

# Probabilistic Rough Classification in Information Systems with Fuzzy Decision Attributes

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**Abstract:** In 2004, G.Ganesan et.al introduced the concept of indexing any information system with fuzzy decision attributes using a threshold. These indices are based on the two way approach of Pawlak's rough sets. However, the approach of Pawlak lacks in quantifying the importance of any basic granule which involve in the approximation. Later, Y.Y.Yao discussed a new Probabilistic Rough Set model which appropriately quantifies the appropriate basic granules. In this paper, we extended the work of G.Ganesan et.al., for the Probabilistic Rough Set Model to improve the efficiency of rough indices in the information system with fuzzy decision attributes.

**Keywords:** information system, rough set, probabilistic rough set, rough index

## 1. Introduction

Z.Pawlak's concept of Rough Sets [4,5] finds various technical applications in the areas such as Knowledge Discovery, Acquisition etc. This incompletes any input/concept in terms of union of **Basic Categories** in two ways. However, this approach lacks with the level of involvement of the basic categories defined. To succeed, W.Ziarko defined Variable Precision Rough Sets in 1993, which mainly deals with the approximations based on the degrees of contributions of the Basic Categories in defining the approximations for a given input. Afterwards, Bing Zhou, YY Yao, Slezak etc have contributed in extending this model through Probabilistic approaches.

The researchers such as Dubois, Prade, Nakamura, Biswas etc at the same time have been prominent noteworthy in hybridizing rough and fuzzy models for real time applications. In 2005, G.Ganesan et.al, discussed the importance of defining the thresholds in rough fuzzy computing in 2008. The indexing terminology in information systems using these thresholds has been introduced by G.Ganesan with fuzzy decision attributes. Recently, Yiyu Yao and Bing Zhou discussed the Naïve Bayesian Rough Set

Model in [8] and earlier to this, the initial approach in this regard was discussed in [7] by Slezak. In this paper, we extended the work of G.Ganesan et. al., on rough indexing to the information systems functions with Probabilistic Naïve Bayesian Rough Set Model.

## 2. Decision Theoretic And Probabilistic Rough Sets

The Rough Sets [4,5] theory gives two way approximations namely lower and upper approximations for a given input. For given finite universe of discourse U and an equivalence

relation E, we define the equivalence class of any  $x \in U$  to be  $[x] = \{y \in U / xEy\}$ . The family of equivalence classes  $U/E = \{[x]_E | x \in U\}$  is a partition of the universe U. For a given concept C, Pawlak defined the lower approximation as  $\underline{apr}_E(C) = \{x \in U / [x]_E \subseteq C\}$  and upper approximation as  $\overline{apr}_E(C) = \{x \in U / [x]_E \cap C \neq \Phi\}$ . According to Pawlak, for a given concept C, three disjoint regions can be defined namely positive, negative and boundary regions which are defined as follows:

Positive Region:  $POS_E(C) = \{x \in U / [x]_E \subseteq C\}$

Boundary Region:  $BND_E(C) = \{x \in U / [x]_E \cap C \neq \Phi \wedge [x]_E \not\subseteq C\}$

Negative region :  $NEG_E(C) = \{x \in U / [x]_E \cap C = \Phi\}$

parameterized rough set model, probabilistic rough set model and generalized rough set model were generalized by many researchers on this approach. Understanding the limitations of Pawlak's restrictive model

By considering degrees of overlap between equivalence classes and a concept C to be approximated and is viewed as the conditional probability of an object belongs to C given that the object is in [x] defined rough membership function. In 1994, by Pawlak and Skowron [6] (for simplicity, we denote  $[x]_E$  with [x]) which is given as

$$\Pr\left(\frac{C}{[x]}\right) = \frac{|C \cap [x]|}{|[x]|}$$

Using the definition quoted above, in [7], the positive, boundary and negative regions are defined as follows:

