

# A New Method for Ranking Exponential Pentagon Fuzzy Numbers with Using Assignment Problem

Dr. S. Chandrasekaran<sup>1</sup>, G. Kokila<sup>2</sup>, Junu Saju<sup>3</sup>

<sup>1</sup>Associate Professor and Head, PG & Research Department of Mathematics, Khadir Mohideen College, Adirampattinam – 614 701,

<sup>2,3</sup>Khadir Mohideen College, Adirampattinam – 614 701

**Abstract:** In this paper, we present a method for ranking of two exponential pentagon numbers. In this study a new cardinality between exponential fuzzy numbers is proposed cardinality in this relatively simple and easier in computation and ranks various types of exponential fuzzy numbers. For this results of the proposed approach to calculate Hungarian Assignment problems.

**Keywords:** Exponential fuzzy numbers ranking method

## 1. Introduction

Ranking fuzzy numbers are an important tool in decision making. In fuzzy decision analysis, fuzzy quantities are used to describe the performances of alternatives in modeling real world problems. Various ranking procedures have been developed since 1976, when the theory of fuzzy sets were first introduced by Zadeh[24]. Ranking fuzzy numbers were first proposed by Jain [25] for decision making in fuzzy subsets. In this paper presenting a new method for ranking exponential fuzzy numbers with using Assignment Problem.

### Preliminaries

#### DEFINITION:

Let  $R$  be the set of real numbers. Then closed interval  $[a, b]$  is said to be an interval number, Where  $a, b \in R$ ,  $a \leq b$ .

#### DEFINITION:

The fuzzy set  $A$  the universe of discourse  $X$  is called a normal fuzzy set implying that there exist at least one  $x \in X$  such that  $\mu_A(x) = 1$ .

#### DEFINITION:

Let  $a = [a_1, a_2]$ ;  $b = [b_1, b_2]$  be two interval numbers. Then the distance between  $(a, b)$  denoted by  $d(a, b)$  is defined by<sup>0.5</sup>  
 $d(a, b) = \int_{-0.5}^0 [(a_1+a_2)/2 + x(a_2-a_1)] - [(b_1+b_2)/2 + x(b_2-b_1)] dx$

#### DEFINITION:

To qualify as a fuzzy number a fuzzy set  $A$  on  $R$  must posses atleast the following three properties :

1.  $A$  must be normal fuzzy set.
2.  $\alpha_A$  must be a closed interval for every  $\alpha \in [0, 1]$
3. The support of  $A_1, O^+A$  must be bounded.

### Ranking of Pentagon Fuzzy Number

Let  $\tilde{A}_p = (a_1, a_2, a_3, a_4, a_5)$  a ranking method is devised based on the following formula.

$$\text{Card } B = wc + w/3e' [(d-e)(1-e')] - [(b-a)(e'-1)]$$

Let  $\tilde{A}_p = (a_1, a_2, a_3, a_4, a_5)$ ;  $\tilde{U}_p = (b_1, b_2, b_3, b_4, b_5)$  be two pentagon fuzzy number then,

$$\tilde{A}_p \geq \tilde{U}_p \Leftrightarrow R(\tilde{A}_p) \geq R(\tilde{U}_p)$$

$$\tilde{A}_p \geq \tilde{U}_p \Leftrightarrow R(\tilde{A}_p) \geq R(\tilde{U}_p)$$

$$\tilde{A}_p \leq \tilde{U}_p \Leftrightarrow R(\tilde{A}_p) \leq R(\tilde{U}_p)$$

### Graphical Representation

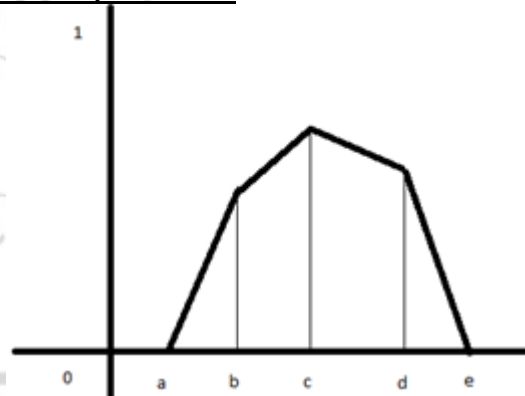


Figure (a): Graphical Representation of Pentagon Fuzzy Number

### Definition

Generally, a generalized fuzzy number  $A$  is described as any fuzzy subsets of the real line  $R$ , Whose membership function  $\mu_n$  satisfies the following conditions,

