{\mu C_p}{k} \quad (\text{The Prandtl number}),$$

$$G = \frac{g \beta d (T_0^* - T_e^*)}{V_0 U_0} \quad (\text{The Grashof number}),$$

$$\alpha = \frac{Q d \nu}{V_0 k} \quad (\text{The heat source parameter}),$$

$$h = \frac{L_1}{d} \quad (\text{The slip parameter}), S = \frac{V_0 d}{\nu} \quad (\text{The suction}$$

parameter), $M = B_0 d \sqrt{\frac{\sigma}{\mu}}$ (The Hartmann number),

And get the equations as:

$$v_y + w_z = 0 \quad (8)$$

$$\nu u_y + w u_z = G \theta + \frac{1}{S} (u_{yy}^2 + u_{zz}^2) - \frac{M^2}{S} u \quad (9)$$

$$\nu v_y + w v_z = -p_y + \frac{1}{S} (v_{yy}^2 + v_{zz}^2) \quad (10)$$

$$\nu w_y + w w_z = -p_z + \frac{1}{S} (w_{yy}^2 + w_{zz}^2) - \frac{M^2}{S} w \quad (11)$$

$$v \theta_y + w \theta_z = \frac{1}{PS} (\theta_{yy}^2 + \theta_{zz}^2) + \frac{\alpha}{P} \theta \quad (12)$$

The corresponding boundary conditions become:

$$y = 0: u = 0, v(z) = 1 + \varepsilon \cos \pi z, w = 0, \theta = 1$$

$$y = 1: u = 1 + h u_y, v = 1, w = 0, \theta = m \quad (13)$$

3. Solution of the Problem

Since the amplitude of the injection velocity ε ($\ll 1$) is very small, we now assume the solution in the following form:

$$f(y, z) = f_0(y) + \varepsilon f_1(y, z) + \varepsilon^2 f_2(y, z) + \dots \quad (14)$$

where f stands for u, v, w, p and θ . When $\varepsilon = 0$, the problem reduces to the two dimensional flow with constant injection and suction at the respective plates and are governed by the following equations:

$$v_{0y} = 0 \quad (15)$$

$$v_0 u_{0y} = G \theta_0 + \frac{1}{S} u_{0yy} - \frac{M^2}{S} u_0 \quad (16)$$

$$v_0 \theta_{0y} = \frac{1}{PS} \theta_{0yy} + \frac{\alpha}{P} \theta_0 \quad (17)$$

with boundary conditions:

$$y = 0: u_0 = 0, v_0 = 1, w_0 = 0, \theta_0 = 1$$

$$y = 1: u_0 = 1 + h u_{0y}, v_0 = 1, w_0 = 0, \theta_0 = m \quad (18)$$

The solution of this two dimensional problem is

$$u_0 = c_3 e^{\pi_1 y} + c_4 e^{\pi_2 y} + z_1 e^{\pi_1 y} + z_2 e^{\pi_2 y} \quad (19)$$

$$\theta_0 = c_1 e^{\pi_1 y} + c_2 e^{\pi_2 y} \quad (20)$$

$$\text{with } v_0 = 1, w_0 = 0, p_0 = \text{constant.} \quad (21)$$

When $\varepsilon \neq 0$, substituting Eq.(14) in Eqs.(8)-(12) and comparing the coefficients of identical powers of ε , neglecting those of $\varepsilon^2, \varepsilon^3$ etc., the following first order equations are obtained:

$$v_{1y} + w_{1z} = 0 \quad (22)$$

$$u_{1y} + v_1 u_0 = G \theta_1 + \frac{1}{S} (u_{1yy} + u_{1zz}) - \frac{M^2}{S} u_1 \quad (23)$$

$$v_{1y} = -p_{1y} + \frac{1}{S} (v_{1yy} + v_{1zz}) \quad (24)$$

$$w_{1y} = -p_{1z} + \frac{1}{S} (w_{1yy} + w_{1zz}) - \frac{M^2}{S} w_1 \quad (25)$$

$$\theta_{1y} + v_1 \theta_{0y} = \frac{1}{PS} (\theta_{1yy} + \theta_{1zz}) + \frac{\alpha}{P} \theta_1 \quad (26)$$

with boundary conditions

$$y = 0: u_1 = 0, v_1 = \cos \pi z, w_1 = 0, \theta_1 = 0$$

$$y = 1: u_1 = h u_{1y}, v_1 = 0, w_1 = 0, \theta_1 = 0 \quad (27)$$

These are the linear partial differential equations which describe the three-dimensional flow.

