International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2013): 6.14 | Impact Factor (2013): 4.438

Heat and Mass Transfer Effects on the Unsteady MHD Flow past an Oscillating Infinite Vertical Plate with Variable Temperature through the Porous Medium

Prof. (Dr). P. T. Hemamalini[1], N. Suresh Kumar[2]

[1]HOD, Department of science and humanities, Karpagam University, Coimbatore- 641021.TN, India

[2]Research Scholar in Department of Mathematics, Karpagam University, Coimbatore-641021, TN, India

Abstract: ***The objective of the paper is to study the effect of the MHD flow past an oscillating infinite vertical plate through the porous medium taking account of the variable temperature with Heat and Mass transfer. The dimensionless governing equations are solved by the Laplace Transform technique. The velocity profiles are studied for the different physical parameters like phase angle, magnetic parameter, Thermal Grashof number, Modified Grashof number, Permeability parameter, Prandtl number, Schmidt number and time.***

Keywords: Heat Transfer, Mass Transfer, MHD, Oscillating plate and Porous Medium.

1. Introduction

MHD is the Science of motion of electrically conducting fluid in the presence of magnetic field. There are numerous examples of application of the MHD principles including the MHD pumps and MHD flow meters etc. The dynamo and motor are the classical examples of MHD principle. The MHD principles find its applications in the medicine and biology.

Free convection affects on the Flow a vertical Oscillating plate studied by Soundalgekr [1]. Effect of the free convection currents and Mass transfer on the flow past a vertical oscillating plate was analyzed are studied by Soundalgekr [2]. Chen Yuh. and Moutsogloye [3] investigated the Combined heat and mass transfer in the mixed convection along a vertical and inclined plate. Lin and Wu [4] studied the combined heat and mass transfer by the Laminar Natural Convection from a vertical plate. The MHD effect on the heat and mass transfer in a flow of the viscous fluid with an induced magnetic field was investigated by Singh and Atul Singh [5].

Muthucumaraswamy et al [6] studied the mass transfer effect on the vertical Oscillating plate with heat the flux. Muthucumaraswamy et al [7] presented the Heat transfer effects on the flow past which is an exponentially accelerated vertical plate with variable Temperature. The behavior of the Skin Friction in a case of heat and mass transfer in an Oscillating plate in the porous media was analyzed by Ramana Murthy et al [8]. MHD Free convection Flow between the two parallel porous walls with varying Temperature was analyzed by Rushi Kumar Gangadhar [9]. Heat and Mass Transfer Effect on the flow past an Oscillating infinite Vertical plate with the variable Temperature through the Porous Media was presented by Saraswat Amit et al [10].Heat Source and Mass Transfer Effect on the MHD Flow of an Elasto – Viscous Fluid through a Porous Medium was studied by Rajesh [11]. Free Convection Effect on the flow past an impulsively started an Oscillating infinite plate was analyzed by Revankar [12].

MHD Free Convection Flow Visco – Elastic fluid past an infinite vertical porous plate was presented by Chowdhury and Islam [13]. Effect of the Mass Transfer on the Flow past an Oscillating Infinite Vertical Plate with constant Heat Flux was presented by Soundalgekar et al [14]. Unsteady MHD Flow past a Vertical Oscillating Plate with Thermal Radiation and variable Mass Diffusion was analyzed by Ramana Reddy et al [15]. MHD Free Convection Flow of an incompressible Viscous Dissipative fluid in an Infinite Vertical Oscillating Plate with the constant Heat Flux was investigated by Srihari et al [16]. The present study deals with the Investigation of an unsteady flow of an incompressible viscous fluid with heat and mass Transfer Effects on the MHD Flow past an Oscillating Infinite Vertical Plate with Variable Temperature through the porous media.

2. Formulation of the Problem

The unsteady flow of an incompressible electrically conducting viscous fluid which is initially at the rest past is an infinite vertical plate with the variable temperature through a porous medium has been considered. The flow is assumed to be in x-direction, which takes the vertical plate in the upward direction. The y-axis is taken as normal to the plate. Magnetic field of the uniform strength B_0 is applied normal to the plate and the induced magnetic field is neglected. Initially the plate and the fluid are at the same Temperature T' with the same the concentration level C' at all the points. At the time of $t' > 0$ the plate starts Oscillating in its own plane with a velocity $u = u_o \cos \omega' t'$. The plate Temperature is raised to T'_w and the levels of concentration near the plate are raised to C'_w linearly with time t. Then by the usual Boussinesq's approximation the unsteady flow is governed by the following equation.