- I.  $\mu_A$  is a continuous mapping from  $R$  to the closed interval  $[0, 1]$ .
- II.  $\mu_A(x) = 0, -\infty < x \leq a$ .
- III.  $\mu_A(x) = L_1(x)$  is strictly increasing on  $[a, b]$ .
- IV.  $\mu_A(x) = L_2(x)$  is strictly increasing on  $[b, c]$ .
- V.  $\mu_A(x) = W, x = c$ .
- VI.  $\mu_A(x) = R_1(x)$  is strictly decreasing on  $[c, d]$ .
- VII.  $\mu_A(x) = R_2(x)$  is strictly decreasing on  $[d, \infty]$ .

Where  $0 \leq w \leq 1$ ,  $a, b$  are real numbers,  $c, d$  are positive real numbers. we denote this type of generalized exponential fuzzy number as  $B = (a, b, c, d, e; w)_E$ . Especially, when  $w = 1$ , we denoted it as  $B = (a, b, c, d, e; w)_E$  exponential fuzzy number, Based on the integral value of graded mean h-level as follows.

However, these fuzzy numbers always have a fix range as  $[c, d]$ . Here, we define their general forms as follows:

$$f_E(x) = \begin{cases} w e^{-[(b-x)/(b-a)]} & \text{for } a \leq x \leq b \\ w e^{-[(c-x)/(c-b)]} & \text{for } b \leq x \leq c \\ w & x = c \\ w e^{-[(x-c)/(d-c)]} & \text{for } c \leq x \leq d \\ w e^{-[(x-d)/(e-d)]} & \text{for } x > e \end{cases}$$

Where  $0 < w \leq 1$ ,  $a, b$  are real numbers and  $c, d, e$  are positive real numbers.

Now let monotonic functions be

$$L_1(x) = w e^{-[(b-x)/(b-a)]}$$

$$L_2(x) = w e^{-[(c-x)/(c-b)]}$$

$$R_1(x) = w e^{-[(x-c)/(d-c)]}$$

$$R_2(x) = w e^{-[(x-d)/(e-d)]}$$

**Proposition**

Cardinality of an exponential pentagon fuzzy number  $B$  characterized by (2.1) is the value of the integral.

$$\text{Card } B = w/e' \{ [(b-a)(e'-1)] + [(c-b)(e'-1)] + [(c-d)(e'-1)] + [(e-d)] \} + w$$

Proof:

$$\begin{aligned} \text{Card } B &= \int_a^b B(x) dx \\ &= \int_a^b w e^{-\left(\frac{b-x}{b-a}\right)} dx + \int_b^c w e^{-\left(\frac{c-x}{c-b}\right)} dx + \\ &\int_c^d w e^{-\left(\frac{x-c}{d-c}\right)} dx + \int_e^{\infty} w e^{-\left(\frac{x-d}{e-d}\right)} dx + w \end{aligned} \quad (1)$$

$$= w[(b-a)(1-1/e')] + w[(c-b)(1-1/e')] + w[(c-d)(1/e' - 1)] + w[(e-d)(1/e)]$$

$$\text{Card } B = w/e' [(b-a)(e'-1)] + [(c-b)(e'-1)] + [(c-d)(e'-1)] + [(e-d)] + w$$

**Proposition 2**

If  $A$  is an exponential pentagon fuzzy number with light tails then

$$\text{Card } B = wc + w/3e' [(d-e)(1-e')] - [(b-a)(e'-1)] d b$$

$$\text{Card } B = wc + 1/3 \left[ \int w e^{-[(x-d)/(e-d)]} dx - \int w e^{-[(b-x)/(b-a)]} dx \right] e a$$

$$= wc + 1/3 [w(d-e)(1-1/e') - (b-a)(1/e'-1)]$$

$$= wc + 1/3e' [w(d-e)(e'-1) - w(b-a)(e'-1)]$$

$$\text{Card } B = wc + w/3e' [(d-e)(e'-1) - (b-a)(e'-1)]$$

**Proposed Approach**

In this section some important results that are useful for the proposed approach are proved.

