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A New Approach to Solve Fuzzy Travelling Salesman Problems by using Ranking Functions

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Abstract: ***Fuzzy number can be used to solve many real life problems like Travelling Salesman, Assignment Problems and so on. In this paper a new method is proposed for solving Travelling Salesman problems using Transitive Fuzzy Numbers. The transitive trapezoidal fuzzy numbers are to solve a general travelling salesman problem with an optimal solution. The efficiency of this method is proved by solving a numerical problem.***

Keywords: Ranking, Heptagon Fuzzy numbers, Zero suffix method

1. Introduction

The ordinary form of travelling salesman problem, a map of cities is given to the salesman problem, and he has to visit all the cities only once and return to the starting point to complete the tour. There are different methods to solve travelling salesman problem. But this method solve easily and minimum number of iteration. This paper is organized as follows: In section 2 Basic definitions and transitive trapezoidal fuzzy numbers. In section 3 formulation of fuzzy travelling salesman problems are presented.

1.1 Transitive Trapezoidal Fuzzy Number

If $a^{(1)} \sim a^{(4)}$, then the trapezoidal fuzzy number A = ($a^{(1)}, a^{(2)}, a^{(3)}, a^{(4)}$) is called transitive trapezoidal fuzzy number. It is denoted by A = ($a^{(1)}$, $a^{(4)}$), Where $a^{(1)}$ is core (A) , $a^{(4)}$ is left width and right width of c. The parametric form of a transitive trapezoidal fuzzy numbers is represented by
$A = [\, a^{(1)} - a^{(2)}(1-r) \,,\, a^{(1)} + a^{(3)}(1-r) \,,\, a^{(1)} - a^{(1)}(1-r) \,]$

1.2 Ranking of Trapezoidal Fuzzy Number

For every A= ($a^{(1)}$, $a^{(2)}$, $a^{(3)}$, $a^{(4)}$) € F(R) , the ranking function R : F(R) → R by graded mean is defined as

$$R(A) = \left[\frac{a_1 + 2a_2 + a_3 + 3\,a_4}{7}\right]$$

(Therefore $a_1 \sim a_4$)
For any two fuzzy trapezoidal fuzzy numbers A = ($a^{(1)}$, $a^{(2)}$, $a^{(3)}$, $a^{(4)}$) and
B = ($b^{(1)}$, $b^{(2)}$, $b^{(3)}$, $b^{(4)}$) in F(R) , we define orders on F(R) by

i. $A < B \Leftrightarrow R(A) > R(B)$
ii. $A > B \Leftrightarrow R(A) > R(B)$
iii. $A \approx B \Leftrightarrow R(A) \approx R(B)$
iv. $A = -B \Leftrightarrow R(A) + R(B) = 0$

For transitive trapezoidal fuzzy numbers $a^{(1)} \approx a^{(4)}$.

2. Preliminaries

A fuzzy number A is a mapping $\mu_A(x) : R \rightarrow [\,0,1\,]$ with the following properties:

1. μ_A is an upper semi-continuous function on R.
2. $\mu_A(x) = 0$ outside of some interval $[a_1 , b_2]$ € R
3. There are real numbers a_2, b, $a_1 \le a_2 \le b_1 \le b_2$ and
   i. $\mu_A(x)$ is a monotonic increasing function on $[\, a_1 , a_2]$
   ii. $\mu_A(x)$ is a monotonic decreasing function on $[\, b_1 , b_2]$
   iii. $\mu_A(x) = 1$ x in $[a_2 , b_1]$. The set of all fuzzy numbers is denoted by F.

2.1 Trapezoidal Fuzzy Numbers

Let A = (a,b,c,d) is a fuzzy set of the real line R whose membership function $\mu_n(x)$ is defined as

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a} & \text{if} \quad a \le x \le b \\ 1 & \text{if} \quad b \le x \le c \\ \dfrac{d-x}{d-c} & \text{if} \quad c \le x \le d \\ 0 & \text{otherwise} \end{cases}$$

2.2 Linear Programming Formulation of Fuzzy Travelling Salesman Problems.

Suppose a person has to visit n cities. He starts from a particular city, visits each city once and then returns to the starting point. The fuzzy travelling costs from i^{th}city to j^{th} city is given by $\check{C}_{ij}$. The chosen fuzzy travelling salesman problem may be formulated by Maximize (or) Minimize $Z = \Sigma x_{ij}$ j = 1,2,……..n, $j \neq i$ and x_{ij}=1 , i=1,2,……n , $i \neq j$ and j=1

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1,2\ldots n \quad i \neq j$$

Table : Fuzzy Travelling Costs

City → ↓	1	2	…..	j	……	n
1	-	$\tilde{E}_{12}$	……	$\tilde{E}_{ij}$	…….	$\tilde{E}_{1n}$
: :	: :	: :	: :	: :	: :	: :
j	$\tilde{E}_{j1}$	$\tilde{E}_{j2}$	……	-	……	$\tilde{E}_{jn}$
: :	: :	: :	: :	: :	: :	: :
N	$\tilde{E}_{n1}$	$\tilde{E}_{n2}$	……	$\tilde{E}_{nj}$	……	-

Where $\tilde{E}_{ij} = (a_{ij}, b_{ij}, c_{ij} , d_{ij})$

3. Numerical Example

	A	B	C	D
A	-	(4,3,3,4)	(1,4,2,1)	(5,13,1,5)
B	(2,6,3,2)	-	(6,4,2,6)	(2,3,7,2)
C	(5,13,1,5)	(5,6,7,5)	-	(1,4,2,1)
D	(6,12,1,6)	(1,4,2,1)	(2,6,3,2)	-

Solution

Here

$$R(A) = a_1 + \left(\frac{2a_2 + a_3 + 3a_4}{7}\right)$$

Therefore

$$R(A_{12}) = 4 + \left(\frac{2(3) + 2 + 4}{7}\right)$$

= 2.28

Similarly

$R(A_{13}) = 2$; $R(A_{14}) = 6.71$;

$R(A_{21}) = 3.28$; $R(A_{23}) = 4.85$; $R(A_{24}) = 3.28$

$R(A_{31}) = 6.71$; $R(A_{32}) = 5.5$; $R(A_{34}) = 3.28$

Step -1 Row Reduction

	A	B	C	D
A	-	2.28	2	6.71
B	3.28	-	4.85	3.28
C	6.71	5.5	-	2
D	7	2	3.28	-

Step -2 Column Reduction

	A	B	C	D
A	-	0.28	$\boxed{0}$	4.71
B	$\boxed{0}$	-	1.57	28
C	4.71	3.5	-	$\boxed{0}$
D	5	$\boxed{0}$	1.28	-

Therefore The optimum solution is A→C→D→B→D = 9.28

4. Conclusion

In this paper the travelling salesman are considered as imprecise numbers described by fuzzy numbers which are more realistic and general in nature. Numerical examples show that by this method we can have the optimal solution as well as the crisp and fuzzy optimal total cost. This technique occurring real life situations.

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