$$\frac{\partial u}{\partial t'} = g\,\beta\,(T' - T'_\infty) + g\,\beta^*\,(C' - C'_\infty) + \nu\,\frac{\partial^2 u}{\partial y^2} - \nu\,(\frac{u}{k'})$$

$$-\frac{\sigma B_0^2 u}{\rho} \quad (1)$$

$$\rho c_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y^2} \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} \quad (3)$$

With the following initial and boundary conditions

$t' \leq 0$, $u = 0$, $T' = T'_\infty$, $C' = C'_\infty$ for all y.

$t' > 0$, $u = u_o \cos \omega' t'$, $T' = T'_\infty + (T'_w - T'_\infty) A t'$,

$C' = C'_\infty + (C'_w - C'_\infty) A t'$, at $y = 0$.

$u = 0$, $T' \to T'_\infty$, $C' \to C'_\infty$ as $y \to \infty$. (4)

Where $A = \left(\frac{u_0^2}{\nu}\right)$

On introducing the following dimensionless quantities

$$U = \left(\frac{u}{u_0}\right),\ t = \left(\frac{t' u_0^2}{\nu}\right),\ Y = \left(\frac{y u_0}{v}\right),$$

$$\Theta = \left[\frac{T' - T'_\infty}{T'_w - T'_\infty}\right],\ \omega = \left(\frac{\nu \omega'}{u_0^2}\right),\ S_c = \left(\frac{\nu}{D}\right),$$

$$P_r = \left(\frac{\mu c_p}{k}\right),\ G_r = \left[\frac{g \beta \nu (T'_w - T'_\infty)}{u_0^3}\right],\ C = \left[\frac{C' - C'_\infty}{C'_w - C'_\infty}\right]$$

$$G_c = \left[\frac{g \beta^* \nu (C'_w - C'_\infty)}{u_0^3}\right],\ \frac{1}{K} = \left(\frac{u_0^2 k'}{v^2}\right),\ M = \left[\frac{\sigma B_0^2 v}{\rho u_0^2}\right] \quad (5)$$

in equations (1) –(3) leads to

$$\frac{\partial U}{\partial t} = Gr\,\Theta + Gc\,C + \frac{\partial^2 U}{\partial Y^2} - (K + M)\,U \quad (6)$$

$$\frac{\partial \Theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \Theta}{\partial Y^2} \quad (7)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} \quad (8)$$

Initial and boundary conditions in non dimensional form are

$U = 0$, $\Theta = 0$, $C = 0$, for all $Y \leq 0$, $t \leq 0$

$U = \cos \omega t$, $\Theta = t$, $C = t$, at $Y = 0$, $t > 0$

$U = 0$, $\Theta \to 0$, $C \to 0$, as $Y \to \infty$ (9)

3. Method of Solution

The governing equations in exact form are solved by Laplace transform technique. On taking Laplace transform of the equations (6),(7),(8) and (9) we get

$$\frac{d^2 \bar{U}}{d Y^2} - (s + M')\,\bar{U} = -Gr\,\bar{\Theta} - Gc\,\bar{C} \quad (10)$$

$$\frac{d^2 \bar{\Theta}}{d Y^2} - s\,Pr\,\bar{\Theta} = 0 \quad (11)$$

$$\frac{d^2 \bar{C}}{d Y^2} - s\,Sc\,\bar{C} = 0 \quad (12)$$

Initial and boundary conditions are becomes

$\bar{U} = 0$, $\bar{\Theta} = 0$, $\bar{C} = 0$ for all Y , $t \leq 0$

$\bar{U} = \frac{s}{s^2+\omega^2}$, $\bar{\Theta} = \frac{1}{s^2}$, $\bar{C} = \frac{1}{s^2}$, at $Y = 0$, $t > 0$.

$\bar{U} = 0$, $\bar{\Theta} \to 0$, $\bar{C} \to 0$ as $Y \to \infty$, $t > 0$ (13)

On solving the equation (10), (11),(12) with the help of equation (13) we get

$$\bar{\Theta} = \frac{e^{-y\sqrt{sPr}}}{s^2} \quad (14)$$

$$\bar{C} = \frac{e^{-y\sqrt{sSc}}}{s^2} \quad (15)$$

$$\bar{U} = \left[\frac{s}{s^2+\omega^2} + \frac{1}{s^2}\left\{\frac{Gr}{s(Pr-1)-M'} + + \frac{Gc}{s(Sc-1)-M'}\right\}\right] e^{-Y\sqrt{s+M'}} - \frac{1}{s^2}\left\{\frac{Gr\,e^{-Y\sqrt{sPr}}}{s(Pr-1)-M'} + \frac{Gc\,e^{-Y\sqrt{sSc}}}{s(Sc-1)-M'}\right\} \quad (16)$$

Where s is the Laplace transform parameter.

