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Stochastic Model for Hotel Revenue Management

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Abstract: ***This study presents several revenue optimisation models for hotel room reservations for a future target day with multiple-day stays. Assume that the hotel has only one type of room but the unit rate for the room may be different during every booking period and every reservation may cover several days. A stochastic programming model with semi-absolute deviations for measuring the risk for hotel revenue under an uncertain environment is proposed. And this stochastic model can be changed into a linear programming model by applying linearisation techniques. Numerical examples are presented to illustrate the efficiency of these models.***

Keywords: revenue management; semi-absolute deviation; stochastic programming.

1. Introduction

In this research, we propose several models for optimal booking of hotel rooms for a future target day. Assume that the hotel has only one type of rooms, the rate for the room may be different for different booking periods, and every reservation may cover several days, though the length of stay during each period may be different. First, a basic deterministic linear programming model for solving rooms' booking is formulated. In this model, the demand is assumed to be known. Then, a stochastic programming model with risk measure is proposed because of uncertainty of demand, and a stochastic programming model with semi-absolute deviations, considering cancellations and no-shows, is also provided.

2. Literature Review

Recently, the revenue management strategies in other industries have been studied. Kimms and Muller-Bungart (2007) propose a planning problem at a broadcasting company. Mangani (2007) derives an optimal ratio between advertising and sales income when a publisher maximises its profits on advertising space and product price. A mobile TV service bundle problem-based pricing strategy is provided by Rautio, Anttila and Tuominen (2007). A number of human-related factors in the design of an efficient revenue management system in a local subsidiary of Multinational Corporation are highlighted by Zarraga-Oberty and Bonache (2007). And a comprehensive review of the recent development of the revenue management in different industries and some important areas that warrant further research are provided by Chiang, Chen and Xu (2007). More different revenue management strategies including pricing, auctions, capacity control, overbooking and forecasting are discussed in their research.

However, most of the above-mentioned models do not consider the risk of fluctuating revenues, except in Lai and Ng (2005). As we know, demand for hotel rooms is uncertain and, therefore, a decision maker may face several demand scenarios in the decision making process. He (or she) needs to consider the risk involved under different scenarios. In this paper, an optimal strategy for renting hotel rooms is provided for situations when a decision maker faces random customer arrivals. Because of uncertain demand, the decision maker may face different demand scenarios; a stochastic programming model with semi-absolute deviations to measure a hotel's revenue risk is formulated, and a stochastic programming model that considers cancellations and no-shows is also provided.

3. Deterministic Linear Programming Models

Liu,Lai,Wang (2008) had showed that one characteristic of the hotel revenue management is that the products under review are perishable. If hotel rooms are not rented on the target day, revenue earned will be zero. So the rooms should be reserved in advance and, especially when supply exceeds demand, the decision maker should determine the right number of rooms to be booked in advance, so as to maximise revenue. During every reservation period, when a potential customer requests a room, the manager has to decide whether or not to rent it in advance to that potential customer. When making this decision, the manager does not know how many additional potential customers, who may be higher paying, will arrive on that particular day. So the decision that the hotel room manager faces is whether to accept a reservation request. This research presents several revenue optimisation models for hotel room reservations for a future target day with multiple-day stays. The booking horizon is divided into several periods. Assume that the hotel has only one type of rooms but the unit rate for the room may be different during every booking period and every reservation may cover several days. Major notations for parameters and variables used in this paper are as follows:

- C is the room capacity of the hotel.
- T is the index for the number of periods in advance of the booking dates, $t = 1, \ldots,$
- T. Period $t = T$ is the start of the booking horizon and period $t = 1$ is the end of the reservation period. In this paper, the time span of each period may not be equal, but period $t = 1$ covers only one day that represents the walk-in day.
- i is the index for the number of days that the customer will stay in the hotel, $i = 1, \ldots,$
- In other words, the customer will stay in the hotel room, which he (or she) has reserved, for i days.
- xti is the number of rooms that the hotel books out during period t for i days.
- Dti is the booking demand, which is reserved during period t for i days.
- rti is the revenue gained per booking made in period t for i days.

- C, xti and Dti are integers, and xti is the decision variable in this paper.