**Numerical Example**

To solve the following Assignment Problem of minimal cost by using the Proposed Approach method.

$$\begin{pmatrix} (0.2, 0.5, 0.3, 0.4, 0.1; 0.35) & (0.1, 0.3, 0.4, 0.5, 0.6; 0.2) \\ (0.2, 0.3, 0.5, 0.4, 0.1; 0.1) & (0.1, 0.2, 0.3, 0.5, 0.6; 0.4) \\ (0.1, 0.2, 0.3, 0.5, 0.6; 0.21) & (0.2, 0.4, 0.6, 0.8, 0.9; 0.5) \\ (0.3, 0.4, 0.5, 0.6, 0.7; 0.28) & (0.1, 0.15, 0.2, 0.25, 0.6, 0.48) \\ (0.1, 0.2, 0.3, 0.5, 0.6; 0.4) & (0.2, 0.3, 0.5, 0.4, 0.1; 0.1) \\ (0.2, 0.4, 0.7, 0.75, 0.8; 0.39) & (0.1, 0.15, 0.2, 0.25, 0.6; 0.48) \end{pmatrix}$$

$$\begin{pmatrix} (0.1, 0.2, 0.3, 0.4, 0.6; 0.51) & (0.1, 0.2, 0.4, 0.1, 0.7; 0.5) \\ (0.1, 0.2, 0.4, 0.6, 0.7; 0.3) & (0.1, 0.2, 0.3, 0.4, 0.6; 0.25) \end{pmatrix}$$

$$\text{Card } B = wc + w/3e' [(d-e)(1-e')] - [(b-a)(e'-1)]$$

Let  $b_{11} = (0.2, 0.5, 0.3, 0.4, 0.1; 0.35)$  then

Here  $a=0.2$

$b=0.5$

$c=0.3$

$d=0.4$

$e=0.1$

$w=0.35$

$e'=2.72$

$$\text{Card } b_{11} = (0.35 \cdot 0.3) + (0.35 / (3 \cdot 2.72)) [(0.4 - 0.1)(-1.72) - (0.3)(1.72)]$$

$$\text{Card } b_{11} = 0.06$$

Similarly

$$\text{Card } b_{12} = 0.07 \quad \text{Card } b_{13} = 0.04 \quad \text{Card } b_{14} = 0.1 \quad \text{Card } b_{21} = 0.07$$

$$\text{Card } b_{22} = 0.27 \quad \text{Card } b_{23} = 0.12 \quad \text{Card } b_{24} = 0.05$$

$$\text{Card } b_{31} = 0.12 \quad \text{Card } b_{32} = 0.04 \quad \text{Card } b_{33} = 0.26 \quad \text{Card } b_{34} = 0.05$$

$$\text{Card } b_{41} = 0.03 \quad \text{Card } b_{42} = 0.2 \quad \text{Card } b_{43} = 0.16 \quad \text{Card } b_{44} = 0.15$$

$$\text{Card } B_{ij} = \begin{pmatrix} 0.06 & 0.07 & 0.04 & 0.1 \\ 0.07 & 0.27 & 0.12 & 0.05 \\ 0.12 & 0.04 & 0.26 & 0.05 \\ 0.03 & 0.2 & 0.16 & 0.15 \end{pmatrix}$$

$$\text{Row Reduction: } = \begin{pmatrix} 0.02 & 0.03 & 0 & 0.06 \\ 0.02 & 0.22 & 0.07 & 0 \\ 0.08 & 0 & 0.22 & 0.01 \\ 0 & 0.17 & 0.13 & 0.12 \end{pmatrix}$$

$$\text{Column Reduction: } = \begin{pmatrix} 0.02 & 0.03 & 0 & 0.06 \\ 0.02 & 0.22 & 0.07 & 0 \\ 0.08 & 0 & 0.22 & 0.01 \\ 0 & 0.17 & 0.13 & 0.12 \end{pmatrix}$$

Therefore

**Card  $B_{ij} = 0.16$**

**Table 1:** A Comparison of Assignment through Ranking with different approaches

<i>Approaches</i>	<i>Results for Ranking Using Assignment Problem</i>
Salim Rezvani	$R(A) = 0.27$
Amitkumar, Pushpinder Singh and Amirpreet Kumar(2010)	$R(A) = 0.2$
Rajeshwari and Sahaya Sudha.A	$R(A) = 0.23$
Salim Rezvani(2014)	$R(A) = 1$
S.Rezvani , M.Molani and M.Ebrahimi (2013)	$R(A) = 0.2$
<b>Proposed Approach</b>	<b><math>R(A) = 0.16</math></b>

## 2. Conclusion

A method for comparison generalized exponential ranking fuzzy numbers provide correct ordering pentagon fuzzy numbers. This proposed ranking method used to solve the real life problem simply and easily.

