the main flow component u_1, and the temperature field θ_1. We assume v_1, w_1 and p_1 of the following form:

$$v_1(y,z) = v_{11}(y)\cos\pi z \quad (28)$$

$$w_1(y,z) = -\frac{1}{\pi}v'_{11}(y)\sin\pi z \quad (29)$$

$$p_1(y,z) = p_{11}(y)\cos\pi z \quad (30)$$

Where a prime denotes differentiation with respect to y. Equations (28) and (29) have been so choosen that the continuity Eq.(22) is satisfied. Substituting these equations into Eqs.(24) and (25) and applying the corresponding transformed boundary conditions, we get the solutions of v_1, w_1 and p_1 as

$$v_1(y,z) = \left[A_1e^{s_1y} + A_2e^{s_2y} + A_3e^{s_3y} + A_4e^{s_4y}\right]\cos\pi z \quad (31)$$

$$w_1(y,z) = -\frac{1}{\pi}[A_1s_1e^{s_1y} + A_2s_2e^{s_2y} + A_3s_3e^{s_3y} + A_4s_4e^{s_4y}]\sin\pi z \quad (32)$$

$$p_1(y,z) = \frac{1}{S}[A_1\left(s_1^2 - Ss_1 - \pi^2\right)e^{s_1y} + A_2\left(s_2^2 - Ss_2 - \pi^2\right)e^{s_2y} + A_3\left(s_3^2 - Ss_3 - \pi^2\right)e^{s_3y} + A_4\left(s_4^2 - Ss_4 - \pi^2\right)e^{s_4y}]\cos\pi z \quad (33)$$

Now assuming u_1 and θ_1 as

$$u_1(y,z) = u_{11}(y)\cos\pi z \quad (34)$$

$$\theta_1(y,z) = \theta_{11}(y)\cos\pi z \quad (35)$$

and substituting in Eqs.(23) and (26), we obtain the following equations:

$$u''_{11} - Su'_{11} - \left(\pi^2 + M^2\right)u_{11} = Sv_{11}u'_0 - SG\theta_{11} \quad (36)$$

$$\theta''_{11} - SP\theta'_{11} + \left(S\alpha - \pi^2\right)\theta_{11} = v_{11}SP\theta'_0 \quad (37)$$

with the corresponding boundary conditions:

$$\begin{aligned} y = 0:&\ u_{11} = 0,\ \theta_{11} = 0 \\ y = 1:&\ u_{11} = h_1u'_{11},\ \theta_{11} = 0 \end{aligned} \quad (38)$$

Substitution of u_{11} and θ_{11} obtained from Eqs.(36) and (37) under boundary conditions Eq.(38) in Eqs.(34) and (35), we get

$$u_1(y,z) = [A_7e^{r_3y} + A_8e^{r_4y} + h_{26}e^{t_2y} + h_9e^{(s_1+r_1)y} + h_{10}e^{(s_2+r_1)y} + h_{11}e^{(s_3+r_1)y} + h_{12}e^{(s_4+r_1)y} + h_{13}e^{(s_1+r_2)y} + h_{14}e^{(s_2+r_2)y} + h_{15}e^{(s_3+r_2)y} + h_{16}e^{(s_4+r_2)y} + h_{17}e^{(s_1+x_{11})y} + h_{18}e^{(s_2+x_{11})y} + h_{19}e^{(s_3+x_{11})y} + h_{20}e^{(s_4+x_{11})y} + h_{21}e^{(s_1+x_{22})y} + h_{22}e^{(s_2+x_{22})y} + h_{23}e^{(s_3+x_{22})y} + h_{24}e^{(s_4+x_{22})y}]\cos\pi z \quad (39)$$

$$\theta_1(y,z) = [A_5e^{t_1y} + A_6e^{t_2y} + h_1e^{(s_1+x_{11})y} + h_2e^{(s_2+x_{11})y} + h_3e^{(s_3+x_{11})y} + h_4e^{(s_4+x_{11})y} + h_5e^{(s_1+x_{22})y} + h_6e^{(s_2+x_{22})y} + h_7e^{(s_3+x_{22})y} + h_8e^{(s_4+x_{22})y}]\cos\pi z \quad (40)$$

Constants of integration and other constants are not reported here for the sake of brevity.