The basic mathematical model for hotel room booking strategy is

$$\max \sum_{t=1}^{T}\sum_{i=1}^{I} r_{ti}x_{ti}$$

$$\text{s.t.} \quad \sum_{t=1}^{T}\sum_{i=1}^{I} x_{ti} \le C$$

$$x_{ti} \le D_{ti}, \quad t=1,\dots,T; i=1,\dots,I$$

$$x_{ti} \ge 0, \quad t=1,\dots,T; i=1,\dots,I,$$

4. Stochastic Programming Models with Semi-Absolute Deviations

Liu,Lai,Wang (2008) had showed the stochastic model for hotel management as below:

Assume that a decision maker faces a set of scenarios $s \in \Omega = \{1,\dots,S\}$ with unknown parameters and for each scenario, the corresponding probability is p_s, such that $p_s \ge 0$ and $\sum_{s=1}^{S} p_s = 1$. Then:

- D_{ti}^{s} is the demand during period t for i days in scenario s.
- r_{ti}^{s} is the revenue gained per booking made in period t for i days in scenario s.

A stochastic programming model can be obtained; and in this model, we use a semi-absolute deviation to measure the risk of revenue falling below the expected revenue. Denote this as

$$\max \sum_{s=1}^{S} p_s \sum_{t=1}^{T}\sum_{i=1}^{I} r_{ti}^{s}x_{ti} - \lambda \sum_{s=1}^{S} p_s \left| \min\left\{0, \sum_{t=1}^{T}\sum_{i=1}^{I} r_{ti}^{s}x_{ti} - \sum_{s=1}^{S} p_s \sum_{t=1}^{T}\sum_{i=1}^{I} r_{ti}^{s}x_{ti}\right\}\right|$$

$$-\sum_{s=1}^{S} p_s \sum_{t=1}^{T}\sum_{i=1}^{I} w_{ti} \left|\min\left\{0, D_{ti}^{s} - x_{ti}\right\}\right| \qquad (3)$$

$$\text{s.t.} \quad \sum_{t=1}^{T}\sum_{i=1}^{I} x_{ti} \le C$$

$$x_{ti} \le \max_{s}\left\{D_{ti}^{s}\right\}, \quad t=1,\dots,T; i=1,\dots,I; s=1,\dots,S$$

$$x_{ti} \ge 0, \quad t=1,\dots,T; i=1,\dots,I$$

where λ and w_{ti} are non-negative weighting factors.

The first term in the objective function of Equation (3) is the expected revenue of the hotel, and the second term is the semi-absolute deviation of the revenue. We use the semi-absolute deviation model to measure the hotel's revenue risk, without considering the risk of exceeding the expected revenue. Parameter can be regarded as a risk a version factor (risk trade-off factor) for the decision maker. Different values of the risk factor represent the different risk aversions among decision makers. In the following section, we will show that the expected revenue declines when the risk-aversion factor increases.

The semi-absolute deviation in the third term of Equation (3) is a model robustness measurement, without considering booking numbers below the corresponding demand, and the parameters wti are the penalty factors for constraint violations. By using semi-absolute deviation values as penalties, the model can generate solutions which are robust in all scenarios. The decision maker may improve room occupancy by changing the corresponding weighting wti. In order to solve this model, we change it into a linear programming model, using a linearisation method as shown below.

If we take into account cancellations and no-shows and assume that the probabilities of cancellations and no-shows are Pc and Pn, respectively, then we can change the first constraint of Equation (3) to

$$\sum_{t=1}^{T}\sum_{i=1}^{I} (1-P_c-P_n)x_{ti} \le C.$$

A stochastic programming model for solving room booking strategy, considering cancellations and no-shows, can be described as

$$\max \sum_{s=1}^{S} p_s \sum_{t=1}^{T}\sum_{i=1}^{I} r_{ti}^{s}x_{ti} - \lambda \sum_{s=1}^{S} p_s \left| \min\left\{0, \sum_{t=1}^{T}\sum_{i=1}^{I} r_{ti}^{s}x_{ti} - \sum_{s+1}^{S} p_s \sum_{t=1}^{T}\sum_{i=1}^{I} r_{ti}^{s}x_{ti}\right\}\right|$$

$$-\sum_{s=1}^{S} p_s \sum_{t=1}^{T}\sum_{i=1}^{I} w_{ti} \left|\min\left\{0, D_{ti}^{s} - x_{ti}\right\}\right|$$

