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Design and Simulation of Auto Tuning of PID Controller using MRAC Technique for Coupled Tanks System

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Abstract: ***In chemical and petroleum industries, liquid level controllers are used very frequently. In control laboratory coupled tanks system is used to design liquid level controllers. The coupled tanks system is a nonlinear and dynamic plant. However, conventional fixed gain and normal feedback controller are unable to give desired performance. To overcome this difficulty, controller must be adaptive in nature. In this paper an auto tuning of proportional integral derivative (PID) controller based on model reference adaptive control (MRAC) by using MIT rule is designed. The proposed controller typically consists of a linear time-invariant Proportional, Integral and Derivative (PID) controller with an adjustment mechanism which tunes the PID parameters in such a way that error between reference model's output and actual plant's output will be minimised. The proposed algorithm is applied on coupled tanks system. Simulation results show the effectiveness of proposed controller.***

Keywords: PID Controller, MRAC, MIT rule and Coupled Tanks System.

1. Introduction

The controlling of liquid level is basic problem in industries like petroleum industry, chemical industry, water treatment plant and paper making plant etc. Mixing of different chemicals and management of liquid level are performed by coupled tanks systems. It is essential for control framework designers to realize that how it functions and controls the fluid level in tanks. The issue of level control in coupled tanks procedures is framework motion and communicating trademark. To comprehend this, numerous control systems have been utilized like PID Controller, fuzzy logic control and two degree of freedom (2-DOF) PID etc. [1], [2] and [3]. However, the time varying environmental condition and inherent nonlinear properties in system frequently reduces the performance of PID. This means that gains of proportional, integral and derivative terms are not being fixed because it is unable to give robust performance. Therefore further adaptive algorithms are necessary for robustness. In these past years, several methods and algorithms are introduced for PID parameters tuning like PID gain scheduling, Ziegler–Nichols' algorithm, Ant algorithm and Fuzzy PID tuning etc.

In this paper design and analysis of PID tuning technique using MRAC and MIT rule is done. This technique provides online automatic tuning of PID parameters. Adaptation algorithm tunes the PID parameters until the performance is satisfactory and then the system continues with updated parameters.

The rest of paper is written in following way. In section-2, mathematical modeling of coupled tank system is analyzed. In section-3, MRAC and MIT gradient approach is presented. Section-4, deals with designing and analysis part of PID tuning algorithm using MRAC concept. Simulation and results are discussed in section -5. Finally, section-6 draws the conclusion.

2. Coupled Tanks System

The schematic block diagram of coupled tank is shown if Fig.1.

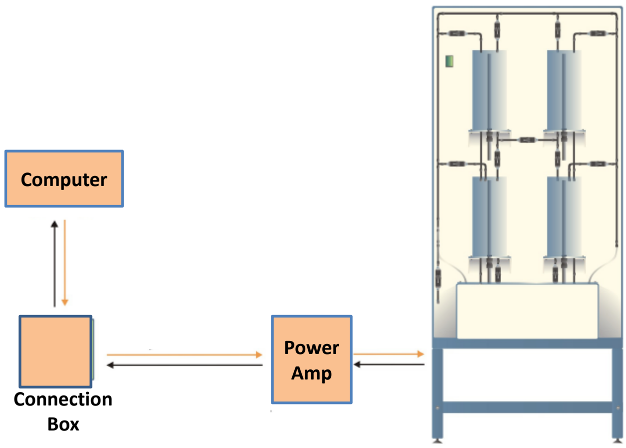

Figure 1: Schematic block diagram of coupled tank process

As shown in Figure 1, the Coupled Tanks unit comprises of four tanks set on an apparatus. Fifth supply tank is set at the base. In the store two submersible pumps are put, which pump the water on summon to the tanks. The water streams uninhibitedly to the base tanks through the configurable hole. The way the water moves through the setup can be arranged from numerous points of view with manual valves [6]. The objective of this problem is control the water level in tanks. In this section a mathematical modeling of two coupled tanks process is discussed. The phenomenological model of two tanks is shown in Fig. 2.

