

# Design of Cosine Modulated Filter Bank Using Unconstrained Optimization Technique

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**Abstract:** This paper proposes a unique method for designing of cosine modulated-QMF bank using unconstrained optimization technique. Objective function of this QMF bank is a quadratic equation form by combining the total pass band error, mean square error of band pass and stop band. We are using the unconstrained optimization method for minimizing the objective function. Result shows that the performance is better than the existing method.

**Keywords:** Prototype filter Cosine modulation, high band width, NOI

## 1. Introduction

These day researchers are giving a lot of attention to the QMF bank because of its wide range of application in the field of digital signal processing. Cosine modulated QMF bank is one of the important subclass of the Multi-rate filter banks. It is being extensively used because of fewer complexity, ease in designing and low cost. Some applications of QMF banks are sub-band coding of speech and image signals [1–6], speech and image compression [7,8], trans-multiplexers [9–11], equalization of wireless communication channels [12], source coding for audio and video signals [13], design of wavelet bases [14], sub-band acoustic echo cancellation [15], and discrete multi-tone modulation systems [16].

The most frequently used filter bank among all the M-band filter banks is cosine-modulated filter bank (CMFB) because the designing is easy and more realizable than that of any other filter banks [3, 6]. In CMFB all the filters are cosine modulated version of a low-pass prototype filter. So the design of whole filter bank reduces to the design of a single prototype filter. Implementation of CMFB consists of one prototype and discrete cosine transforms (DCT). Near perfect reconstruction (NPR) finite impulse response (FIR) CMFB avoids computation of large matrix sets. There are two types of CMFB one is with perfect reconstruction [7] and the other is pseudo-QMF [8]. Unlike PR filter banks, in pseudo-QMF aliasing is canceled approximately and the distortion is approximately a delay [3, 6] and the approximation improves with the filter order. It could be also called a special case of perfect reconstruction QMF bank.

It is found in many signals that their energy is dominantly concentrated in a particular region of frequency. To save the bandwidth in such signals, signal can be compressed and decimated. But simple way of compression affects the quality of the signal.

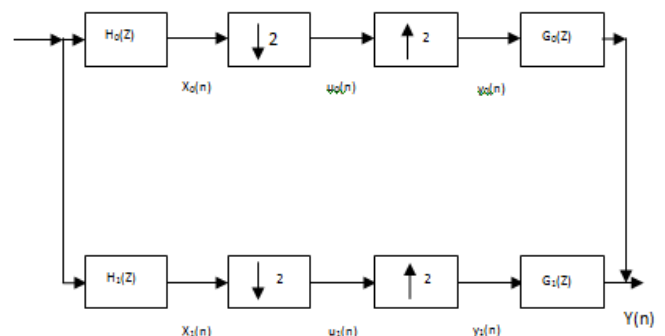


Figure 1: Basic structure of two channel QMF

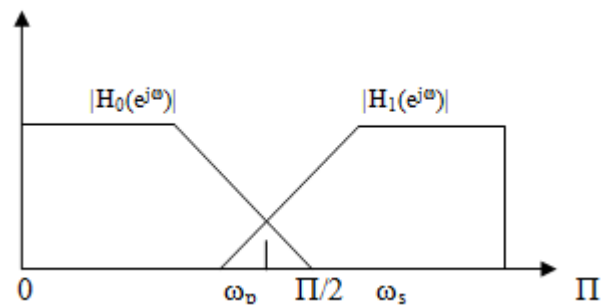


Figure 2: Frequency response of the filter

So Quadrature Mirror Filter (QMF) comes as a solution to this problem as it saves the bandwidth and increases efficiency without compromising the quality of the signal.

Figure 1 shows the typical two channel QMF bank and figure 2 shows the frequency response of this filter. This filter splits the signal into two sub-bands using high-pass and low-pass filters  $H_1(Z)$  and  $H_0(Z)$  respectively. These signals are decimated, coded and transmitted on the transmitter side. These signals are decoded, interpolated and then finally passed through the filters to recombine the signals. The reconstructed signal is not exact replica of the transmitted signal; it suffers from three types of distortions: amplitude distortion (AMD), phase distortion (PHD) and aliasing distortion (ALD).

Most of the researchers stresses on the elimination or minimization of these distortions to obtain the perfect

reconstruction (PR) or near perfect reconstruction (NPR) system [4].

Aliasing can be cancelled completely by choosing the appropriate synthesis filters with respect to the analysis filter phase distortion can be eliminated by using the linear phase FIR filters [5-6]. Amplitude distortion can be reduced using computer aided techniques [7].