$$POS(C) = \left\{x \in U / \Pr\left(\frac{C}{[x]}\right) = 1\right\}$$

$$BND(C) = \left\{ x \in U / 0 < \Pr\left(\frac{C}{[x]}\right) < 1 \right\}$$

$$NEG(C) = \left\{ x \in U / \Pr\left(\frac{C}{[x]}\right) = 0 \right\}$$

In 2009, Greco et.al [3] discussed the parameterized roughest model by generalizing the above said definitions. In this model, two thresholds namely  $\alpha$  and  $\beta$  are used to define the probabilistic regions and the positive, boundary and negative regions are modified as follows:

$$POS_{(\alpha,\beta)}(C) = \left\{ x \in U / \Pr\left(\frac{C}{[x]}\right) \geq \alpha \right\}$$

$$BND_{(\alpha,\beta)}(C) = \left\{ x \in U / \beta < \Pr\left(\frac{C}{[x]}\right) < \alpha \right\}$$

$$NEG_{(\alpha,\beta)}(C) = \left\{ x \in U / \Pr\left(\frac{C}{[x]}\right) \leq \beta \right\}$$

These Probabilistic regions will lead three way decisions namely acceptance, deferment and rejection respectively for any object  $x$  in  $U$ . But, however, in several cases, it is easy to compute the probability of the existence of a category  $[x]$  for a given concept  $C$  using  $\Pr\left(\frac{[x]}{C}\right) = \frac{|[x] \cap C|}{|C|}$

Hence as in [ ], by Baye's Theorem, the Positive, Boundary and Negative Regions are given by

$$POS_{(\alpha',\beta')}^B(C) = \left\{ x \in U / \log \frac{\Pr([x]/C)}{\Pr([x]/C^c)} \geq \alpha' \right\}$$

$$BND_{(\alpha',\beta')}^B(C) = \left\{ x \in U / \beta' < \log \frac{\Pr([x]/C)}{\Pr([x]/C^c)} < \alpha' \right\}$$

$$NEG_{(\alpha',\beta')}^B(C) = \left\{ x \in U / \log \frac{\Pr([x]/C)}{\Pr([x]/C^c)} \leq \beta' \right\}$$

$$\text{where } \alpha' = \log \frac{\Pr(C^c)}{\Pr(C)} + \log \frac{\alpha}{1-\alpha}$$

$$\text{and } \beta' = \log \frac{\Pr(C^c)}{\Pr(C)} + \log \frac{\beta}{1-\beta}$$

Now, we shall discuss the conventional approach on dealing the fuzzy sets to approximate under rough computing, which was discussed in [1]

### 3. Analysis of Fuzzy Set Using a Threshold

Consider a set  $D$ , called **R-domain** [1], satisfying the following properties:

- $D \subset (0,1)$
- If a fuzzy concept  $C$  is under computation, eliminate the values  $\mu_C(x)$  and  $\mu_{C^c}(x) \forall x \in U$  from the domain  $D$ , if they exist.
- After the computation using  $C$ , the values removed in (b) may be included in  $D$  provided  $A$  must not involve in further computation

Consider the universe of discourse  $U = \{x_1, x_2, \dots, x_n\}$ . Let  $\alpha, \alpha_1, \alpha_2, \beta$  be the thresholds assume one of the values from the domain  $D$ , where  $D$  is constructed using the fuzzy concepts  $A$  and  $B$ . For a given threshold  $\alpha$  and a fuzzy set  $A$ ,

the Strong  $\alpha$ -Cut is given by  $A[\alpha] = \{x \in U / \mu_A(x) > \alpha\}$ . The union and intersection of fuzzy sets [10] are by the maximum and minimum of corresponding membership values respectively.

In 1972, Zadeh[9] introduced the concept of hedges. In fuzzy logic, in order to improve the efficiency of fuzziness, the concept of concentration and dilation were introduced by him.

For example, for the linguistic variable 'low' with the membership function  $\alpha$ , the hedges 'very' and 'very very' emphasis the efficiency of the variable with the corresponding membership values  $\alpha^2$  and  $\alpha^4$ . They are called **concentration**, whereas the hedges 'slightly' and 'more slightly' dilutes the efficiency of the linguistic variables with the membership values with the corresponding membership values  $\alpha^{1/2}$  and  $\alpha^{1/4}$ . They are called **dilation**.