Now after knowing the velocity field, we can calculate the skin friction components τ_x and τ_z in the main flow and transverse directions, respectively as

$$\tau(x) = \frac{d\tau_x^*}{\mu U_0} = \left(\frac{du_0}{dy}\right)_{y=0} + \varepsilon\left(\frac{du_{11}}{dy}\right)_{y=0}\cos\pi z$$
$$= c_3r_1 + c_4r_2 + z_1x_{11} + z_2x_{22} + \varepsilon[A_7r_3 + A_8r_4 + h_{26}t_2 + h_9(s_1+r_1) + h_{10}(s_2+r_1) + h_{11}(s_3+r_1) + h_{12}(s_4+r_1) + h_{13}(s_1+r_2) + h_{14}(s_2+r_2) + h_{15}(s_3+r_2) + h_{16}(s_4+r_2) + h_{17}(s_1+x_{11}) + h_{18}(s_2+x_{11}) + h_{19}(s_3+x_{11}) + h_{20}(s_4+x_{11}) + h_{21}(s_1+x_{22}) + h_{22}(s_2+x_{22}) + h_{23}(s_3+x_{22}) + h_{24}(s_4+x_{22})]\cos\pi z \quad (41)$$

$$\tau(z) = \frac{d\tau_z^*}{\mu U_0} = \varepsilon\left(\frac{dw_1}{dy}\right)_{y=0}$$
$$= -\frac{\varepsilon}{\pi}[A_1s_1^2 + A_2s_2^2 + A_3s_3^2 + A_4s_4^2]\sin\pi z \quad (42)$$

From the temperature field we can obtain the heat transfer coefficient i.e. Nusselt number as:

$$Nu = \frac{dq_w^*}{K\left(T_1^* - T_0^*\right)} = \left(\frac{d\theta_0}{dy}\right)_{y=0} + \varepsilon\left(\frac{d\theta_{11}}{dy}\right)_{y=0}\cos\pi z$$
$$= c_1x_{11} + c_2x_{22} + \varepsilon[A_5t_1 + A_6t_2 + h_1(s_1+x_{11}) + h_2(s_2+x_{11}) + h_3(s_3+x_{11}) + h_4(s_4+x_{11}) + h_5(s_1+x_{22}) + h_6(s_2+x_{22}) + h_7(s_3+x_{22}) + h_8(s_4+x_{22})]\cos\pi z \quad (43)$$

4. Results and Discussion

The effect of magnetic field on three dimensional convection flow of a viscous incompressible fluid through a vertical channel with transpiration cooling is analyzed. In order to study the effects of different parameters appearing in the flow problem, we have carried out analytical calculations for the main flow velocity distribution (u), temperature field (θ), cross flow velocity (w), skin friction $(\tau(x), \tau(z))$ and rate of heat transfer (Nu).

The main flow velocity profiles are presented in Fig. 1 to 3. From Fig. 1 it is interesting to note that when magnetic field strength M increased the velocity decreased near the stationary plate while that near the moving plate velocity is increased. The effects of a transverse magnetic field on an electrically conducting fluid gives rise to a resistive-type force called Lorentz force similar to drag force this force has the tendency to slow down the motion of the fluid in the channel. In Fig. 2 it can be interpreted that velocity increases with increase in Grashof number G and injection parameter S near the stationary plate but decreases near the moving plate. Fig. 3 shows that the velocity decreases throughout the channel with the increase of the slip parameter h. Also from this figure it is shown that when heat source parameter α and m are increased the velocity near the stationary plate increased while that near the moving plate is decreased.

Temperature profiles are presented in Fig. 4 to 6. It is evident from Fig. 4 that increase in heat source parameter α increases the temperature field. Fig. 5 shows that the temperature field increases with increase in injection parameter S and m. From Fig. 6 it is clear that temperature field decreases with the increase in Prandtle number P. This is because the fluid is highly conductive for small value of Prandtl number. Physically, if Prandtl number increases, the thermal diffusivity decreases and this phenomenon lead to the decreasing manner of the energy transfer ability that reduces the thermal boundary layer.

