MHD Free Convective Flow Over A Vertical Porous Surface with Ohmic Heating, Thermal Radiation and Chemical Reaction

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Abstract: In the present work, the effect of ohmic heating on steady MHD free convective flow of an incompressible, electrically conducting fluid over a semi-infinite vertical porous plate in the presence of uniform transverse magnetic field, thermal radiation and chemical reaction have been investigated. The basic partial differential equations are transformed into dimensionless form by employing the appropriate transformations and the dimensionless equations are solved analytically using perturbation method. The effects of various parameters on the velocity, temperature and concentration profile are displayed graphically whereas numerical values of skin-friction coefficient, Nusselt number and Sherwood number are presented through Tables.

Keywords: MHD, Free Convection, Radiation, Chemical Reaction, Porous Medium, Ohmic Heating

1. Introduction

In recent year, the study of free convection flow of an electrically conducting fluid past a vertical porous plate under the influence of magnetic field is receiving considerable attention of many researchers because of its various applications in several field such as thermal insulation, drying of porous solid materials, heat exchanges, refrigerators, electrical conductors, polymer production, manufacturing of ceramic, packed-bed catalytic reactors, food processing, cooling of nuclear reactors, enhanced oil recovery, underground energy transport, magnetized plasma flow, cosmic jets and stellar system. The problem of heat and mass transfer by natural convection in a porous medium has been studied by Bejan et al. [1]. Cogley et al. [2] developed differential approximations for radiative heat transfer in a nonlinear equations-grey gas near equilibrium. MHD free convective flow past an accelerated vertical porous plate by finite difference method has studied by Singh [3]. Raptis et al. [4] investigated the radiation and free convection flow past a moving plate. Chamkha [5] examined the unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. Pal et al. [7] have examined perturbation analysis of unsteady magneto hydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction. Heat and mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in a porous medium has been studied by Das et al. [8]. Makinde [10] studied the free convection flow with thermal radiation and mass transfer past a moving vertical porous plate. Ganesan et al. [11] investigated the finite difference analysis of unsteady natural convection MHD flow past an inclined plate with variable surface heat and mass flux. Sharma [13] examined the unsteady effect on MHD free convective and mass transfer flow through porous medium with constant suction and constant heat flux in rotating system. Ahmed [14] studied the effects of unsteady free convective MHD flow through a porous medium bounded by an infinite vertical porous plate.

In the above mentioned studies the effect of viscous dissipation and ohmic heating is not discussed. However it is more realistic to include these two effects in order to explore the impact of magnetic field on the thermal transport in the boundary layer. In the case of electrolytic refining of mixtures or electrolysis, Ohmic heating plays a vital role. Ohmic heating is the generation of excess heat in the fluid either due to direct current or applied magnetic fields. Chen [6] investigate the combined heat and mass transfer in MHD free convection from a vertical surface with ohmic heating and viscous dissipation. Umamaheswar et al. [9] have examined the unsteady MHD free convective visco-elastic fluid flow bounded by an infinite inclined porous plate in the presence of heat source, viscous dissipation and ohmic heating. Sharma et al. [12] studied the effects of variable thermal conductivity, viscous dissipation on steady MHD natural convection flow of low Prandtl fluid on an inclined porous plate with Ohmic heating. Babu et al. [15] have analyzed the mass transfer effects on MHD mixed convective flow from a vertical surface with ohmic heating and viscous dissipation.

The object of the present study is to analyze the effect of ohmic heating on free convective MHD flow with heat and mass transfer over a vertical porous plate in the presence of thermal radiation, viscous dissipation and chemical reaction. The transformed governing equations are solved by using perturbation method. The effects of various governing parameters on velocity, temperature and concentration profile are discussed and shown through graphs and numerical values of skin-friction coefficient, Nusselt number and Sherwood number are presented through Tables.

