Design and Optimization of Slider and Crank Mechanism with Multibody Systems

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Abstract: The slider-crank mechanism is considered as one of the most used mechanism in the mechanical field. It is found in pumps, compressors, steam engines, feeders, crushers, punches and injectors. Furthermore, the slider-crank mechanism is central to diesel and gasoline internal combustion engines, which play an indispensable role in modern living. It mainly consists of crank shaft, slider block and connecting rod. It works on the principle of converting the rotational motion of crank shaft to the translational motion of slider block. Over the past two decades, extensive work has been conducted on the kinematic and dynamic effects of the slider and crank mechanism in multibody mechanical systems. In contrast, little work has been devoted to optimizing the performance of mechanical systems. The slider and crank mechanism simulation model is developed using the design software MSC.ADAMS. Different simulations are performed at different crank speeds to observe the response of the reaction forces at joint R2 (joint between crank shaft and connecting rod). An innovative design-of-experiment (DOE)-based method for optimizing the performance of a mechanical system for different ranges of design parameters is then proposed. Based on the simulation model results the design parameters are predicted by an artificial intelligence technique. This allows for predicting the influence of design parameter changes, in order to optimize joint reaction forces and power requirements of the slider and crank mechanisms.

Keywords: Multibody system, ADAM, Slider Crank Mechanism.

1. Introduction

Multibody dynamics is based on classical mechanics and has a long and detailed history. The simplest multibody system is a free particle which can be treated by Newton’s equations published in 1686. D’Alembert considered a system of constrained rigid bodies where he distinguished between applied and reaction forces. A systematic analysis of constrained mechanical systems was established by Lagrange. Modern methods for the dynamic analysis of constrained multibody systems fall into two main categories: differential algebraic equations (DAEs) and ordinary differential equations (ODEs). DAEs employ a maximal set of variable to describe the motion of the system and use multipliers to model the constraint forces. Premultiplying the constraint reaction-induced dynamic equations by the orthogonal complement matrix to the constraint Jacobian results in the governing equations as ODEs. Numerous advances have been made during the last couple of centuries in theory and in methods of formulating the equations of motion.

The slider and Crank Mechanism is considered one of the most used systems in the mathematical field. The purpose of the mechanism is to convert the linear motion of the piston to rotational motion of the crank shaft. By definition: slider and crank mechanism is one type of four bar linkages which has three revolute joints and one sliding joint. In industry, many applications of planar mechanisms such as mechanism have been found in thousands of devices. A slider–crank mechanism is widely used in gasoline/diesel engines and quick-return machinery. Research works in analysis of the slider–crank mechanism have been investigated due to their significant advantages such as low cost, reduced number of parts, reduced weight and others. It kinematic analysis with multibody dynamics and its parametric optimization has been little studied when compared to the mechanisms.

Assad,(2012) presented the kinematic and dynamic analysis of slider crank mechanism. The slider crank mechanism is simulated in ADAMS software to observe the response of the slider block and the reaction forces at joint R2 (joint between crank shaft and connecting rod). The dynamic analysis has been performed by applying moment of 4.2 Nm at joint R1 (the revolute joint between connecting shaft and connecting plate). The applied moment is removed by imposing rotational motion at joint R1 with angular velocity of 6 rad/sec to perform dynamic analysis. These simulations were performed with different time steps and durations. The friction was assumed to be negligible during these simulations. As a result of this work, the longitudinal response of the slider block is observed with applied moment as well as slider block response along with reaction forces at joint R2 is investigated in case of imposed rotational motion. [11]

Sharma and Ranjan, (2013) analyzed of a four-bar mechanism is undertaken. In the analysis and design of mechanisms, kinematic quantities such as velocities and accelerations are of great engineering importance. Velocities and displacements give an insight into the functional behavior of the mechanism. The accelerations, on the other hand, are related to forces. The main theme of this paper are the modelling, computer-aided dynamic force analysis and simulation of four-bar planar mechanisms composed of rigid bodies and mass less force and torque producing elements. Modelling of planar four-bar mechanisms will be done by using the ADAMS software. By this software we can simulate their link at different positions and find the velocity and acceleration graph and compared with analytical
equations. Motions of the rigid bodies are predicted by numerically integrating Differential-Algebraic Equations (DAEs). ADAMS is more reliable software because it considers mass, centre of mass location and inertia properties on the links.[12]