Most of the researchers stresses on the elimination or minimization of these distortions to obtain the perfect reconstruction (PR) or near perfect reconstruction (NPR) system [8].

## 2. Basic Analysis of Filter Bank

General equation for the two channels QMF is given as

$$Y(Z) = \frac{1}{2} [H_0(Z)G_0(Z) + H_1(Z)G_1(Z)]X(Z) + \frac{1}{2} [H_0(-Z)G_0(Z) + H_1(-Z)G_1(Z)]X(-Z) \quad (1)$$

The second term in the above equation represents the alias component in the output.

Alias component  $X(-Z)$  can be removed by choosing the synthesis filter such that to make the second term zero such as

$$G_0(Z) = H_1(-Z) \text{ and } G_1(Z) = -H_0(-Z) \quad (2)$$

After removing the alias component from the equation (1) overall transfer function is given by

$$T(Z) = \frac{Y(Z)}{X(Z)} = \frac{1}{2} [H_0(Z)G_0(Z) + H_1(Z)G_1(Z)] \quad (3)$$

$Y(n)$  suffers from amplitude distortion (AMD) (if  $|T(Z)|$  is not constant for all value of  $\omega$ ), and it would suffer from phase distortion (PHD) (if  $T(Z)$  doesn't has linear phase). So to eliminate AMD  $|T(Z)|$  should be all pass and to eliminate PHD  $T(Z)$  should be linear phase.

\*When all the filters involved are linear-phase filters transfer function of the QMF  $T(Z)$  would behave like linear phase filter that would remove the phase distortion in the QMF.

The relation of analysis and synthesis to each other is given as [9-10]

$$H_1(Z) = H_0(Z), G_0(Z) = H_0(Z), G_1(Z) = -H_1(Z) = -H_0(-Z) \quad (4)$$

Using equation (3) and (4), we get

$$T(Z) = \frac{1}{2} [H_0^2(Z) - H_0^2(-Z)] \quad (5)$$

So it's clear from the equation (5) now to get the linear-phase transfer function we need to have linear-phase low-pass linear phase filter.

One of the most frequently used filter bank among all the M-band filter banks is cosine-modulated filter bank (CMFB) because the designing is easier and more realizable than that of other filter banks [11-12].

It is one of the efficient techniques which provide minimum computational cost for the design of filter banks. In CMFB all the filters are cosine modulated version of a low-pass prototype filter. In this technique all the analysis filters and synthesis filters are simultaneously generated by the low-pass prototype filter. So the design of whole filter bank reduces to the design of a single prototype filter [13-18]. Implementation of CMFB consists of one prototype and discrete cosines transform (DCT).

All of these filters can be designed using either the near perfect reconstruction (NPR) or perfect reconstruction (PR) [19]. Many prominent authors has been studied the cosine modulated filter banks with PR and NPR conditions and find that NPR type is more realizable, computationally efficient and more simple in comparison to the PR [14],[20-21]

## 3. Formulation of Design Problem

Design of cosine modulated-QMF banks start with the design of the low-pass prototype filter. All the other filters are then obtained by the cosine modulation of this low-pass prototype filter [22]. We take linear-phase FIR filter for the basic low-pass prototype filter. Our aim is to minimize the objective function of the prototype filter, which is in our case is a quadratic equation formed with some real constants and total pass band error, mean square error of band pass and stop band of the filter.

Objective function  $v_i$  is given by the equation as

$$v_i = (L * E_p) + (j * E_s) + k * E_t \quad (6)$$

Where  $L$ ,  $j$  and  $k$  are real constants, and  $E_p$ ,  $E_s$  and  $E_t$  are the measure of mean-square pass-band error, mean-square stop band error and total error. Where  $E_p$ ,  $E_s$  and  $E_t$  can be expressed as

$$E_p = b_i * q * b_i' \quad (7)$$

Where

$$q = \frac{1}{\pi} \int_0^{\omega_p} (1-c)(1-c)' d\omega \quad (8)$$

$E_s$  is the stop band energy of low-pass prototype filter between  $\omega_s$  to  $\pi$  is given by

$$E_s = b_i * p_1 * b_i' \quad (9)$$

Where

$$p_1 = \frac{1}{\pi} \int_{\omega_s}^{\pi} c * c' d\omega \quad (10)$$

$E_t$  is the square error of the prototype filter given as

$$E_t = ((b_i * d) - H_r)^2 \quad (11)$$

Where

$$c = [\cos(\omega((N-1)/2)) \dots \cos(\omega/2)]^T \quad (12)$$

$d=c$  evaluated at  $\omega = \pi/2$ ,  $H_{r1}$  is the amplitude function and

We optimize the given objective function using Marquardt optimization method to get the minimum value.