In order to solve these equations we shall first consider the Eqs.(22), (24) and (25) for cross flow, being independent of

the main flow component u_1 , and the temperature field θ_1 . We assume v_1, w_1 and p_1 of the following form:

$$v_1(y, z) = v_{11}(y) \cos \pi z \quad (28)$$

$$w_1(y, z) = -\frac{1}{\pi} v'_{11}(y) \sin \pi z \quad (29)$$

$$p_1(y, z) = p_{11}(y) \cos \pi z \quad (30)$$

Where a prime denotes differentiation with respect to y . Equations (28) and (29) have been so chosen that the continuity Eq.(22) is satisfied. Substituting these equations into Eqs.(24) and (25) and applying the corresponding transformed boundary conditions, we get the solutions of v_1, w_1 and p_1 as

$$v_1(y, z) = [A_1 e^{s_1 y} + A_2 e^{s_2 y} + A_3 e^{s_3 y} + A_4 e^{s_4 y}] \cos \pi z \quad (31)$$

$$w_1(y, z) = -\frac{1}{\pi} [A_1 s_1 e^{s_1 y} + A_2 s_2 e^{s_2 y} + A_3 s_3 e^{s_3 y} + A_4 s_4 e^{s_4 y}] \sin \pi z \quad (32)$$

$$p_1(y, z) = \frac{1}{S} [A_1 (s_1^2 - S s_1 - \pi^2) e^{s_1 y} + A_2 (s_2^2 - S s_2 - \pi^2) e^{s_2 y} + A_3 (s_3^2 - S s_3 - \pi^2) e^{s_3 y} + A_4 (s_4^2 - S s_4 - \pi^2) e^{s_4 y}] \cos \pi z \quad (33)$$

Now assuming u_1 and θ_1 as

$$u_1(y, z) = u_{11}(y) \cos \pi z \quad (34)$$

$$\theta_1(y, z) = \theta_{11}(y) \cos \pi z \quad (35)$$

and substituting in Eqs.(23) and (26), we obtain the following equations:

$$u''_{11} - S u'_{11} - (\pi^2 + M^2) u_{11} = S v_{11} u'_0 - S G \theta_{11} \quad (36)$$

$$\theta''_{11} - S P \theta'_{11} + (S \alpha - \pi^2) \theta_{11} = v_{11} S P \theta'_0 \quad (37)$$

with the corresponding boundary conditions:

$$\begin{aligned} y=0: & u_{11} = 0, \theta_{11} = 0 \\ y=1: & u_{11} = h_1 u'_{11}, \theta_{11} = 0 \end{aligned} \quad (38)$$

Substitution of u_{11} and θ_{11} obtained from Eqs.(36) and (37) under boundary conditions Eq.(38) in Eqs.(34) and (35), we get

$$\begin{aligned} u_1(y, z) = & [A_7 e^{s_3 y} + A_8 e^{s_4 y} + h_{26} e^{t_2 y} + h_9 e^{(s_1+r_1)y} + h_{10} e^{(s_2+r_1)y} \\ & + h_{11} e^{(s_3+r_1)y} + h_{12} e^{(s_4+r_1)y} + h_{13} e^{(s_1+r_2)y} + h_{14} e^{(s_2+r_2)y} + h_{15} e^{(s_3+r_2)y} \\ & + h_{16} e^{(s_4+r_2)y} + h_{17} e^{(s_1+x_{11})y} + h_{18} e^{(s_2+x_{11})y} + h_{19} e^{(s_3+x_{11})y} \\ & + h_{20} e^{(s_4+x_{11})y} + h_{21} e^{(s_1+x_{22})y} + h_{22} e^{(s_2+x_{22})y} + h_{23} e^{(s_3+x_{22})y} \\ & + h_{24} e^{(s_4+x_{22})y}] \cos \pi z \quad (39) \end{aligned}$$

$$\begin{aligned} \theta_1(y, z) = & [A_5 e^{t_1 y} + A_6 e^{t_2 y} + h_4 e^{(s_1+x_{11})y} + h_2 e^{(s_2+x_{11})y} \\ & + h_3 e^{(s_3+x_{11})y} + h_4 e^{(s_4+x_{11})y} + h_5 e^{(s_1+x_{22})y} + h_6 e^{(s_2+x_{22})y} \\ & + h_7 e^{(s_3+x_{22})y} + h_8 e^{(s_4+x_{22})y}] \cos \pi z \quad (40) \end{aligned}$$

Constants of integration and other constants are not reported here for the sake of brevity.