2. Formulation of the Problem:

Consider a steady two-dimensional flow of a viscous incompressible, electrically conducting and radiating fluid over a semi-infinite vertical porous plate. The $x$ -axis is taken along the plate and $y$ -axis is normal to it. A uniform
magnetic field of strength $B_0$ is applied in a direction perpendicular to the plate. The radiative heat flux in the $x$-direction is considered negligible in comparison that in $y$-direction. All fluid properties are assumed to be constant except for the density in the body force term. Under the following assumption, the governing equation of continuity, momentum, energy and concentration can be written as:

$$
\frac{\partial \nu^v}{\partial y} = 0
$$

(1)

$$
v \cdot \frac{\partial \nu^u}{\partial y} = \nu \frac{\partial^2 \nu^u}{\partial y^2} + g \beta_r \left( T - T_u \right) +
$$

$$
g \beta_r \left( C - C_o \right) - \frac{\sigma B^2_o}{\rho} \nu^u \frac{\nu^u}{k_p}
$$

(2)

$$
\frac{\nu^v}{\partial y} = \frac{k}{\rho c_p} \left( \frac{\partial T}{\partial y} \right) + \left( \frac{\nu^u}{c_p} \frac{\partial \nu^u}{\partial y} \right) - \frac{1}{\rho c_p} \frac{\partial q_i}{\partial y} +
$$

$$
\frac{\sigma B^2_o}{\rho c_p} \nu^u \frac{\partial \nu^u}{\partial y}
$$

(3)

$$
\frac{\nu^v}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_c C
$$

(4)

where $u$ and $v$ are the velocity component, $\mu$ is the viscosity, $g$ is the acceleration due to gravitation, $\beta_r$ is coefficient of volume expansion, $\beta_r$ is the coefficient of volume expansion with concentration, $\nu$ is kinematic viscosity, $T'$ is the temperature, $T_u$ is the fluid temperature far away from the wall, $C'$ is the fluid concentration, $C_o$ is the fluid concentration far away from the wall, $\sigma$ is electrical conductivity, $\rho$ is fluid density, $k_p$ is permeability of porous medium, $k_c$ is rate of chemical reaction, $c_p$ is specific heat at constant pressure, $q_i$ is radiative flux and $k$ is thermal conductivity.

It is assumed that the level of species concentration is very low; hence the heat generated due to chemical reaction is neglected.

The governing equations in non-dimensional form are:

$$
\frac{u^*}{v_0} + \frac{v^*}{v_0} = 0, \frac{\nu^v}{\nu_0}, \frac{\theta^*}{T_0}, \frac{C}{C_o}, \frac{Pr}{\mu c_p}, \frac{k_c}{k_p}
$$

$$
Sc = \frac{v}{D} = \frac{\sigma B^2_o}{\rho \nu_0}, F = \frac{4 \nu v}{k_v}, Gr = \frac{v g \beta_r \left( T - T_u \right)}{v_0} k_o \nu_0, \frac{v k}{v_0}
$$

(9)

$$
Gm = \frac{v g \beta_r \left( C - C_o \right)}{v_0} k_o, \frac{v^2}{c_p \left( T - T_u \right)}
$$

(10)

where $Gr$ is the Grashof number, $Pr$ is the Prandtl number, $M$ is the Magnetic parameter, $F$ is the Radiation parameter, $Sc$ is the Schmidt number, $K$ is Permeability parameter, $k_o$ is Chemical reaction parameter, $Gm$ is mass Grashof number and $E$ is Eckert number.

The corresponding boundary conditions in non-dimensional form are:

$$
y = 0: u = 0, \theta = 1, C = 1
$$

(13)

$$
y \to \infty: u \to 0, \theta \to 0, C \to 0
$$

(14)

3. Solution

To solve the equations (10), (11) and (12) we assume the solution in the following form:

$$
u = u_0 + E \theta + o \left( E^2 \right)
$$

(15)

$$
\theta = \theta_0 + E \theta_0 + o \left( E^2 \right)
$$

(16)

$$
C = C_o + E C_o + o \left( E^2 \right)
$$

(17)

Substituting equations (15)-(17) into equations (10)-(12) and equating the harmonic and non-harmonic terms and neglecting the coefficient of $o \left( E^2 \right)$, the following equations are obtained.