### Cosine Modulated Filter Bank

After designing the low-pass filter all filters of analysis and synthesis sections are obtained by cosine modulation of the prototype filter using the following relations.

$$H(k, n) = 2 * (\text{real}(a(k)) * n1 + \text{imag}(a(k)) * n2) * h(n) \quad (13)$$

Where  $n1$  and  $n2$  is given as

$$n1 = \cos(\pi * (2 * k - 1) * (2 * n - 1) / (4 * nbands)) \quad (14)$$

$$n2 = \sin(\pi * (2 * k - 1) * (2 * n - 1) / (4 * nbands)) \quad (15)$$

Equation (14) and (15) represents the analysis and synthesis filters of the filter bank respectively. And  $n$  bands is the number of bands in filter banks. Where,  $n$  varies from 0 to  $N-1$  and  $k$  varies from 0 to  $M-1$ .

There are two main advantages of cosine modulated filter banks are, first one is the cost of whole analysis filters reduces to the cost of one prototype filter plus the cost of modulating and second one is we need not optimize so many parameters. The number of parameters need to be optimize becomes fewer.

### 4. Design Algorithm

Here we are optimizing the objective function using Marquardt optimization method. If  $b_t$  is the minimum value at the  $t^{\text{th}}$  step then  $b_{t+1}$  at  $t+1^{\text{th}}$  step can be calculated by using Marquardt optimization method as follows

$$b_{t+1} = b_t - [J_t]^{-1} \nabla \phi_t \quad (16)$$

Where

$$J_t = H_t + a_t I, \quad (17)$$

Where  $\nabla \phi_t$  is the gradient of the objective function,  $H_t$  is the Hessian matrix,  $a_t$  is a  $t^{\text{th}}$  stage iteration constant and  $I$  is  $N/2 \times N/2$  identity matrix

Where

$$H_t = 2(\phi + \beta D) \quad (18)$$

Where  $\beta$  is a constant and

$$D = d * d' \quad (d = \cos(\omega(((N-1)/2) - N/2)) \text{ at } \omega = \pi/2) \text{ and}$$

$$\nabla \phi_t = 2(\phi + \beta D)h \quad (19)$$

- (1) Set initial values of  $\alpha$ ,  $e_1$ ,  $a > 1$ ,  $b < 1$ , and  $N$ .
- (2) Set initial design vector  $h_0 = [h_0(0)h_0(1)h_0(2) \dots h_0(N/2-1)]^T$  such that energy is unit remains within a specified tolerance, i.e.,

$$u = \left| 1 - 2 \sum h_0^2(k) \right| < e_1 \quad (20)$$

- (3) Set the iteration number  $t = 0$ , and  $b_t = 2h_t$ .
- (4) Compute  $\nabla \phi_t$  using Eq. (19) at the design vector  $b_t$ .
- (5) Compute the Hessian matrix  $H$  using Eq. (18) and the matrix  $J_t$  using Eq. (17).
- (6) Compute the new or improved approximation 
$$b_{t+1} = b_t - [J_t]^{-1} \nabla \phi_t$$
- (7) Compute the value as given in (20)  $u$ , at the point  $b_{t+1}$ , and if value is not satisfied then choose the optimum point as  $b_t$ , stop the procedure and go to step (10).
- (8) If the condition at (19) is satisfied at the point  $b_{t+1}$ , compute the objective function  $\phi_{t+1}$  at the point  $b_{t+1}$ . If  $\phi_{t+1} \geq \phi_t$ , choose the optimum point as  $b_t$ , stop the procedure and go to step (10). If  $\phi_{t+1} < \phi_t$ , set  $\phi_t = \phi_{t+1}$  and  $b_t = b_{t+1}$ .
- (9) Set the new iteration number as  $t = t + 1$ ,  $a = a \times b$  and go to step (3).
- (10) Compute  $h_0 = (1/2)b_t$  and stop the procedure.