Now after knowing the velocity field, we can calculate the skin friction components τ_x and τ_z in the main flow and transverse directions, respectively as

$$\begin{aligned} \tau(x) = \frac{d\tau_x^*}{\mu U_0} = & \left(\frac{du_0}{dy} \right)_{y=0} + \varepsilon \left(\frac{du_{11}}{dy} \right)_{y=0} \cos \pi z \\ = & c_3 r_1 + c_4 r_2 + z_1 x_{11} + z_2 x_{22} + \varepsilon [A_7 r_3 + A_8 r_4 + h_{26} t_2 \\ & + h_9 (s_1 + r_1) + h_{10} (s_2 + r_1) + h_{11} (s_3 + r_1) + h_{12} (s_4 + r_1) \\ & + h_{13} (s_1 + r_2) + h_{14} (s_2 + r_2) + h_{15} (s_3 + r_2) + h_{16} (s_4 + r_2) \\ & + h_{17} (s_1 + x_{11}) + h_{18} (s_2 + x_{11}) + h_{19} (s_3 + x_{11}) + h_{20} (s_4 + x_{11}) \\ & + h_{21} (s_1 + x_{22}) + h_{22} (s_2 + x_{22}) + h_{23} (s_3 + x_{22}) \\ & + h_{24} (s_4 + x_{22})] \cos \pi z \quad (41) \end{aligned}$$

$$\begin{aligned} \tau(z) = \frac{d\tau_z^*}{\mu U_0} = & \varepsilon \left(\frac{dw_1}{dy} \right)_{y=0} \\ = & -\frac{\varepsilon}{\pi} [A_1 s_1^2 + A_2 s_2^2 + A_3 s_3^2 + A_4 s_4^2] \sin \pi z \quad (42) \end{aligned}$$

From the temperature field we can obtain the heat transfer coefficient i.e. Nusselt number as:

$$\begin{aligned} Nu = \frac{dq_w^*}{K(T_1^* - T_0^*)} = & \left(\frac{d\theta_0}{dy} \right)_{y=0} + \varepsilon \left(\frac{d\theta_{11}}{dy} \right)_{y=0} \cos \pi z \\ = & c_1 x_{11} + c_2 x_{22} + \varepsilon [A_5 t_1 + A_6 t_2 + h_1 (s_1 + x_{11}) + h_2 (s_2 + x_{11}) \\ & + h_3 (s_3 + x_{11}) + h_4 (s_4 + x_{11}) + h_5 (s_1 + x_{22}) + h_6 (s_2 + x_{22}) \\ & + h_7 (s_3 + x_{22}) + h_8 (s_4 + x_{22})] \cos \pi z \quad (43) \end{aligned}$$

4. Results and Discussion

The effect of magnetic field on three dimensional convection flow of a viscous incompressible fluid through a vertical channel with transpiration cooling is analyzed. In order to study the effects of different parameters appearing in the flow problem, we have carried out analytical calculations for the main flow velocity distribution (u), temperature field (θ), cross flow velocity (w), skin friction ($\tau(x), \tau(z)$) and rate of heat transfer (Nu).

The main flow velocity profiles are presented in Fig. 1 to 3. From Fig. 1 it is interesting to note that when magnetic field strength M increased the velocity decreased near the stationary plate while that near the moving plate velocity is increased. The effects of a transverse magnetic field on an electrically conducting fluid gives rise to a resistive-type force called Lorentz force similar to drag force this force has the tendency to slow down the motion of the fluid in the channel. In Fig. 2 it can be interpreted that velocity increases with increase in Grashof number G and injection parameter S near the stationary plate but decreases near the moving plate. Fig. 3 shows that the velocity decreases throughout the channel with the increase of the slip parameter h . Also from

this figure it is shown that when heat source parameter α and m are increased the velocity near the stationary plate increased while that near the moving plate is decreased.

