Proof

Let $f:(X,\tau) \longrightarrow (Y,\acute{\tau})$ be s-coc-continuous, open onto function and (X,τ) is s-coc-locally connected. To prove $(Y,\acute{\tau})$ is locally connected, let $y \in Y$ and U open set in Y such that $\in U$. Since f s-coc-continuous then $f^{-1}(U)$ is s-coc-open sets in Y. Since X is s-coc-locally connected then there is V s-coc-connected open set such that $x \in V \subseteq f^{-1}(U)$. Since f open then $f\,(V)$ open in Y, since V is s-coc-connected then $f\,(V)$ *connected* by proposition (3.9). Thus $(x) = y \in f\,(V)\ \subseteq U$. Then Y is locally connected.

Remark (3.7)

The s-coc-continuous image of s-coc-locally connected need not to be s-coc-locally connected.

Example (3.9)

Let $X = \{1,2,3,\dots\}, \tau\ discrete\ topology, Y = \{a,b\}, \tau' = \{\emptyset, Y, \{a\}\}$. Define $f:(X,\tau) \longrightarrow (Y,\tau')f(x) = \begin{cases} a & if\ x = 1 \\ b & if\ x \neq 1 \end{cases}$, since open sets in Y are Y, $\{a\}$ and $f^{-1}(Y) = X, f^{-1}(\{a\}) = \{1\}$ s-coc-open in X.Thus f s-coc-continuous and X s-coc-locallyconnected. But $b \in Y\ and\ \{b\}$ s-coc-open set in Y such that $b \in \{b\}$ and there exists no s-coc-connected open set V such that b$\in V \subseteq \{b\}$. Thus Y is not s-coc-locally connected.

Proposition (3.20)

The s-coć-continuous, open onto image of s-coc-locally connected space is s-coc-locally connected.

Proof

Let $f:(X,\tau) \longrightarrow (Y,\acute{\tau})$ be s-coć-continuous, open onto function and (X,τ) is s-coc-locally connected. To prove $(Y,\acute{\tau})$ is s-coc-locally connected. Let $y \in Y$ and U s-coc-open set in Y such that y $\in U$**.** Since f onto and $y \in Y$ there is $x \in X$ such that$f(x) = y$**.** Since f s-coć-continuous then $f^{-1}(U)$ s-coc-open sets in $\boldsymbol{X}$**.** Since X is s-coc-locally connected then there is V s-coc-connected open set in X such that $\in V \subseteq f^{-1}(U)$**.** Since f open then $f\,(V)$ open in Y and connected by proposition (3.10)**.** Then y= $f(x) \in f\,(V)\ \subseteq U$**.** Thus Y is s-coc-locally connected.

Definition (3.11) [12]

A space (X,τ) is said to be extremely disconnected if the closure of every open subset of the X is open in X.

Definition (3.12)

A space (X,τ) is said to be s-coc-extremely disconnected if the closure of every open subset of the X is s-coc-open in X .

Remark (3.8)

Every extremely disconnected space is s-coc-extremely. But the convers is not true.

Example (3.10)

Let $X = \{1,2,3,4,\dots\}, \tau = \{\emptyset, X, \{1\}, \{2\}, \{1,2\}\}$.Then τ^{sk} is discrete topologyon X. Then every subset in X is s-coc-open and s-coc-closed set.Thus if A open in X, then $\overline{A}$ is s-coc-open set. Thus X is s-coc-extremely disconnected. But $A = \{1\}$ open set and $\overline{A} = \{1,3,4,5,\dots\}$ not open. Then X is not extremely disconnected.

Proposition (3.21)

For a topological space (X,τ) if the closure of every s-coc-open set is s-coc-open, then X is s-coc-extremely disconnected.

Proof

Let U is open set of X. Then U is s-coc-open set. Since the closure of s-coc-open set is s-coc-open set. Thus X is s-coc-extremely disconnected.

Proposition (3.22)

If $X\ and\ Y$ s-coc-extremely disconnected then $X \times Y$ s-coc-extremely disconnected.

Proof

Let $W = A \times B$ open in $X \times Y$. Then A, B open in X, Y.Since X, Y s-coc-extremely then $\overline{A}, \overline{B}$ s-coc-open in $X, Y\ \ then\ \overline{W} = \overline{A \times B} = \overline{A} \times \overline{B}$ s-coc-open set by proposition (1.3).Then $\overline{A} \times \overline{B}$ s-coc-open setin $X \times Y$. Then $X \times Y$ s-coc-extremely disconnected.

Remark (3.9)

If X s-coc-extremely disconnected then X need not to be s-coc-connected for example .

Example (3.11)

Let $X = R$ and U usual Topology on R.Since $R = (-\infty,0)\cup[0,\infty)$ and$(-\infty,0)\cup[0,\infty) = \emptyset$ and $(-\infty,0),[0,\infty)$ s-coc-open sets .Thus (R,U) s-coc-disconnected.But for every (a,b) open set in R.Thus $\overline{(a,b)} = [a,b]$ is s-coc-open set in R. Therefore X is s-coc-extremely.

Proposition (3.23)

If X is s-coc-connected then X is not s-coc-extremely disconnected.

Proof

Let X s-coc-connected and X is s-coc-extremely disconnected. To get contradiction.Then for all A open set we get $\overline{A}$ s-coc-open. Since $\overline{A}$ closed set then $\overline{A}$ s-coc-closed.Then X is not s-coc-connected by proposition (3.2) (if X is s-coc-connected then the only s-coc-clopen sets are$\emptyset, X$). Therefore X is not s-coc-extremely disconnected

Not that

If X is s-coc-extremely disconnected then X need not to be s-coc-locally connected for the following example.

Example (3.12)

Let $X = \{1,2,3,\dots\}, \tau = \{\Phi, X, \{1,2\}\}$, *since* $\{1,2\}$ open set in X and $\overline{\{1,2\}} = X$ s-coc-open set. Then X is s-coc-extremely disconnected.But $1 \in \{1,2\}$ open and there is no V s-coc-connected open set such that $1 \in V \subseteq \{1,2\}$. Then X is not s-coc-locally connected.