Using the definitions of fuzzy sets mentioned above, the following properties were derived in [1].

- $A[\alpha_1] \cup A[\alpha_2] = A[\alpha]$  where  $\alpha = \min(\alpha_1, \alpha_2)$
- $A[\alpha_1] \cap A[\alpha_2] = A[\alpha]$  where  $\alpha = \max(\alpha_1, \alpha_2)$
- $(A \cup B)[\alpha] = A[\alpha] \cup B[\alpha]$
- $(A \cap B)[\alpha] = A[\alpha] \cap B[\alpha]$
- $A^c[\alpha] = A[1-\alpha]^c$
- $(A \cup B)^c[\alpha] = A^c[\alpha] \cap B^c[\alpha]$
- $(A \cap B)^c[\alpha] = A^c[\alpha] \cup B^c[\alpha]$

Using the mathematical tool derived as above, in [1], rough set approach on fuzzy sets using a threshold is introduced as discussed below.

### 3.1 Rough Approximations on fuzzy sets using $\alpha$

Let  $\Psi$  be any partition of  $U$ , say  $\{B_1, B_2, \dots, B_t\}$ . For the given fuzzy concept, the lower and upper approximations with respect to  $\alpha$  can be defined as  ${}_aC = \underline{C[\alpha]}$  and  ${}^u C = \overline{C[\alpha]}$  respectively.

#### 3.1.1 Propositions

Here, by using the properties of rough sets, the following propositions [ ] can be obtained.

- ${}_a(A \cup B) = {}_aA \cup {}_aB$
- ${}_a(A \cap B) = {}_aA \cap {}_aB$
- ${}_a(A \cup B) \supseteq {}_aA \cup {}_aB$
- ${}_a(A \cap B) \subseteq {}_aA \cap {}_aB$
- ${}_a(A^c) = ({}_aA)^c$
- ${}_a(A^c) = ({}_aA)^c$

Now, we shall hybridize the concepts dealt in the above two sections which gives the approach of dealing a fuzzy concepts under Naïve Bayesian Probabilistic Rough Sets.

### 4. Naïve Bayesian Probabilistic Rough Sets Model for A Fuzzy Concept

Since, in the above both sections, the same threshold  $\alpha$  has been used, for different purposes, to make the homogeneity,

in this paper, we replace the threshold  $\alpha$  to obtain a Strong Cut on fuzzy sets with  $\delta$ .

Hence, for a given fuzzy concept  $F$  with the threshold  $\delta$ , the probabilistic positive, boundary and negative regions are respectively defined on the approximation space  $U/E$  as

$$POS_{\delta}(F) = \left\{ x \in U / \Pr\left(\frac{F[\delta]}{[x]}\right) = 1 \right\}$$

$$BND_{\delta}(F) = \left\{ x \in U / 0 < \Pr\left(\frac{F[\delta]}{[x]}\right) < 1 \right\}$$

$$NEG_{\delta}(F) = \left\{ x \in U / \Pr\left(\frac{F[\delta]}{[x]}\right) = 0 \right\}$$

For given parameters  $\alpha$  and  $\beta$ , the regions of the parameterized rough sets model are given by

$$POS_{(\alpha, \beta, \delta)}(F) = \left\{ x \in U / \Pr\left(\frac{F[\delta]}{[x]}\right) \geq \alpha \right\}$$

$$BND_{(\alpha, \beta, \delta)}(F) = \left\{ x \in U / \beta < \Pr\left(\frac{F[\delta]}{[x]}\right) < \alpha \right\}$$

$$NEG_{(\alpha, \beta, \delta)}(F) = \left\{ x \in U / \Pr\left(\frac{F[\delta]}{[x]}\right) \leq \beta \right\}$$

and the Regions of Naïve Bayesian Rough Sets Model are given by

$$POS_{(\alpha', \beta', \delta)}^B(F) = \left\{ x \in U / \log \frac{\Pr([x] / F[\delta])}{\Pr([x] / (F[\delta])^c)} \geq \alpha' \right\}$$