Temperature profiles are presented in Fig. 4 to 6. It is evident from Fig. 4 that increase in heat source parameter α increases the temperature field. Fig. 5 shows that the temperature field increases with increase in injection parameter S and m . From Fig. 6 it is clear that temperature field decreases with the increase in Prandtl number P . This is because the fluid is highly conductive for small value of Prandtl number. Physically, if Prandtl number increases, the thermal diffusivity decreases and this phenomenon lead to the decreasing manner of the energy transfer ability that reduces the thermal boundary layer.

The cross flow velocity component (w) is due to transverse sinusoidal injection velocity distribution applied through the plate at rest. This secondary flow component is shown in Fig. 7. It is interesting to note that magnetic field prevents back flow throughout the flow field.

The variations of skin friction components at the left plate versus suction parameter in the main flow and transverse directions are shown in Fig. 8 and 9. It was found from Fig. 8 $\tau(x)$ decreases with an increase in m, α, z up to the middle and thereafter $\tau(x)$ increases with increasing m, α, z . Also from this figure it is clear that the $\tau(x)$ decreases with the increase in slip parameter h . Fig. 9 shows that when z increases, $\tau(z)$ also increases.

Fig. 10 illustrates the variation of heat transfer Nu is plotted against the suction parameter S . It is clear from the figure that rate of heat transfer tends to increase by increasing the magnitude of suction parameter. The figure also showing that when strength of heat source parameter α and m are increased, the Nusselt number is increased.

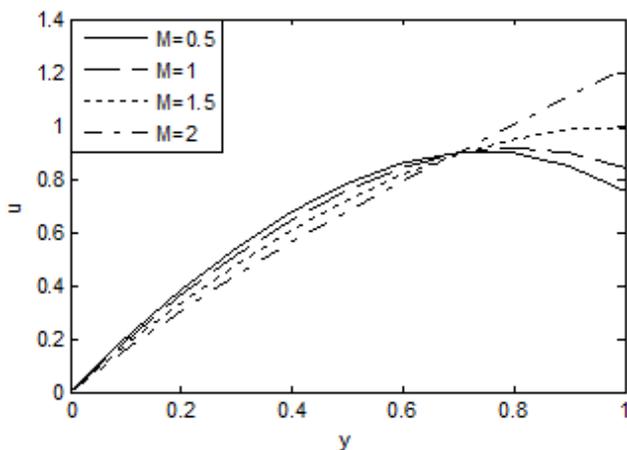


Figure 1: Main flow velocity distribution u plotted against y for different values of M and $\varepsilon=0.01, P=0.71, z=0.25, G=5, h=0.2, m=2, \alpha=1, S=0.5$.

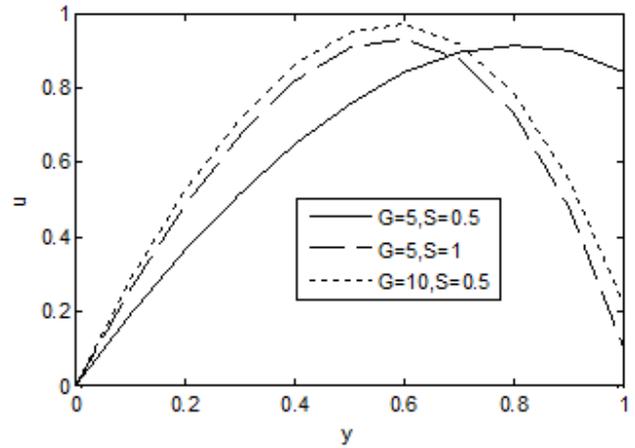


Figure 2: Main flow velocity distribution u plotted against y for different values of G, S and $M=1, h=0.2, m=2, \alpha=1, \varepsilon=0.01, P=0.71, z=0.25$.

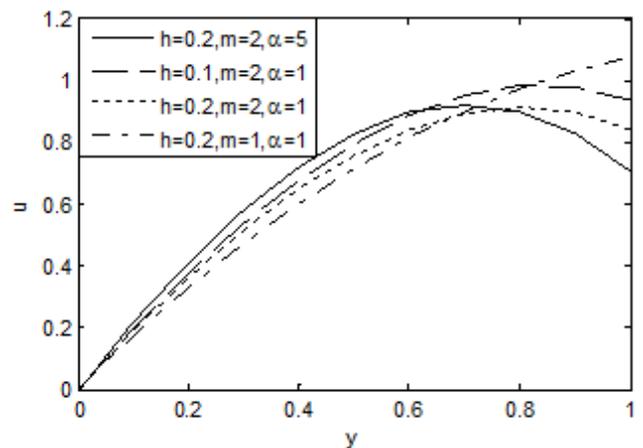


Figure 3: Main flow velocity distribution u plotted against y for different values of h, m, α and $M=1, \varepsilon=0.01, P=0.71, z=0.25, G=5, S=0.5$.