The cross flow velocity component (w) is due to transverse sinusoidal injection velocity distribution applied through the plate at rest. This secondary flow component is shown in Fig. 7. It is interesting to note that magnetic field prevents back flow throughout the flow field.

The variations of skin friction components at the left plate versus suction parameter in the main flow and transverse directions are shown in Fig. 8 and 9. It was found from Fig. 8 $\tau(x)$ decreases with an increase in m, α, z up to the middle and thereafter $\tau(x)$ increases with increasing m, α, z Also from this figure it is clear that the $\tau(x)$ decreases with the increase in slip parameter h. Fig. 9 shows that when z increases, $\tau(z)$ also increases.

Fig. 10 illustrates the variation of heat transfer Nu is plotted against the suction parameter S. It is clear from the figure that rate of heat transfer tends to increase by increasing the magnitude of suction parameter. The figure also showing that when strength of heat source parameter α and m are increased, the Nusselt number is increased.

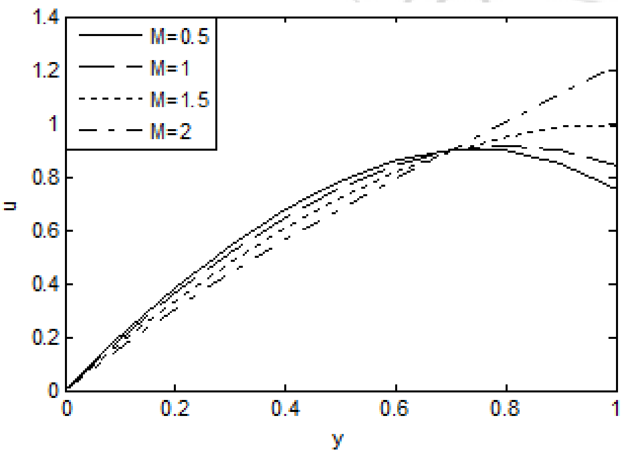

Figure 1: Main flow velocity distribution u plotted against y for different values of M and ε =0.01, P =0.71, z =0.25, G =5, h =0.2, m =2, α =1, S =0.5.

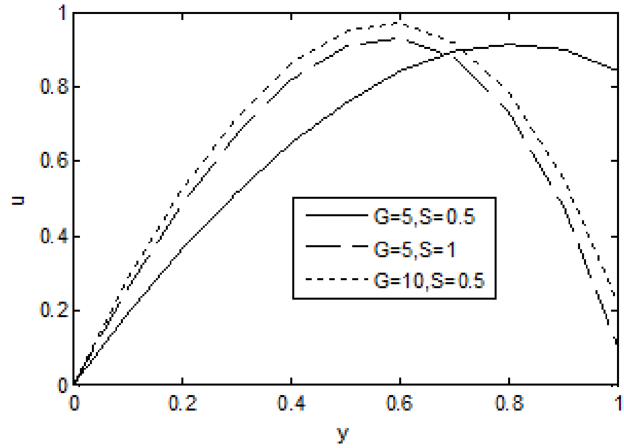

Figure 2: Main flow velocity distribution u plotted against y for different values of G, S and M =1, h =0.2, m =2, α =1, ε =0.01, P =0.71, z =0.25.

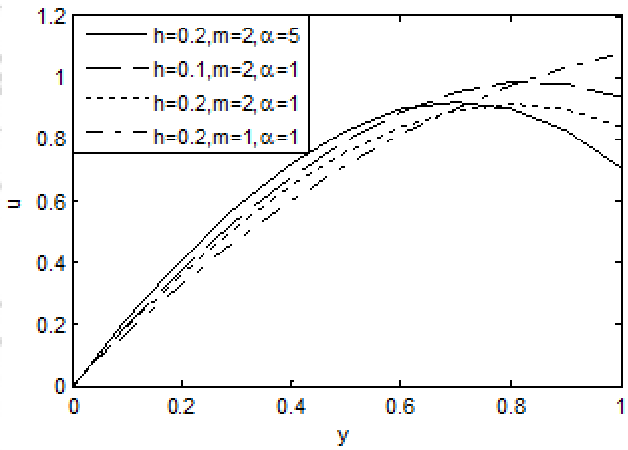

Figure 3: Main flow velocity distribution u plotted against y for different values of h, m, α and M =1, ε =0.01, P =0.71, z =0.25, G =5, S =0.5