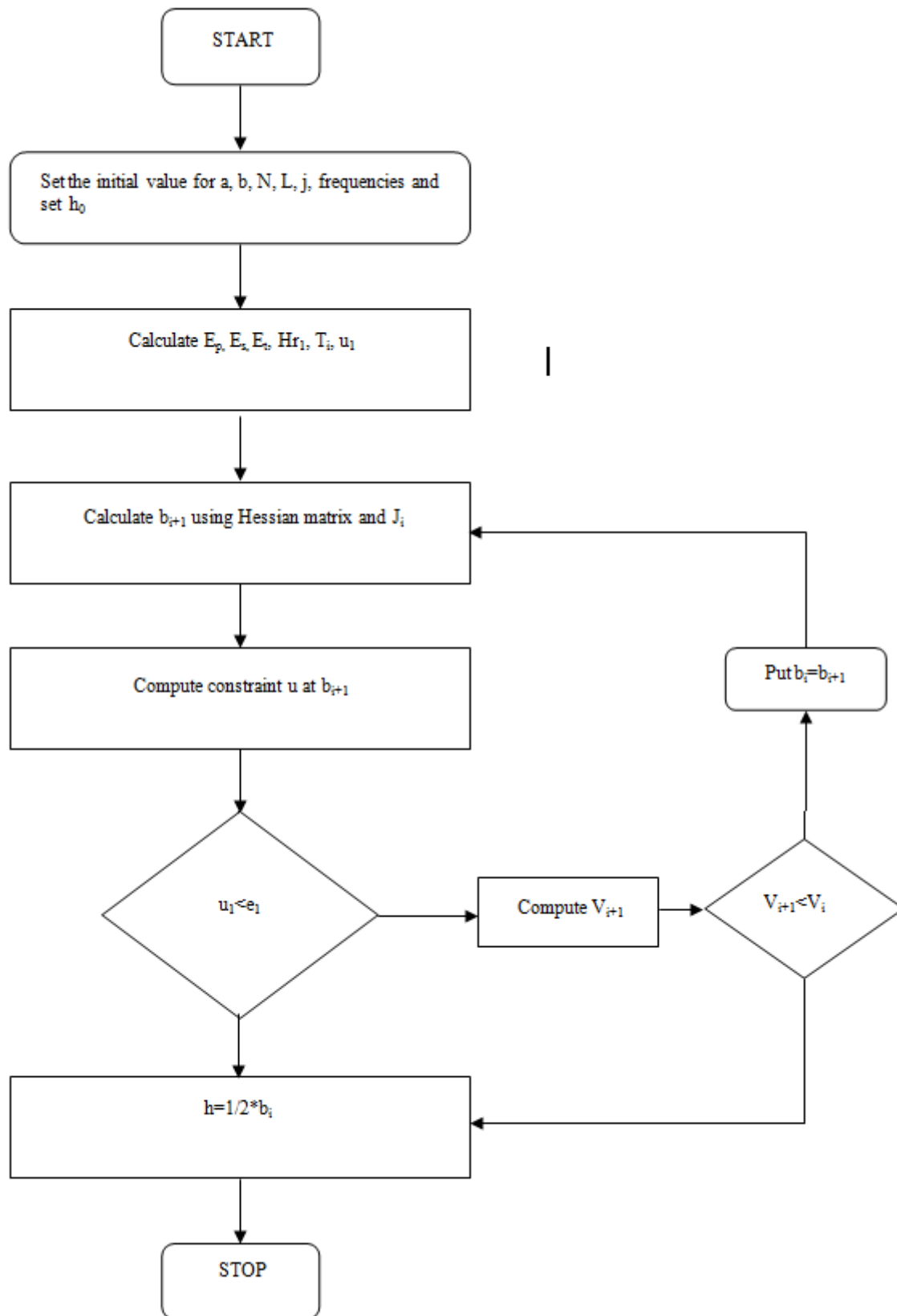
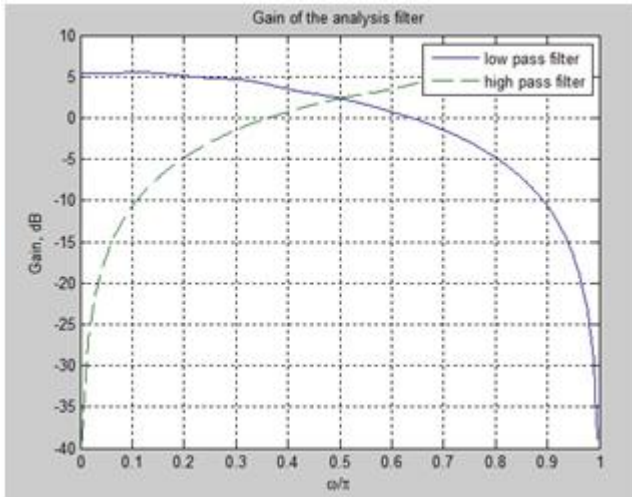