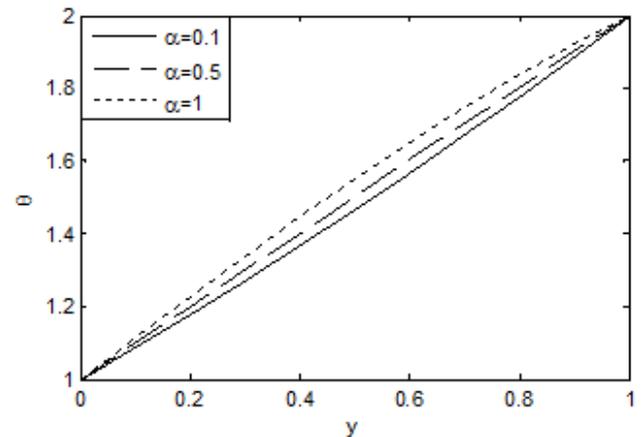


Figure 4: Temperature distribution θ plotted against y for different values of α and $M=1, \varepsilon=0.01, P=0.71, z=0.25, G=5, S=0.5, h=0.2, m=2$.

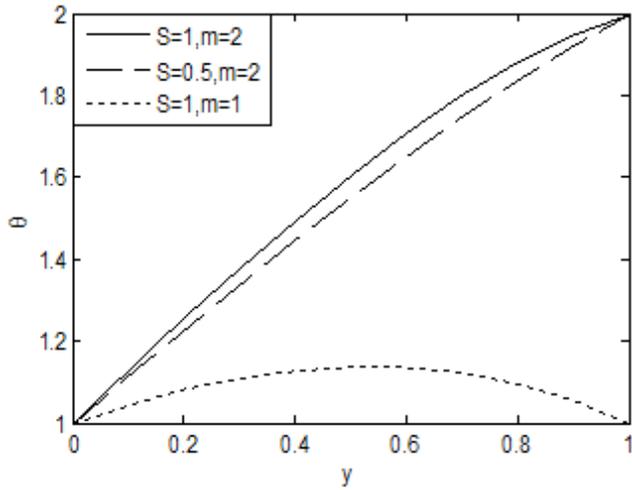


Figure 5: Temperature distribution θ plotted against y for different values of S, m and $h=0.2, \alpha=1, M=1, \varepsilon=0.01, P=0.71, z=0.25, G=5$.

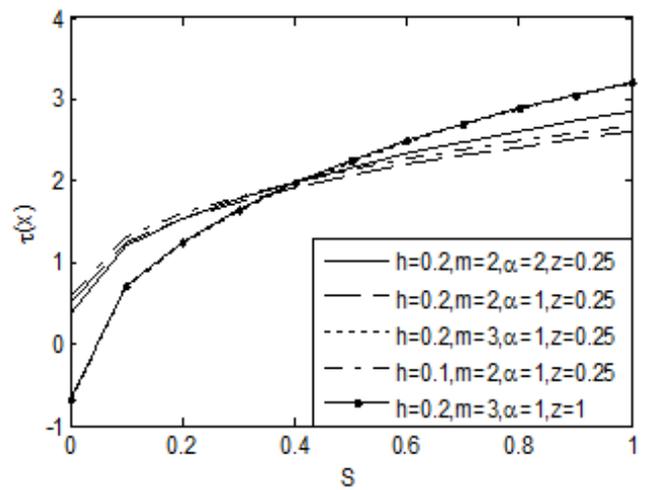


Figure 8: Skin friction $\tau(x)$ plotted against S for different values of h, m, α, z and $M=1, \varepsilon=0.01, P=0.71, G=5, S=0.5$.