Zero order terms:

$$
u_0' + u_0'' = -Gr \theta_0 - Gm C_o
$$

(18)

$$
\theta_0' + Pr \theta_0' = -Pr u_0'' - Pr F \theta_0
$$

(19)

$$
C_0' + Sc C_o' = k_o Sc C_o
$$

(20)

First order terms:

$$
u_1' + u_1'' = -Gr \theta_1 - Gm C_1
$$

(21)

$$
\theta_1' + Pr \theta_1' = -Pr u_0'' - Pr F \theta_1
$$

(22)

$$
C_1' + Sc C_o' = k_o Sc C_o
$$

(23)

The corresponding boundary conditions are:

$$
y = 0: u_0 = 0, u_1 = 0, \theta_0 = 1, \theta_1 = 0, C_0 = 1, C_1 = 0
$$

(24)

The solution of above equations which satisfies the boundary conditions are given as follow:

$$
C_0 = e^{-k \gamma}
$$

(25)

$$
\theta_0 = e^{-k \gamma}
$$

(26)

$$
u_0 = \left( -A_1 - A_2 \right) e^{-k \gamma} + A_1 e^{-k \gamma} + A_2 e^{-k \gamma}
$$

(27)

$$
C_1 = 0
$$

(28)
The effect of Schmidt number is, \( \theta_y = A_y e^{-ky} + A_y e^{-k_y} + A_y e^{-k_y} + A_y e^{-k_y} \) and \( \theta_y = A_y e^{-ky} + A_y e^{-k_y} + A_y e^{-k_y} + A_y e^{-k_y} \) is, \( \theta_y = A_y e^{-2ky} + A_y e^{-2k_y} + A_y e^{-2k_y} + A_y e^{-2k_y} \) is, \( \theta_y = A_y e^{-2ky} + A_y e^{-2k_y} + A_y e^{-2k_y} + A_y e^{-2k_y} \) on the concentration. It is leads to decrease in velocity, 

It is found that velocity decreases with increase in Prandtl number \( \Pr \). The effect of Prandtl number \( \Pr \) leads to decrease in velocity in both the case of air and water. This is due to the fact that application of transverse magnetic field will result in a resistive force known as Lorentz force which tends to resist the fluid flow and thus reducing its velocity. Velocity profile for different values of mass Grashof number \( Gm \) and Grashof number \( Gr \) are shown in the Fig.3 and 4.

It is observed that velocity increases with increase in the value of \( Gm \) and \( Gr \). Fig.5 and 6 illustrates the effect of Prandtl number \( Pr \) and Schmidt number \( Sc \) on fluid velocity. It is found that velocity decreases with increase in \( Pr \) and \( Sc \). This happens due to dominance of thermal diffusion and molecular diffusion over the viscous diffusion. The effect of radiation parameter \( F \) on velocity is shown by the Fig.7. It is found from the Fig.7 that fluid velocity decreases with increase in \( F \). Fig.8 illustrates the variation in temperature with Prandtl number \( Pr \) and radiation parameter \( F \).

From the Fig.8 we observe that temperature decreases with increases in the value of \( Pr \). This is due to the reason that increasing value of \( Pr \) lead to a decrease in the boundary layer and in general lower average temperature within the boundary layer. From the Fig.8 it is also observed that with increase in the value of \( F \) leads to decrease in temperature. This is due to the reason that radiation parameter defines the ratio of the thermal conduction relative to the thermal radiation, the increase in the radiation parameter decreases the thermal radiation, and hence decreases the temperature at large values of \( F \). The effect of magnetic parameter \( M \) on temperature is shown by the Fig.9. From the Fig.9 we observe that temperature increases with increases in the value of \( M \). This is due to the reason that magnetic field creates a Lorentz force that increases with \( M \). The fluid exhibits a resistance to this force by increasing the friction between its layers by which temperature increases. Fig. 10 depicts the effect of permeability parameter \( K \) on temperature. It is clear from the Fig.10 that temperature decreases with increase in the value of \( K \). The effect of Schmidt number \( Sc \) on temperature is shown by the Fig.11. It is observed from the Fig.11 that with increase in the value of \( Sc \) there is a decrease in the temperature. Fig.12 illustrates the effect of chemical reaction parameter \( k_i \) on the concentration. It is observed from the Fig.12 that concentration decreases with increase in the value of \( k_i \). The influence of Schmidt number \( Sc \) on the concentration is illustrated in Fig. 13. It is observed from the Fig.13 that increase in \( Sc \) leads to decrease the concentration of the fluid medium. This is due to the reason that an increase in \( Sc \) causes a decrease in the molecular diffusion \( D \).