$$BND_{(\alpha', \beta', \delta)}^B(F) = \left\{ x \in U / \beta' < \log \frac{\Pr([x] / F[\delta])}{\Pr([x] / (F[\delta])^c)} < \alpha' \right\}$$

$$NEG_{(\alpha', \beta', \delta)}^B(F) = \left\{ x \in U / \log \frac{\Pr([x] / F[\delta])}{\Pr([x] / (F[\delta])^c)} \leq \beta' \right\}$$

$$\text{where } \alpha' = \log \frac{\Pr(C^c)}{\Pr(C)} + \log \frac{\alpha}{1 - \alpha}$$

$$\text{and } \beta' = \log \frac{\Pr(C^c)}{\Pr(C)} + \log \frac{\beta}{1 - \beta}$$

## 5. Rough Indices

Let  $U$  be the universe of discourse and  $\alpha$  be any value in  $(0,1)$ . Let  $X = \{W_1, W_2, \dots, W_n\}$  be any partition defined on  $U$ . For any fuzzy set  $A$  define  $A[\alpha] = \{x \in U / \mu_A(x) > \alpha\}$  where  $\alpha$  is chosen from  $R$ -domain satisfying the property that  $\text{dil}^n(\alpha)$  and  $\text{con}^n(\alpha)$  are the members of  $R$ -Domain for any positive integer  $n$  [ $\text{dil}$  represents dilation and  $\text{con}$  represents concentration]. The lower and upper approximations  $A_{\alpha}$  and  $A^{\alpha}$  are given by  $A_{\alpha} = (A[\alpha])_{\sim}$  and  $A^{\alpha} = (A[\alpha])^{\sim}$  respectively [1].

The following algorithm as in [2] illustrates the method of indexing the elements of  $U$ , by using the lower and upper approximations of the given fuzzy set  $A$

Let  $M$  denote the largest number under consideration such that  $n+M$  is always positive and  $n-M$  is always negative for any integer  $n$ .

### 5.1 Algorithms

Algorithm rough index  $(x, A, \alpha)$

//Algorithm to obtain rough index of  $x$  an element of universe of discourse

//Algorithm returns the rough index

1. Let  $x\_index$  be an integer initialized to 0
2. Pick the equivalence class  $K$  containing  $x$ .

If  $\mu_A(y) = 0$  for all  $y \in K$

begin

$x\_index = -M$

goto 6

end

end

3. If  $\mu_A(x) = 1$

begin

If  $\mu_A(y) = 1$  for all  $y \in K$

begin

$x\_index = M$

goto 6

end

end

4. compute  $A_{\alpha}$  and  $A^{\alpha}$

5. If  $x \in A_{\alpha}$

begin

$x\_index = M$

while  $(x \in A_{\alpha})$

begin

$\alpha = \text{dil}(\alpha)$  //dilation of  $\alpha$

$x\_index = x\_index + 1$

compute  $A_{\alpha}$

end

end

else

if  $x \notin A^{\alpha}$

begin

$x\_index = -M$

while  $(x \notin A^{\alpha})$

begin

$\alpha = \text{con}(\alpha)$  //concentration of  $\alpha$

$x\_index = x\_index - 1$

compute  $A^{\alpha}$

end

end

else

Let  $\beta = \alpha$

compute  $A^{\beta}$

while  $(x \notin A_{\alpha} \text{ and } x \in A^{\beta})$

begin

$\alpha = \text{con}(\alpha)$  // concentration of  $\alpha$

$\beta = \text{dil}(\beta)$  // dilation of  $\beta$

compute  $A_{\alpha}, A^{\beta}$

$x\_index = x\_index + 1$

end

if  $x \in A_{\alpha}$  then

$x\_index = -x\_index$

end

6. return  $x\_index$

Consider the universe of discourse  $U=\{a,b,c,d,e,f,g,h\}$  with the partition  $X=\{\{a,e,f\},\{b,g\},\{c,h\},\{d\}\}$ . Let  $\alpha=0.5$ .