## References

- [1] S.Abbasbandy, T.Hajjari, A new approach for ranking of trapezoidal fuzzy numbers, *Computers and Mathematics with Applications*, 57(2009) 413-419
- [2] G.Bortolan, R.Degani, A review of some methods for ranking fuzzy subsets, *Fuzzy Sets and Systems*, 15(1) (1985) 1-19
- [3] S.Bodjanova, Median value and median interval of a fuzzy number, *Information Sciences*, 172(2005) 73-89
- [4] S. H. Chen, Ranking fuzzy numbers with maximizing set and minimizing set , *Fuzzy Sets and Systems*, 17(2) (1985) 113-129
- [5] S.J.Chen, S.M.Chen, Fuzzy risk analysis based on the ranking of generalized trapezoidal fuzzy numbers, *Applied Intelligence*, 26(2007) 1-11
- [6] Shan-Hou Chen, Guo-Chin Li, Representation, Ranking and Distance of Fuzzy Number eith Exponential Membership Function Using Graded mean Integeration Method, *Tamsui Oxford journal of Mathematical Sciences* , 16 (2000) 123-131.
- [7] CH.Cheng, A new approach for ranking fuzzy numbers by distance method , *Fuzzy Sets and Systems*, 95(3) (1998) 307-317
- [8] Ta-Chung Chu, Chung- Tsen Tsao, Ranking fuzzy numbers with an area between the centroid point and original point, *Computers and Mathematics with Applications*, 43(1-2) (2002) 111-117
- [9] D.Dubois, H.Prade, The mean value of a fuzzy number , *Fuzzy Sets and Systems*, 24(3)(1987) 279-300
- [10] R.Jain, Decision making in the presence of fuzzy variables. *IEEE Tranactions on Systems, Man and Cybernetics*, 6(10)(1976) 698-703
- [11] C.Liang, J.Wu, J.Zhang, Ranking indices and rules for fuzzy numbers based on gravity centerpoint, Paper presented at the 6<sup>th</sup> world congress on Intelligent Control and Automation, Dalian, China, (2006) 21-23
- [12] A.Kumar, P.Singh, A.Kaur, P.Kaur, RM approach for ranking of generalized trapezoidal fuzzy numbers, *Fuzzy Information and Engineering*, 2(1) (2010) 37-47
- [13] S.Rezvani, Graded Mean Representation Method with Triangular Fuzzy Number, *World Applied Sciences Journal*, 11 (7) (2010) 871-876
- [14] S.Rezvani, Multiplication Operation on Trapezoidal Fuzzy Numbers , *Journal of Physical Sciences*, 15(2011) 17-26
- [15] S.Rezvani, A New Method for Ranking in Perimeters of two Generalised Trapezoidal Fuzzy Numbers , *International Journal of Applied Operational Research*, 2(3)(2012) 83-90
- [16] S.Rezvani, A New approach Ranking of Exponential Trapezoidal Fuzzy Numbers , *Journal of Physical Sciences* , 16(2012) 45-57
- [17] S.Rezvani, Ranking Exponential Trapezoidal fuzzy numbers by Median Value
- [18] S.Rezvani, A New Method for Ranking in Areas of two Generalized Trapezoidal Fuzzy Numbers , *International Journal of Fuzzy Logic Systems (IJFLS)*, 3(1) (2013) 17-24
- [19] S.Rezvani, Ranking Generalized Trapezoidal Fuzzy Numbers with Euclidean Distance by the Incentre of Centroids, *Mathematica Aeterna*, 3(2) (2013) 103-114
- [20] S.Rezvani, Ranking Method of Trapezoidal Intuitionistic Fuzzy Numbers , *Annals of Fuzzy Mathematics and Informatics*, 5(3)(2013) 515-523
- [21] R.Saneifard, A modified method for defuzzification by probability density function , *Journal of Applied Sciences Research*, 7(2)(2011) 102-110
- [22] Shyi-Ming Chen, Jim-Ho Chen, Fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads , *Expert Systems with applications*, 36 (3) (2009) 2033-2042
- [23] Y.J.Wang, H.S.Lee, The revised method of ranking fuzzy numbers with an area between the centroid and original points , *Computers and Mathematics with Applications*, 55(2008) 2033-2042
- [24] L.A.Zadeh , Fuzzy set , *Information and Control* , 8(3)(1965) 338-353
- [25] R.Jain, Decision making in the presence of fuzzy variables , *IEEE Tranactions on systems , Man and Cybernetics*, 6(10)(1976) 698-703