**Skin friction:**

The expression for skin friction coefficient \( (C_f) \) is,

\[
C_f = \frac{-\tau}{\rho u y^2 \nu} = \left( \frac{\partial u}{\partial y} \right)_{y=0} = k_y (A_k + A_y) - A_k k_y + E (-A_k k_y - A_y k_y - 2A_y k_y - 2A_y k_y - A_k k_y - A_k k_y - A_y k_y - A_y k_y)
\]

**Nusselt number:**

The expression for Nusselt number \( (Nu) \) is,

\[
Nu = -\left( \frac{\partial \theta}{\partial y} \right)_{y=0} = k_y + E (k_y A_y + 2k_y A_y + 2k_y A_y + 2k_y A_y + A_y A_y + A_y A_y + A_y A_y)
\]

**Sherwood number:**

The expression for the Sherwood number \( (Sh) \) is,

\[
Sh = -\left( \frac{\partial C}{\partial y} \right)_{y=0} = k_i
\]

### 4. Results and Discussion

In order to get a physical insight into the problem, we have carried out numerical calculation for non-dimensional velocity, temperature and concentration. The obtained results for velocity, temperature and concentration have been presented graphically in Fig.1-13 for several sets of values of the pertinent parameters such as Grashof number \( Gm \), mass Grashof number \( Gm \), Magnetic parameter \( M \), Radiation parameter \( F \), Prandtl number \( Pr \), Eckert number \( E \), Chemical reaction parameter \( k_i \), Permeability parameter \( K \) and Schmidt number \( Sc \). To be realistic the values of \( Pr \) have been chosen as 0.71 for air and 7 for water.

Fig.1 demonstrates the effect of permeability parameter \( K \) on fluid velocity. It is observed from the Fig.1 that velocity increases with increase in \( K \) in both the case of generative reaction and destructive reaction. This is due to the reason that there is a decrease in the resistance of porous medium with the increase of \( K \) which tends to accelerate fluid flow. Fig.2 presents the velocity profile for different values of magnetic parameter \( M \). It is found that velocity decreases with increase in \( M \) in both the case of air and water. This is due to the fact that application of transverse magnetic field will result in a resistive force known as Lorentz force which tends to resist the fluid flow and thus reducing its velocity. Velocity profile for different values of mass Grashof number \( Gm \) and Grashof number \( Gr \) are shown in the Fig.3 and 4.
Figure 1: Velocity versus \( y \) for different value of \( K \) when
\[ Pr = 0.71, Gr = 5, Gm = 5, M = 2, F = 0.5, Sc = 0.22, E = 0.01 \]

Figure 2: Velocity versus \( y \) for different value of \( M \) when
\[ k_0 = 1, Gr = 5, Gm = 5, K = 1, F = 0.5, Sc = 0.22, E = 0.01 \]

Figure 3: Velocity versus \( y \) for different values of \( Gm \)
when \( M = 2, F = 0.5, Gr = 5, Pr = 0.71, K = 1, k_0 = 1, Sc = 0.22, E = 0.01 \)

Figure 4: Velocity versus \( y \) for different values of \( Gr \) when
\[ M = 2, F = 0.5, Gm = 5, Pr = 0.71, K = 1, k_0 = 1, Sc = 0.22, E = 0.01 \]

Figure 5: Velocity versus \( y \) for different values of \( Pr \) when
\[ M = 2, F = 0.5, Gr = 5, Gm = 5, K = 1, k_0 = 1, Sc = 0.22, E = 0.01 \]

Figure 6: Velocity versus \( y \) for different values of \( Sc \) when
\[ M = 2, F = 0.5, Gr = 5, Gm = 5, Pr = 0.71, K = 1, k_0 = 1, Sc = 0.22, E = 0.01 \]
Figure 7: Velocity versus $y$ for different values of $F$ when $M = 2, Sc = 0.22, Gr = 5, Gm = 5,$ $K = 1, k_0 = 1, Pr = 0.71, E = 0.01$

Figure 8: Temperature versus $y$ for different values of $Pr$ and $F$ when $M = 2, Sc = 0.22, Gr = 5, Gm = 5,$ $K = 1, k_0 = 1, E = 0.01$

Figure 9: Temperature versus $y$ for different values of $M$ when $Sc = 0.22, F = 0.5, Gr = 5, Gm = 5,$ $K = 1, k_0 = 1, Pr = 0.71, E = 0.01$

Figure 10: Temperature versus $y$ for different values of $K$ when $M = 2, Sc = 0.22, Gr = 5, Gm = 5,$ $F = 0.5, k_0 = 1, Pr = 0.71, E = 0.01$