Consider the fuzzy set  $\{(a,0.6),(b,0.4), (c,0.8), (d,0.24), (e,0.44), (f,0.56), (g,0.98), (h,0.77)\}$ . By the above algorithm, 'b' can be indexed by -1 and d can be indexed by -2-M. Similarly, other values of U can be indexed. These indices are called rough indices.

Sometimes, the elements of different equivalence classes may have same rough index but in the above it is to be noted that the elements of same equivalence classes have the same rough indices. Clearly, it depends upon the choice of  $\alpha$  and the fuzzy set taken under consideration. But,, it is obvious that the elements of the same equivalence classes will have the same rough indices and therefore, instead of indexing the elements of U, one may follow the given algorithm for rough indexing the equivalence classes. Now, this algorithm shall be modify for three way approach on rough sets as follows:

```

Algorithm Three_Way_rough index (x,A, $\delta$ )
//Algorithm returns the Three_Way_rough index of x
1. Let x_index be an integer initialized to 0
2. Pick the equivalence class K containing x.
If  $\mu_A(y)=0$  for all  $y \in K$ 
begin
x_index=-M
goto 6
end
3. If  $\mu_A(x)=1$ 
begin
If  $\mu_A(y)=1$  for all  $y \in K$ 
begin
x_index=M
goto 6
end
end
4. compute  $POS_\delta(A)$ ,  $BND_\delta(A)$  and  $NEG_\delta(A)$ 
5. If  $x \in POS_\delta(A)$ 
begin
x_index=M
while ( $x \in POS_\delta(A)$ )
begin
 $\alpha=dil(\delta)$  //dilation of  $\delta$ 
x_index=x_index+1
compute $POS_\delta(A)$ 
end
end
else
if  $x \in NEG_\delta(A)$ 
begin
x_index=-M
while ( $x \in NEG_\delta(A)$ )
begin
 $\delta=con(\delta)$  //concentration of  $\delta$ 
x_index=x_index-1
compute $NEG_\delta(A)$ 
end
end
else

```

```

Let  $\gamma=\delta$ 
compute $NEG_\gamma(A)$ 
while ( $x \notin (POS_\delta(A) \cup NEG_\gamma(A))$ )
begin
 $\delta=con(\delta)$  // concentration of  $\delta$ 
 $\gamma=dil(\gamma)$  // dilation of  $\gamma$ 
compute $POS_\delta(A) \cup NEG_\gamma(A)$ 
x_index=x_index+1
end
if  $x \in POS_\delta(A)$  then
x_index= - x_index
end
6. return x_index

```

This algorithm can be illustrated in the same manner as mentioned in the previous example. Now, we parameterize the algorithm using parameters  $\alpha$  and  $\beta$ .

```

Algorithm Naïve Bayesian_rough index (x,A, $\alpha,\beta,\delta$ )
//Algorithm returns Naïve Bayesian_rough index of x
1. Let x_index be an integer initialized to 0
2. Pick the equivalence class K containing x.
If  $\mu_A(y)=0$  for all  $y \in K$ 
begin
x_index=-M
goto 6
end
3. If  $\mu_A(x)=1$ 
begin
If  $\mu_A(y)=1$  for all  $y \in K$ 
begin
x_index=M
goto 6
end
end
4. compute  $POS^B_{(\alpha',\beta',\delta)}(A)$ ,  $BND^B_{(\alpha',\beta',\delta)}(A)$  and  $NEG^B_{(\alpha',\beta',\delta)}(A)$ 
5. If  $x \in POS^B_{(\alpha',\beta',\delta)}(A)$ 
begin
x_index=M
while ( $x \in POS^B_{(\alpha',\beta',\delta)}(A)$ )
begin
 $\alpha=dil(\delta)$  //dilation of  $\delta$ 
x_index=x_index+1
compute $POS^B_{(\alpha',\beta',\delta)}(A)$ 
end
end
else
if  $x \in NEG^B_{(\alpha',\beta',\delta)}(A)$ 
begin
x_index=-M
while ( $x \in NEG^B_{(\alpha',\beta',\delta)}(A)$ )
begin
 $\delta=con(\delta)$  //concentration of  $\delta$ 
x_index=x_index-1
compute $NEG^B_{(\alpha',\beta',\delta)}(A)$ 
end
end
else
Let  $\gamma=\delta$ 