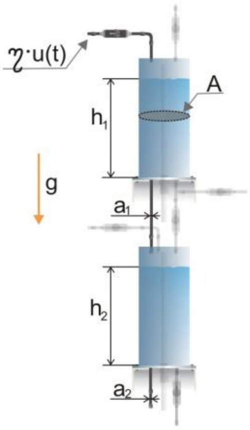

Figure 2: Phenomenological models of two coupled tanks

The coupled tanks system is a nonlinear plant. In the Fig. 2, h_1 and h_2 are the water levels in tank1 and tank2 respectively. $u(t)$ is defined as a control signal in voltage term.

$$\frac{dh_1(t)}{dt} = -\frac{\alpha_1}{A}\sqrt{2.g.h_1(t)} + \eta.u(t) \tag{1}$$

$$\frac{dh_2(t)}{dt} = \frac{\alpha_1}{A}\sqrt{2.g.h_1(t)} - \frac{\alpha_2}{A}\sqrt{2.g.h_2(t)} \tag{2}$$

Where,
α_1 = tank1 outlet area, α_2 = tank2 outlet area, η = constant relating the control voltage with the water flow from the pump, g = gravitational constant, A = cross-sectional area of the tanks.

After linearization of (1) and (2):

$$\frac{d\Delta h_1(t)}{dt} = \frac{d}{dh_1}\left(-\frac{\alpha_1}{A}\sqrt{2.g.h_1(t)}\right) + \eta.\Delta u(t) \tag{3}$$

$$\frac{d\Delta h_2(t)}{dt} = \frac{d}{dh_1}\left(\frac{\alpha_1}{A}\sqrt{2.g.h_1(t)}\right).\Delta h_1(t) - \frac{d}{dh_2}\left(\frac{\alpha_2}{A}\sqrt{2.g.h_2(t)}\right).\Delta h_2(t) \tag{4}$$

Taking the Laplace transformation of (3) and (4), the following results are obtained.

$$s\Delta H_1(s) = -\left(\frac{\alpha_1}{A}\right)^2 \frac{g}{\eta.u_0}.\Delta H_1(s) + \eta.\Delta U(s) \tag{5}$$

$$s\Delta H_2(s) = \left(\frac{\alpha_1}{A}\right)^2 \frac{g}{\eta.u_0}.\Delta H_1(s) - \left(\frac{\alpha_2}{A}\right)^2 \frac{g}{\eta.u_0}.\Delta H_2(s) \tag{6}$$

Consequently the respective transfer functions are as follows:

$$\frac{\Delta H_1(s)}{\Delta U(s)} = \frac{\eta}{s + \left(\frac{\alpha_1}{A}\right)^2 \frac{g}{\eta u_o}} \tag{7}$$

$$\frac{\Delta H_2(s)}{\Delta H_1(s)} = \frac{\left(\frac{\alpha_1}{A}\right)^2 \frac{g}{\eta u_o}}{s + \left(\frac{\alpha_2}{A}\right)^2 \frac{g}{\eta u_o}} \tag{8}$$

Where, u_0 =2.7

The above equations (7) and (8) are showing the transfer function of tank1 and tank2 respectively.

3. Model Reference Adaptive Control (MRAC)

The MRAC is a reference model based adaptive control system. The adjustment mechanism is designed using MIT rule. The block diagram of MRAC is shown in Fig. 3.