Figure 11: Temperature versus $y$ for different values of $Sc$ when $F = 0.5, k_0 = 1, Pr = 0.71, E = 0.01$ $M = 2, K = 1, Gr = 5, Gm = 5$

Figure 12: Concentration versus $y$ for different values of $k_0$ when $Pr = 0.71, Sc = 0.22, Gr = 5, M = 2$ $K = 1, E = 0.01, Gm = 5, F = 0.5$
for different values of $P_r$, $k$, $E$, and $r$. Table 1 shows the effects of Magnetic parameter $M$, Radiation parameter $F$, Prandtl number $P_r$, Eckert number $E$, Chemical reaction parameter $k_0$, Permeability parameter $K$, and Schmidt number $Sc$ on the skin-friction coefficient $C_f$, Nusselt number $Nu$ and the Sherwood number $Sh$. From Table 1 it is observed that Sherwood number increases with increase in $Sc$ and $k_0$, whereas it remains unaffected for $E$. A variation in Nusselt number is shown in the Table 2. From Table 2 it is noticed that Nusselt number decrease with increase in $M$, $K$ and $k_0$, whereas it increases with increase in radiation parameter $F$. From Table 3 it is observed that skin-friction coefficient decreases with increase in the value of $M$ and $F$. But for $K$ and $k_0$ it shows reverse effect.

Table 1: Numerical values of Sherwood number ($Sh$) for various values of physical parameters when $M = 2, Gr = 5, Gm = 5, P_r = 0.71, K = 1$

<table>
<thead>
<tr>
<th>$E$</th>
<th>$k_0$</th>
<th>$Sc$</th>
<th>$Sh$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.04</td>
<td>0.22</td>
<td>0.3555</td>
</tr>
<tr>
<td>0.05</td>
<td>0.04</td>
<td>0.22</td>
<td>0.3555</td>
</tr>
<tr>
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<td>0.04</td>
<td>0.22</td>
<td>0.3555</td>
</tr>
<tr>
<td>0.05</td>
<td>0.04</td>
<td>0.60</td>
<td>0.9272</td>
</tr>
<tr>
<td>0.05</td>
<td>0.04</td>
<td>0.78</td>
<td>1.3747</td>
</tr>
<tr>
<td>0.05</td>
<td>0</td>
<td>0.22</td>
<td>0.3300</td>
</tr>
<tr>
<td>0.05</td>
<td>-0.04</td>
<td>0.22</td>
<td>0.2965</td>
</tr>
</tbody>
</table>

Table 2: Numerical values of Nusselt number ($Nu$) for various values of physical parameters when $Gr = 5, Gm = 5, P_r = 0.71, E = 0.01$

<table>
<thead>
<tr>
<th>$M$</th>
<th>$F$</th>
<th>$k_0$</th>
<th>$K$</th>
<th>$Nu$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.04</td>
<td>0.1</td>
<td>1.3664</td>
</tr>
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<td>0.1</td>
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</tr>
<tr>
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</tr>
<tr>
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<td>1.7429</td>
</tr>
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Table 3: Numerical values of skin-friction coefficient ($C_f$) for various values of physical parameters when $Gr = 5, Gm = 5, P_r = 0.71, E = 0.01$

<table>
<thead>
<tr>
<th>$M$</th>
<th>$F$</th>
<th>$k_0$</th>
<th>$K$</th>
<th>$C_f$</th>
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</tr>
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</tr>
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<td>1</td>
<td>4.9253</td>
</tr>
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5. Conclusion

In this paper, the effect of ohmic heating on MHD free convection flow along a vertical porous plate in the presence of chemical reaction and thermal radiation has been investigated. From the present study the following conclusion can be drawn:

1. The velocity increase with increase in $Gr$, $K$ and $Gm$, whereas it decreases with increase in $Pr$, $M$, $F$ and $Sc$.
2. Temperature increases with increase in $M$, whereas it decreases with increase in $Pr$, $F$, $Sc$ and $K$.
3. Concentration of the fluid decreases with increase in $k_0$ and $Sc$.
4. The rate of mass transfer in terms of Sherwood number increases with increase in $k_0$ and $Sc$.
5. The rate of heat transfer in terms of Nusselt number decreases with increase in $M$, $K$ and $k_0$, whereas it increases with increase in $F$.
6. Skin friction coefficient decreases with increase in the value of $M$, $F$, $k_0$, while it increase with increase in $K$.

Reference


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