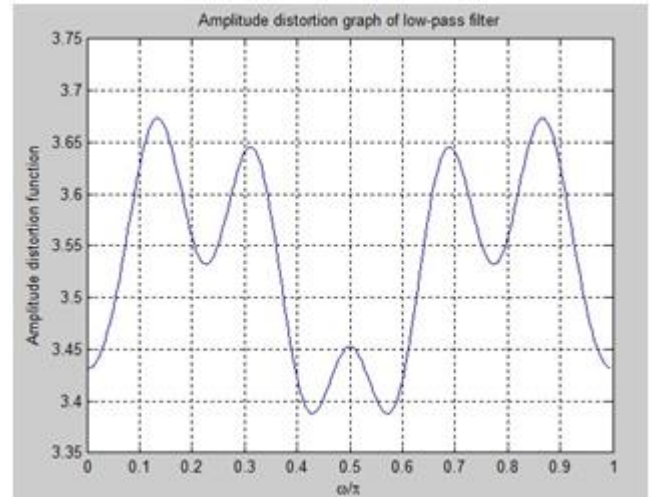
Figure 3: Flow chart for the optimization process

## 5. Results and Discussion

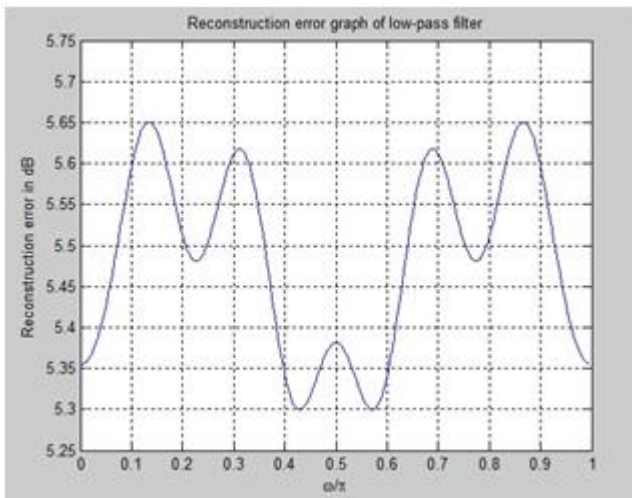
A two channel Quadrature mirror filter with cosine modulation is presented here with proposed algorithm using MATLAB.



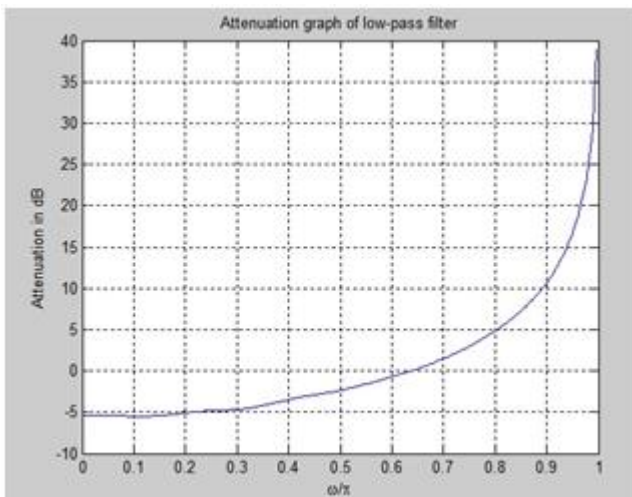
**Figure 4(a):** Gain of the analysis filter



**Figure 4(d):** Amplitude distortion function graph



**Figure 4(b):** Reconstruction error graph



**Figure 4(c):** Attenuation graph of analysis filter

Effectiveness of the proposed algorithm is shown in terms of sharp cut off, broader bandwidth, less NOI and less CPU time required. Graphs here shown is at  $N=32$ .

Figure 4(a) and 4(b), show the gain and reconstruction error graph of for the analysis filter respectively. While figure 4(c) and 4(d) gives the attenuation and distortion graph for the same respectively.

Table 1 gives performance summary of the proposed filter at the different filter orders.

**Table 1:** Performance Parameters of the Filter

Filter order	Bands	CPU time	NOI	$A_s$
20	2	0.0371	91	-2.4093
32	2	0.12458	91	-2.3966
40	2	0.102295	91	-2.3962
60	2	0.056411	91	-2.3975

Simulation result show that the proposed method gives the better result in terms of computation time, number of iteration and stop-band edge attenuation.

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