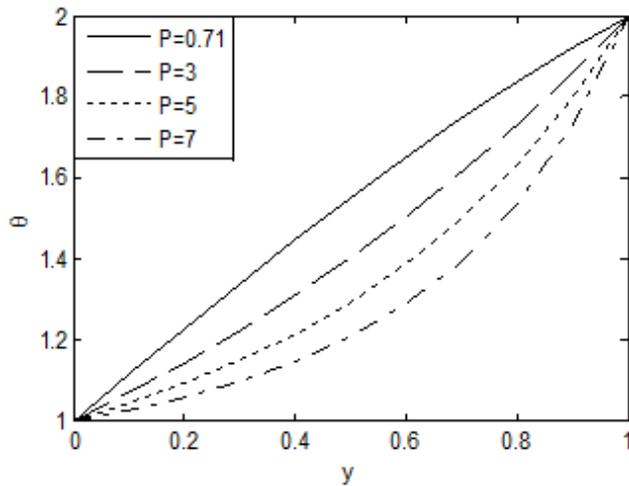


Figure 6: Temperature distribution θ plotted against y for different values of P and $h=0.2, \alpha=1, M=1, \varepsilon=0.01, m=2, z=0.25, G=5, S=0.5$.

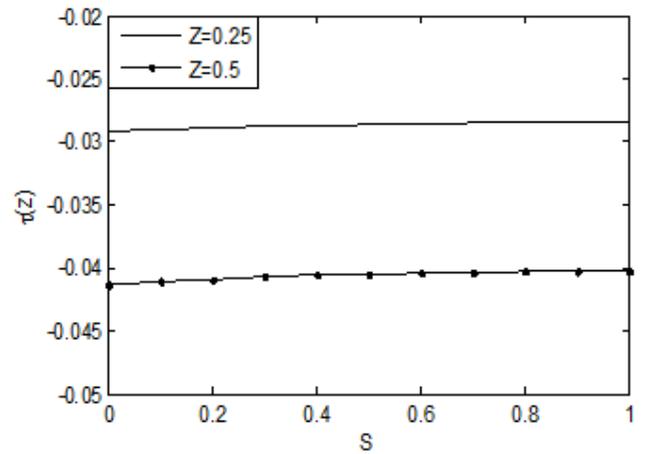


Figure 9: Skin friction $\tau(y)$ plotted against S for different values of z and $M=1, \varepsilon=0.01, P=0.71, G=5, S=0.5, h=0.2, m=2, \alpha=1$.

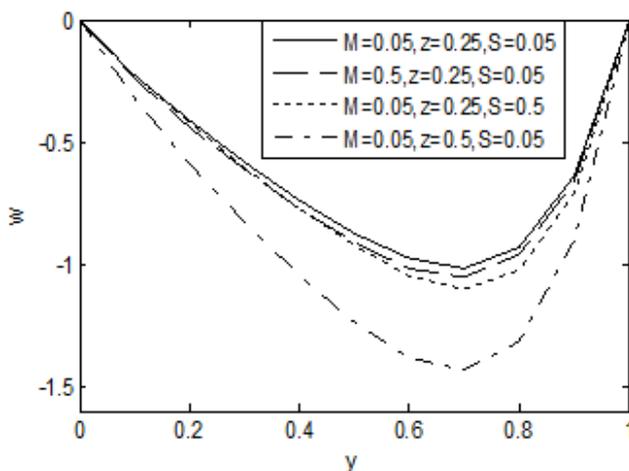


Figure 7: Cross flow velocity w plotted against y for different values of M, z, S and $h=0.2, \alpha=1, \varepsilon=0.01, m=2, P=0.71, G=5$.

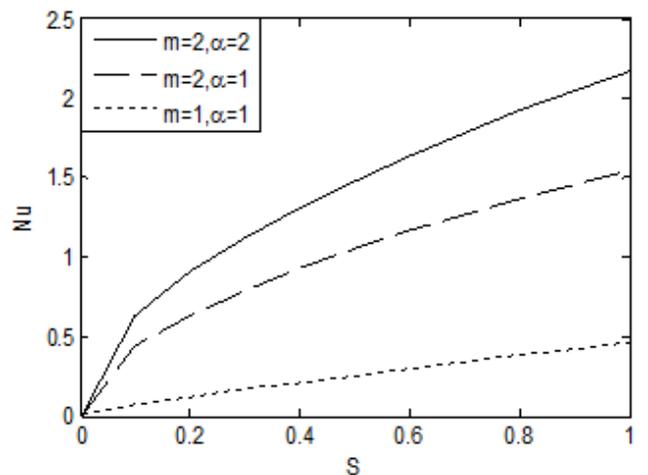


Figure 10: Nusselt number Nu plotted against S for different values of m, α and $M=1, \varepsilon=0.01, P=0.71, G=5, S=0.5, h=0.2, z=0.25$.

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