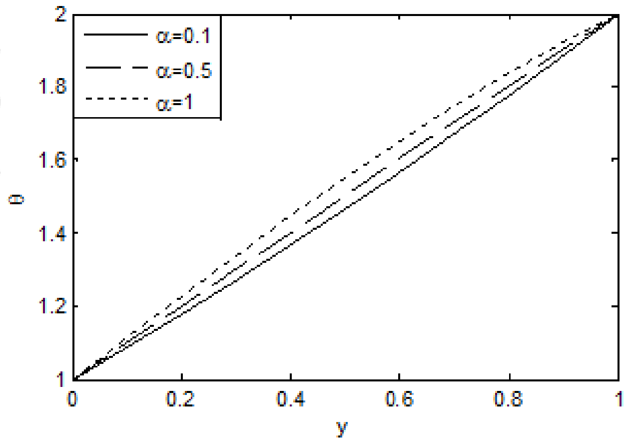

Figure 4: Temperature distribution θ plotted against y for different values of α and M =1, ε =0.01, P =0.71, z =0.25, G =5, S =0.5, h =0.2, m =2.

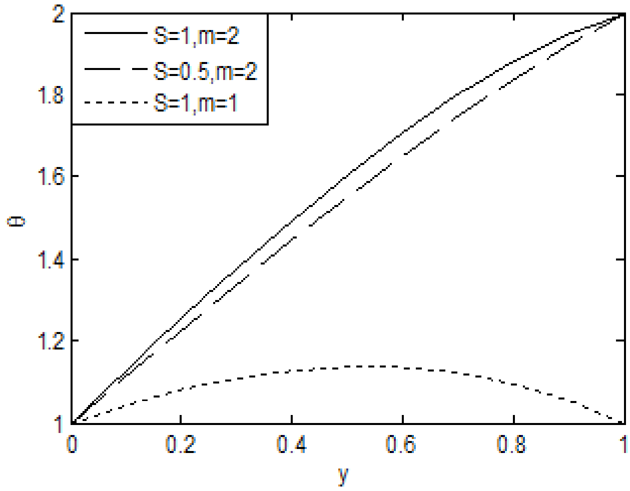

Figure 5: Temperature distribution θ plotted against y for different values of S, m and h =0.2, α =1, M =1, ε =0.01, P =0.71, z =0.25, G =5.

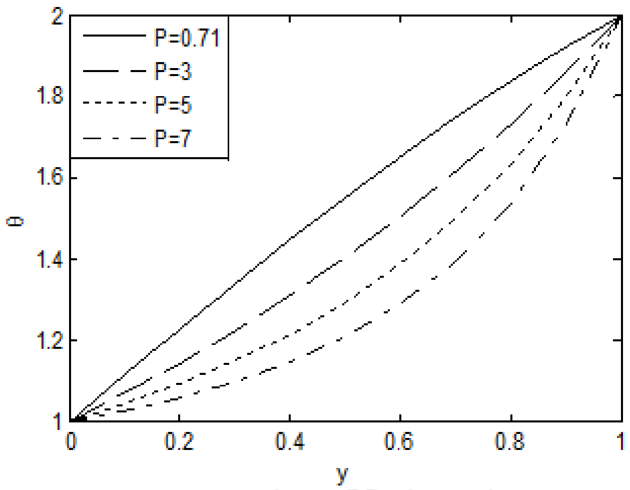

Figure 6: Temperature distribution θ plotted against y for different values of P and h =0.2, α =1, M =1, ε =0.01, m =2, z =0.25, G =5, S =0.5