Considering the triangle ABC, we can set up the following two equations and the third one required for the problem to be determinate is the driver equation specifying constant angular velocity of the crank:

\[ l_1 \cos(\theta_1) + l_2 \cos(\theta_2) = x \quad \text{(1)} \]
\[ l_1 \sin(\theta_1) + l_2 \sin(\theta_2) = 0 \quad \text{...... (2)} \]
\[ \theta_1 = \omega \quad \text{.......................... (3)} \]

Eq. (3) directly gives \( \theta_1 \), in time and solving Eq. (2), we can find \( \theta_2 \):

\[ \theta_2 = \arcsin \left( \frac{l_1}{l_2} \sin(\theta_1) \right) \quad \text{...... (4)} \]

Finally Eq. (1) allows to find \( x \) from \( \theta_1 \) and \( \theta_2 \).

To calculate the forces accurately, we need to find the acceleration of the mass \( \ddot{x} \). Equations that determine the acceleration can be found by differentiation of Eqn. (1) to (3) twice with respect to time. The first differentiation gives us equations that can be used to determine the velocities and, it generally is necessary to determine these first. The first differentiation leads to:

\[ -l_1 \sin(\theta_1) \dot{\theta}_1 - l_2 \sin(\theta_2) \dot{\theta}_2 = \ddot{x} \quad \text{...... (5)} \]
\[ l_1 \cos(\theta_1) \dot{\theta}_1 - l_2 \cos(\theta_2) \dot{\theta}_2 = 0 \quad \text{...... (6)} \]
\[ \ddot{\theta}_1 = \omega \quad \text{.............................. (7)} \]

Where the former two equations can be solved for \( \dot{\theta}_2 \) and \( \ddot{x} \):

\[ \dot{\theta}_2 = \frac{l_1 \cos(\theta_1)}{l_2 \cos(\theta_2)} \dot{\theta}_1 \quad \text{.......................... (8)} \]

\[ \ddot{x} = -l_1 \sin(\theta_1) \dot{\theta}_1 - l_2 \sin(\theta_2) \dot{\theta}_2 = -l_1 \sin(\theta_1) \dot{\theta}_1 - l_2 \sin(\theta_2) \left( \frac{l_1 \cos(\theta_1)}{l_2 \cos(\theta_2)} \right) \dot{\theta}_1 \]

We differentiate Eqn. (5) to (9) with respect to time, leading to

\[ -l_1 \sin(\theta_1) \ddot{\theta}_1 - l_2 \sin(\theta_2) \ddot{\theta}_2 - l_1 \cos(\theta_1) \dot{\theta}_1^2 - l_2 \cos(\theta_2) \dot{\theta}_2^2 = \ddot{x} \quad \text{...... (10)} \]
\[ l_1 \cos(\theta_1) \ddot{\theta}_1 - l_2 \cos(\theta_2) \ddot{\theta}_2 - l_1 \sin(\theta_1) \dot{\theta}_1^2 + l_2 \sin(\theta_2) \dot{\theta}_2^2 = 0 \quad \text{...... (11)} \]

\[ \ddot{\theta}_1 = 0 \quad \text{................................. (12)} \]

Where the former two equations are solved for \( \ddot{\theta}_2 \) and \( \ddot{x} \), yielding

\[ \dot{\theta}_2 = \frac{l_1 \cos(\theta_1)}{l_2 \cos(\theta_2)} \dot{\theta}_1 - l_1 \sin(\theta_1) \dot{\theta}_1^2 + l_2 \sin(\theta_2) \dot{\theta}_2^2 \quad \text{.......................... (13)} \]
\[ \ddot{x} = \left( -l_1 \sin(\theta_1) - l_2 \sin(\theta_2) \right) \left( \frac{l_1 \cos(\theta_1)}{l_2 \cos(\theta_2)} \right) \dot{\theta}_1 - l_1 \cos(\theta_1) \dot{\theta}_1^2 - l_2 \cos(\theta_2) \dot{\theta}_2^2 \quad \text{...... (14)} \]