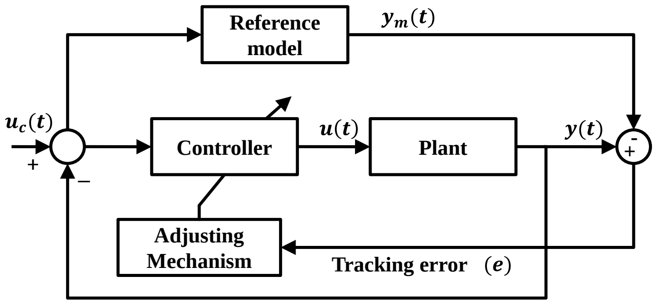

Figure 3: Block diagram of MRAC

In this paper, MIT rule is used for adjustment mechanism. The MRAC control strategy is obtained using gradient approach of MIT rule. According to gradient approach, a cost function $(\psi(\phi))$ is decided in terms of tracking error (e) as shown below. The tracking error is defined as error between output of reference model and output of plant.

$$e = y(t) - y_m(t) \tag{9}$$

$$\psi(\phi) = \frac{1}{2}e^2(\phi) \tag{10}$$

According to the MIT rule, rate of change of ϕ is directly proportional to negative gradient of cost function, as shown in the following equations:

$$\frac{d\phi}{dt} = -\gamma\frac{\partial\psi}{\partial\phi} \tag{11}$$

$$\frac{d\phi}{dx} = -\gamma e\frac{\partial e}{\partial\phi} \tag{12}$$

Where, ϕ = controller parameter vector, e = tracking error, γ = adaptive gain and $\frac{\partial e}{\partial\phi}$ = sensitivity derivative

4. PID Tuning Algorithm using MRAC for Coupled Tanks Process

The proposed algorithm is an adaptive control law in which the value of proportional gain, derivative gain and integral gain are automatically updated in such a way that plant follows the reference model.

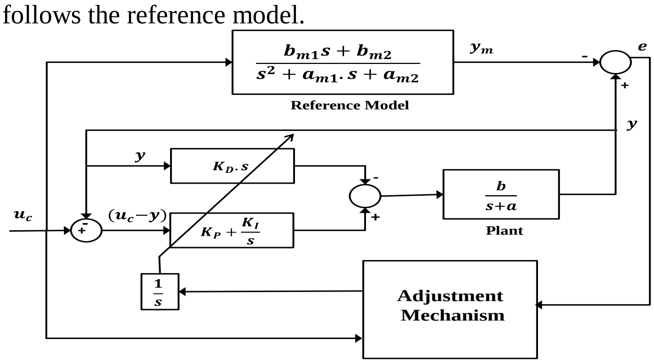

Figure 4: Block Diagram of PID Tuning Algorithm using MRAC

Block diagram of PID tuning algorithm is shown in Fig. 4. Mathematical analysis of the proposed algorithm is described subsequently. Let us consider a second order plant ($G(s)$) having the equation as follows:

$$G(s)=\frac{b}{s+a} \qquad (13)$$

From Fig. 4, control signal is described as

$$u(t)=K_P\left[u_c-y_m\right]+\frac{K_I}{s}\left[u_c-y_m\right]-K_D.s.y_m \qquad (14)$$

After some mathematical calculations, we have obtained following equation.

$$\frac{y(s)}{u_c(s)}=\frac{b(K_P s+K_I)}{(1+bK_D)s^2+(a+bK_P)s+bK_I} \qquad (15)$$

To calculate the parameter equations of PID controller, the following approximations can be made [5].

$$s^2+\frac{(a+bK_P)}{(1+bK_D)}s+\frac{bK_I}{(1+bK_D)}\approx s^2+a_{m1}.s+a_{m2} \qquad (16)$$

According to (13) and (14), reference model can be obtained as

$$\frac{y_m(s)}{u_m(s)}=\frac{b_{m1}s+b_{m2}}{s^2+a_{m1}.s+a_{m2}} \qquad (17)$$

To calculate the tuning equation of PID parameters using MIT rule, equation can be written as

$$\frac{dK_P}{dt}=-\gamma_P\frac{\partial\psi}{\partial K_P}=-\gamma_P\times\frac{\partial\psi}{\partial e}\times\frac{\partial e}{\partial y}\times\frac{\partial y}{\partial K_P} \qquad (18)$$

$$\frac{dK_I}{dt}=-\gamma_I\frac{\partial\psi}{\partial K_I}=-\gamma_I\times\frac{\partial\psi}{\partial e}\times\frac{\partial e}{\partial y}\times\frac{\partial y}{\partial K_I} \qquad (19)$$

$$\frac{dK_D}{dt}=-\gamma_D\frac{\partial\psi}{\partial K_D}=-\gamma_D\times\frac{\partial\psi}{\partial e}\times\frac{\partial e}{\partial y}\times\frac{\partial y}{\partial K_D} \qquad (20)$$