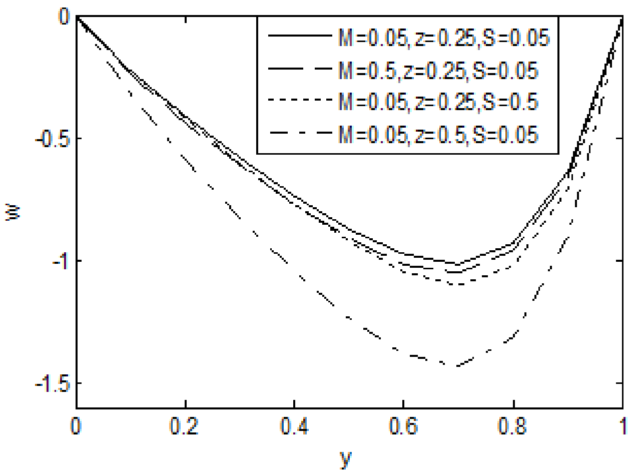

Figure 7: Cross flow velocity w plotted against y for different values of M, z, S and h =0.2, α =1, ε =0.01, m =2, P =0.71, G =5.

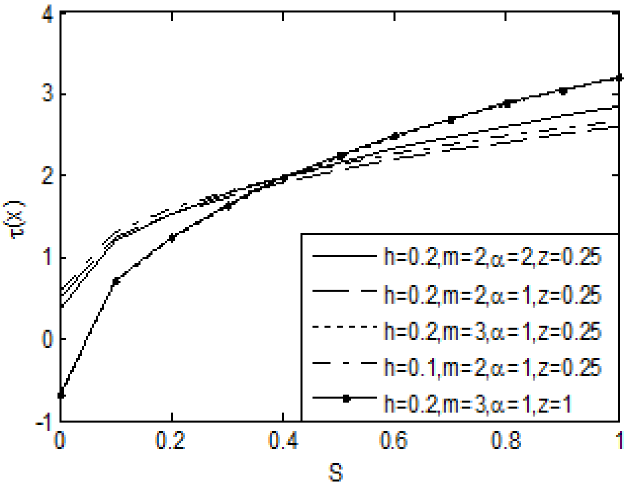

Figure 8: Skin friction $\tau(x)$ plotted against S for different values of h, m, α, z and M =1, ε =0.01, P =0.71, G =5, S =0.5.

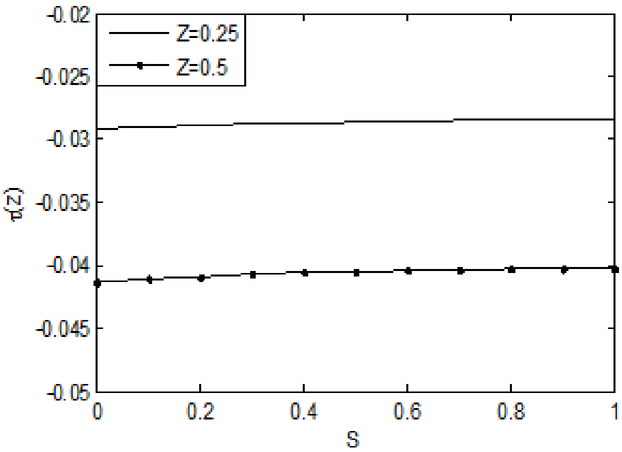

Figure 9: Skin friction $\tau(y)$ plotted against S for different values of z and M =1, ε =0.01, P =0.71, G =5, S =0.5, h =0.2, m =2, α =1

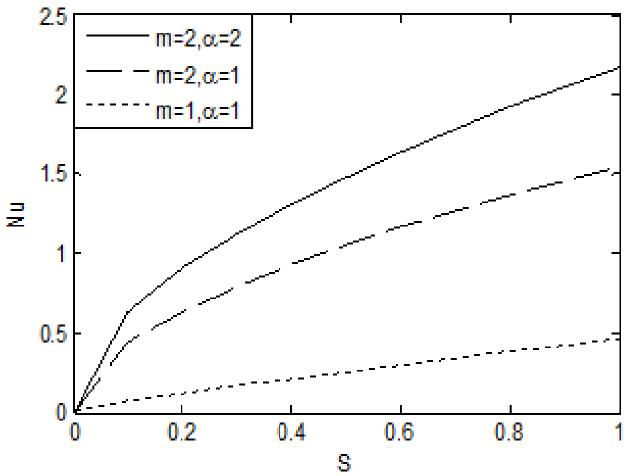

Figure 10: Nussult number Nu plotted against S for different values of m, α and M =1, ε =0.01, P =0.71, G =5, S =0.5, h =0.2, z =0.25.

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