$$\text{s.t.} \quad \sum_{t=1}^{T}\sum_{i=1}^{I} (1-P_c-P_n)x_{ti} \le C$$

$$x_{ti} \le \max_{s}\left\{D_{ti}^{s}\right\}, \quad t=1,\dots,T; i=1,\dots,I; s=1,\dots,S$$

$$x_{ti} \ge 0, \quad t=1,\dots,T; i=1,\dots,I.$$

5. Illustrative Examples

Suppose that the decision maker will set the room rate per unit in each respective booking period fixed by the method most simple, ie assume that the simplest reply was fulfilled, linear, no Stochastic. Suppose room rates remain at 40 dollars, the cancellation fee amounting to 5 Dollar / room, demand remains per day with a capacity of 14 rooms 200. It will be obtained,
Revenue = 14 * 40 Dollar-5 Dollar * Max (0, (200-14)),
Revenue = 382.23 Dollar.

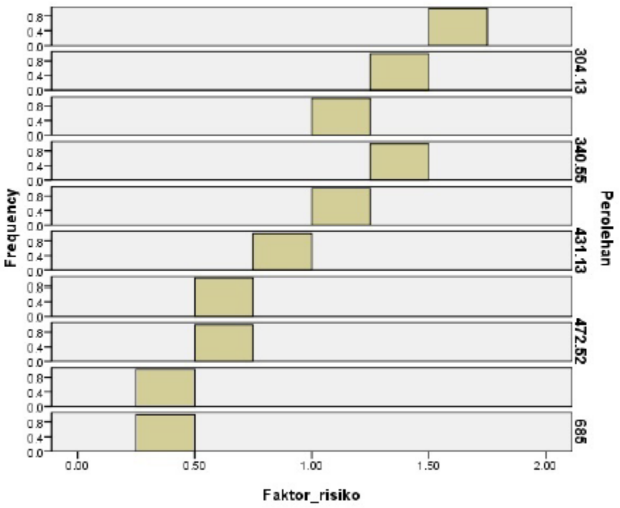
Assume that the decision maker faces two scenarios. Suppose that the decision makers will set the room rate per unit in each each reservation period fixed by the probability 0.76,0.78,0.80, and 0.82 for both scenarios. Risk factors in the sample is equal to one. For ease, all the weight of WTI is set to 1. Under request each scenario is listed in Tables 1 and 2, and the optimal solution obtained by determining the acquisition of the majority (Table 1 and 2).

Tabel 1 Tabel permintaan (skenario 1)

T	i=1	i=2	i=3	i=4	Fixed Order	Perolehan(Dollar)
1	11	17	16	7	11,24	493,80
2	8	14	15	9	14,26	358,24
3	15	17	17	20	14,86	331,07
4	13	13	13	11	12,64	431,13
5	13	12	14	10	12,57	434,18

Tabel 2 Tabel permintaan (skenario 2)

T	i=1	i=2	i=3	i=4	Fixed Order	Perolehan(Dollar)
1	12	12	15	7	7	685
2	12	14	15	17	14,65	340,55
3	10	9	15	6	15,46	304,13
4	16	13	12	13	11,72	472,52
5	7	12	16	6	15,66	295,39



The histogram in Figure 4.2 shows the relationship between factors risks and hotel acquisition obtained. Based on the histogram can concluded that the expected recovery is not comparable with the growth risk factor. It means that if the smaller the risk factor acquisition the greater obtained, and vice versa. It is possible influences several factors, such as use of the cost of electricity, water, number of employees, WiFi / HotSpot, laundry services has only a slight difference for booking a number of x rooms for i days due to the use by the network system. means if more rooms were booked, the greater the gain, and the number empty room getting a bit that indicates the smaller the risk factors.

6. Conclusion

This paper studies a basic deterministic programming model and a stochastic programming model with semi-absolute deviations for solving room reservations on a future target day, for managing hotel revenue. In the stochastic optimisation model, the semi-absolute deviation depicts the degree of the expected revenue deviation, while any revenue exceeding the expected amount is not considered. As discussed in Section 3, this method can be changed to a linear programming model, using linearisation techniques. From the results of the examples illustrated in this paper, we conclude that the expected revenue is not increasing when the risk-aversion factor grows.

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