2. Numerical Method

In this section a computer model for the classic slider-crank mechanism is considered to analyze the behavior of the mechanical system for the Figure 2. The multibody model has four ideal joints. In which three are revolute joints and one translational joint. The revolute joints are existed between the ground and the crank, the crank and coupler and at the slider pin and the translational joint between the ground and coupler. The geometric and inertia properties of each body in this system are shown in Table 1

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**Figure 1:** Slider-Crank Mechanisms with Kinematic Coordinates

According to Figure 1, we introduce three coordinates to describe the configuration of the mechanism. In principle, we only need one coordinate, the most obvious choice being \( \theta_1 \), but since there is not an obvious connection between \( \theta_1 \) and the complete configuration of the mechanism, we use more coordinates, i.e., \( \theta_1 \), \( \theta_2 \) and \( x \). The kinematic analysis now aims at finding the relationship between the three coordinates and other required kinematical information, such as motion of centre of mass of the bodies or the like. In this case, we will primarily be interested in the motion of the slider, being the only mass in the system.
3. Results

An innovative design-of-experiment (DOE) based method for optimizing the performance of a mechanical system for different ranges of design parameters is proposed in Table 2 to optimize the performance of slider and crank mechanism.

Table 2: Properties of Mechanism

<table>
<thead>
<tr>
<th>Body</th>
<th>Length (m)</th>
<th>Height (m)</th>
<th>Depth (m)</th>
<th>Moment of Inertia (kg-m²)</th>
<th>Mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crank</td>
<td>0.31</td>
<td>0.04</td>
<td>0.02</td>
<td>0.4</td>
<td>2</td>
</tr>
<tr>
<td>Coupler</td>
<td>---</td>
<td>0.04</td>
<td>0.02</td>
<td>0.75</td>
<td>6</td>
</tr>
<tr>
<td>Slider</td>
<td>0.2</td>
<td>0.07</td>
<td>0.1</td>
<td>0.75</td>
<td>8</td>
</tr>
<tr>
<td>Base</td>
<td>1.2</td>
<td>0.05</td>
<td>0.1</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Usually the design process is treated as an optimization problem. To each user specified performance requirement is associated a performance index whose value increases with its level of violation. The joint reaction forces and power consumption are considered as input and the outputs are design parameters and crank speed. The data from Table 4 are used to build the NN model.

Table 3: Simulation Results

<table>
<thead>
<tr>
<th>Set</th>
<th>Speed (rpm)</th>
<th>Joint reaction force in X-Direction (N)</th>
<th>Joint reaction force in Y-Direction (N)</th>
<th>Power Consumption (N-m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set-1</td>
<td>1000</td>
<td>1000</td>
<td>750</td>
<td>5000</td>
</tr>
<tr>
<td>Set-1</td>
<td>2000</td>
<td>5000</td>
<td>3000</td>
<td>40,000</td>
</tr>
<tr>
<td>Set-2</td>
<td>3000</td>
<td>10,000</td>
<td>7500</td>
<td>1.5×10⁷</td>
</tr>
<tr>
<td>Set-2</td>
<td>1000</td>
<td>750</td>
<td>600</td>
<td>3000</td>
</tr>
<tr>
<td>Set-2</td>
<td>2000</td>
<td>3000</td>
<td>2300</td>
<td>25,000</td>
</tr>
<tr>
<td>Set-2</td>
<td>3000</td>
<td>7500</td>
<td>5000</td>
<td>1×10⁷</td>
</tr>
<tr>
<td>Set-3</td>
<td>1000</td>
<td>750</td>
<td>350</td>
<td>2500</td>
</tr>
<tr>
<td>Set-3</td>
<td>2000</td>
<td>3000</td>
<td>1500</td>
<td>20,000</td>
</tr>
<tr>
<td>Set-3</td>
<td>3000</td>
<td>6250</td>
<td>3000</td>
<td>70,000</td>
</tr>
<tr>
<td>Set-4</td>
<td>1000</td>
<td>550</td>
<td>225</td>
<td>1100</td>
</tr>
<tr>
<td>Set-4</td>
<td>2000</td>
<td>2400</td>
<td>850</td>
<td>9000</td>
</tr>
<tr>
<td>Set-4</td>
<td>3000</td>
<td>5000</td>
<td>2000</td>
<td>30,000</td>
</tr>
</tbody>
</table>