From solving (17), (18), (19) and (20), we get final PID tuning equations obtained as following.

$$\frac{dK_P}{dt}=-\gamma_P.e.\frac{bs.\left[u_c-y\right]}{(1+bK_D)s^2+(a+bK_P)s+bK_I} \qquad (21)$$

$$\frac{dK_I}{dt}=-\gamma_I.e.\frac{b.\left[u_c-y\right]}{(1+bK_D)s^2+(a+bK_P)s+bK_I} \qquad (22)$$

$$\frac{dK_D}{dt}=\gamma_D.e.\frac{b.s^2\left[y\right]}{(1+bK_D)s^2+(a+bK_P)s+bK_I} \qquad (23)$$

Above three equations show the change in PID parameters with respect to time.

5. Simulation Results

For Simulation, based on (21), (22) and (23), a Simulink model is implemented in MATLAB and applied on coupled tanks process. The Simulink model is presented in Fig. 5.

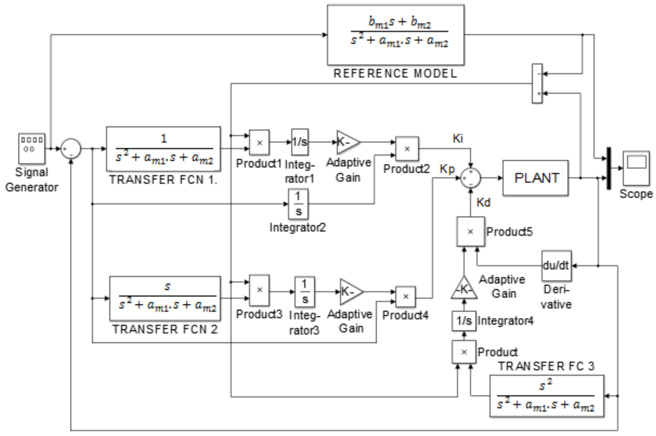

Figure 5: Simulink Model for Coupled Tanks Plant

Transfer function of coupled tanks can be calculated from (7) and (8). The physical parameters of coupled tanks are listed in Table 1.

Table 1: Physical Parameters of Coupled Tanks Process

Physical Parameters	Value
Cross sectional area of tanks (A)	0.01389 m^2
Tank1 outlet area (a_1)	50.265 × 10^{-6} m^2
Tank2 outlet area(a_2)	50.265 × 10^{-6} m^2
Gravitational constant (g)	9.8 m/sec^2
Constant relating the control voltage with water flow from the pump (η)	2.2 × 10^{-3}

By putting these physical parameters in (7) and (8), we get transfer function of $G_1(s)$ and $G_2(s)$ for tank1 and tank2 respectively.

$$G_1(s)=\frac{0.0022}{s+0.02159} \qquad (24)$$

$$G_2(s)=\frac{0.02159}{s+0.02159} \qquad (25)$$

Let the appropriate reference model be

$$G_m(s)=\frac{s+1}{s^2+16s+1} \qquad (26)$$

The simulation is carried out for different values of adaptive gains and results are obtained as shown in Fig. 6.