```

```

computeNEGγ(A)
while
(x ∉ (POSB(α',β',δ)(A) ∪ NEGB(α',β',γ)(A)))
begin
δ=con(δ) // concentration of δ
γ=dil(γ) // dilation of γ
compute
POSB(α',β',δ)(A) ∪ NEGB(α',β',γ)(A)
x_index=x_index+1
end
if x ∈ POSB(α',β',δ)(A) then
x_index= - x_index
end
6. return x_index

```

## 6. Naïve Bayesian Indexing In Information System With Fuzzy Decision Attribute

According to the perspective of Z.Pawlak, any information system is given by  $T=(U, A, C, D)$ , where  $U$  is the universe of discourse,  $A$  is a set of primitive attributes,  $C$  and  $D$  are the subsets of  $A$  called condition and decision features respectively [ $C$  and  $D$  may not exist in a few of the information systems].

Consider an information system with conditional attributes  $C=\{a_1, a_2, \dots, a_n\}$  and decision attributes  $\{d_1, d_2, \dots, d_s\}$  with the records  $U=\{x_1, x_2, \dots, x_m\}$ . For any index key 'a' in  $C$ , the indiscernibility relation is given by  $x_i \approx_{a_k} x_j$  (read as  $x_i$  is related to  $x_j$  with respect to  $a_k$ ) if and only if  $a_k(x_i)=a_k(x_j)$ . Clearly, this indiscernibility relation partitions the universe of discourse  $U$ . However, the procedure of selecting the appropriate minimal attributes [reducts] for effectiveness is not discussed in this paper.

For example, consider the decision table with  $C=\{a, b, c, d\}$  and  $D=\{E\}$ .

	a	b	c	d	E
$x_1$	1	0	2	1	1
$x_2$	1	0	2	0	1
$x_3$	1	2	0	0	2
$x_4$	1	2	2	1	0
$x_5$	2	1	0	0	2
$x_6$	2	1	1	0	2
$x_7$	2	1	2	1	1

Let us consider the index key as 'c'. As  $x_1, x_2, x_4, x_7$  have the values 2;  $x_3, x_5$  have the values 0 and  $x_6$  has the value 1. Hence, the partition on  $U$  with respect to  $c$  can be defined as  $\{\{x_1, x_2, x_4, x_7\}, \{x_3, x_5\}, \{x_6\}\}$ .

However, in real time systems we can find several information systems with fuzzy decision attributes and hence the scope of the algorithms discussed above would be applicable for such information systems. Here, the Naïve Bayesian rough indexing of the data can be derived from the fuzzy decision attribute as discussed in the previous section.

For example, consider knowledge representation of the information system with  $C=\{a, b, c, d\}$  and  $D=\{E\}$  where  $E$  is of fuzzy natured.

	a	b	c	d	$\mu_E(x_i)$
$x_1$	1	0	2	1	0.45
$x_2$	1	0	2	0	0.7
$x_3$	1	2	0	0	0.65
$x_4$	1	2	2	1	0.1
$x_5$	2	1	0	0	0.91
$x_6$	2	1	1	0	0.6
$x_7$	2	1	2	1	0.35

On considering 'c' as the index key, the partition obtained is  $\{\{x_1, x_2, x_4, x_7\}, \{x_3, x_5\}, \{x_6\}\}$ . Let  $\delta=0.5$ . Here,  $E[\delta]=\{x_2, x_3, x_5, x_6\}$ . For a given  $\alpha$  and  $\beta$ , the Naïve Bayesian indexing algorithm would be implemented further.

## 7. Conclusion

In this paper, by using the concept of Naïve Bayesian rough sets the approach of indexing the records of the information system is dealt. These rough indices are useful to analyze and index a database when the fuzzy information about the entire key values is obtained.

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