In this work, a method of artificial neural network applied for the solution of performance indices to predict the output values. The input layer in NN has three nodes which take the values. The input layer in NN has three nodes which take the value of design parameters and crank speed. The data from Table 4 are used to build the NN model.

Table 4: Setting simulation model parameters

<table>
<thead>
<tr>
<th>Input data</th>
<th>Output data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joint reaction force in X-Direction (N)</td>
<td>Joint reaction force in Y-Direction (N)</td>
</tr>
<tr>
<td>1000</td>
<td>750</td>
</tr>
<tr>
<td>5000</td>
<td>3000</td>
</tr>
<tr>
<td>10,000</td>
<td>7500</td>
</tr>
<tr>
<td>750</td>
<td>600</td>
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<tr>
<td>3000</td>
<td>2300</td>
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<tr>
<td>7500</td>
<td>5000</td>
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<td>750</td>
<td>350</td>
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<td>3000</td>
<td>1500</td>
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<tr>
<td>6250</td>
<td>3000</td>
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<tr>
<td>5000</td>
<td>2000</td>
</tr>
</tbody>
</table>

In this work, a method of artificial neural network applied for the solution of performance indices to predict the output values. The input layer in NN has three nodes which take the value of design parameters and crank speed. The data from Table 4 are used to build the NN model.

Finally, numerical results obtained from two application examples with different design parameters, crank speed are presented for the further analysis of the mechanical system. This allows for predicting the influence of design parameter changes, in order to minimize reaction forces, accelerations, and power requirements. Table 3 shows the simulation results for the slider and crank mechanism.
Coupler length (m)

0.15
0.17
0.19
0.21
0.23
0.25
0.27
0.29
0.31
0.33

No.of Points

Figure 3: Crank length

Crank length (m)

0.15
0.17
0.19
0.21
0.23
0.25
0.27
0.29
0.31
0.33

No.of Points

Figure 4: Coupler lengths

4. Conclusions

The NN-model was used to replace the computer simulation experiment as a cost-effective mathematical tool for optimizing the system performance. This research was focused on using the design-of-experiment method to develop a NN-model instead of the computer simulation model. The use of the NN model allowed the prediction of the system’s response at other design points with a significantly lower computational time and cost. For the studied mechanism, the predictions were shown to be within 5% of the actual values from dynamic simulations, for which close to an hour of computational time is to be spent for each simulation. In addition to the use of the NN model for the prediction of the response at different design points, the scheme allows for the visualization of the trends of the response surfaces when the design variables are changed. The global results obtained from this study indicate that the dynamic behavior of the mechanical system is quite sensitive to the crank speed. The contact force is increased when the crank speed increases and the decrease in crank speed tends to make the results more noisy. The method presented in this thesis can be utilized for optimizing the performance of mechanical systems with joint clearances. By utilizing the NN-model, the computer simulation time can be significantly reduced, while the response of the system can be studied and optimized for a range of input design variables. Thereby based on simulation and analysis by MSC Adams view software and optimization based on NN technique we have attained optimized result based on length of crank and coupler. These results are validated using error found in NN optimization tool, MATLAB.

The optimized results of crank and coupler length helps in achieving less amount of power consumption and joint forces at the joint and relatively lesser cost of material.

References