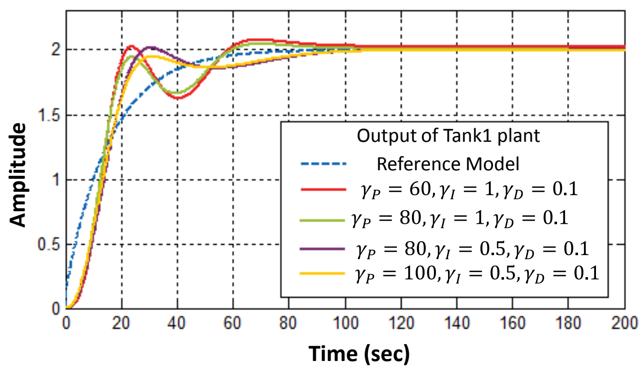

Figure 6: Response of Tank1 with Different Values of Adaptive Gains

Table 2: Control Parameters with Different Values of Adaptive Gain for Tank1

	γ_P	γ_I	γ_D	Overshoot	Rise Time	Settling Time
1	60	1	0.1	1.5%	28.452 sec	82.638 sec
2	80	1	0.1	0%	17.967 sec	88.054 sec
3	80	0.5	0.1	2.6%	21.803 sec	83.904 sec
4	100	0.5	0.1	0%	17.108 sec	80.095 sec

From table 2, we observed that values of $\gamma_P = 100, \gamma_I = 0.5$ and γ_D=0.1 gives best performance comparatively. The simulation is done using transfer function of tank1 and tank2 with the set point as shown in Table 3.

Table 3: Operating set point for simulation

Tank	$t = 0$sec	$t = 200$sec	$t = 400$sec
1	2	4	6
2	2	4	6

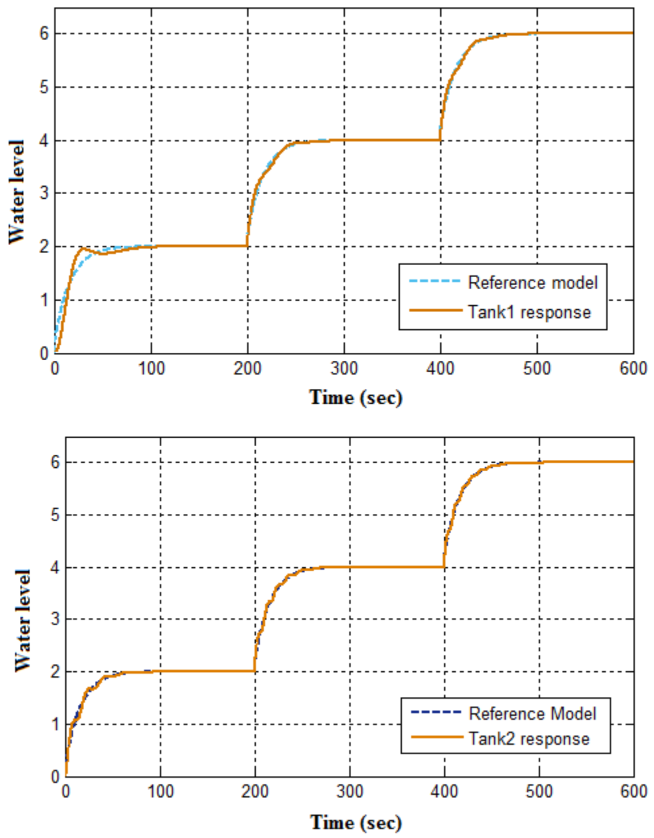

Figure 7: Simulation Results for Tank1 and Tank2

From simulation results, we observaed that proposed controller gives robust performance. From Fig. 7, during first step, adaptation mechanism tunes the PID parameters to track the output of reference model. Results show that updated PID controller is enable to track the reference model quite smoothly.

6. Conclusion

Adaptive control is a widely used approach which deals with plants having uncertain or time-varying parameters. In this paper, we have proposed an adaptive PID controller which tunes the values of proportional gain, integral gain and derivative gain to track the reference model. Mathematical analysis of proposed controller is presented. Simulation results are obtained in MATLAB & SIMULINK and these results demonstrate the adequacy of proposed controller.

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