On taking inverse Laplace transform of equation (14), (15), and (16) we get

$$\Theta = t\left[(1 + 2\eta^2 Pr)\,erfc\left(\eta\sqrt{Pr}\right) - \frac{2}{\sqrt{\pi}} e^{-\eta^2 Pr}\, \eta\sqrt{Pr}\right] \quad (17)$$

$$C = t\left[(1 + 2\eta^2 Sc)\,erfc\left(\eta\sqrt{Sc}\right) - \frac{2}{\sqrt{\pi}} e^{-\eta^2 Sc}\, \eta\sqrt{Sc}\right] \quad (18)$$

$$\begin{aligned}
U = {} & e^{-i\omega t}\left[e^{Y\sqrt{M'-i\omega}}\, erfc\left\{\eta + \sqrt{(M'-i\omega)t}\right\} + e^{-Y\sqrt{M'-i\omega}}\, erfc\left\{\eta - \sqrt{(M'-i\omega)t}\right\}\right] \\
& + e^{i\omega t}\left[e^{Y\sqrt{M'+i\omega}}\, erfc\left\{\eta + \sqrt{(M'+i\omega)t}\right\} + e^{-Y\sqrt{M'+i\omega}}\, erfc\left\{\eta - \sqrt{(M'+i\omega)t}\right\}\right] \\
& + e^{Y\sqrt{M'}}\, erfc\left(\eta + \sqrt{M't}\right)\left[\frac{Gr}{Pr-1}\left(\frac{-1}{2C^2} + \frac{t}{2C} + \frac{Y}{4C\sqrt{M'}}\right) + \frac{Gc}{Sc-1}\left(\frac{-1}{2b^2} + \frac{t}{2b} + \frac{Y}{4b\sqrt{M'}}\right)\right] \\
& + e^{-Y\sqrt{M'}}\, erfc\left(\eta - \sqrt{M't}\right)\left[\frac{Gr}{Pr-1}\left(\frac{-1}{2C^2} + \frac{t}{2C} - \frac{Y}{4C\sqrt{M'}}\right) + \frac{Gc}{Sc-1}\left(\frac{-1}{2b^2} + \frac{t}{2b} - \frac{Y}{4b\sqrt{M'}}\right)\right] \\
& + \frac{Gr\,e^{-ct}}{2(Pr-1)C^2}\left[e^{Y\sqrt{-C\,Pr}}\left\{erfc\left(\eta + \sqrt{-C\,Pr\,t}\right) - erfc\left(\eta\sqrt{Pr} + \sqrt{-Ct}\right)\right\}\right. \\
& \qquad \left. + e^{-Y\sqrt{-C\,Pr}}\left\{erfc\left(\eta - \sqrt{-C\,Pr\,t}\right) - erfc\left(\eta\sqrt{Pr} - \sqrt{-Ct}\right)\right\}\right] \\
& + \frac{Gc\,e^{-bt}}{2(Sc-1)b^2}\left[e^{Y\sqrt{-b\,Sc}}\left\{erfc\left(\eta + \sqrt{-b\,Sc\,t}\right) - erfc\left(\eta\sqrt{Sc} + \sqrt{-bt}\right)\right\}\right. \\
& \qquad \left. + e^{-Y\sqrt{-b\,Sc}}\left\{erfc\left(\eta - \sqrt{-b\,Sc\,t}\right) - erfc\left(\eta\sqrt{Sc} - \sqrt{-bt}\right)\right\}\right] \\
& - \frac{Gr}{Pr-1}\left[erfc\left(\eta\sqrt{Pr}\right)\left\{\frac{-1}{C^2} + \frac{1}{C}\left(t + \frac{Y^2 Pr}{2}\right)\right\} - \frac{Y}{C} e^{\frac{Y^2 Pr}{4t}}\sqrt{\frac{Pr\,t}{\pi}}\right] \\
& - \frac{Gc}{Sc-1}\left[erfc\left(\eta\sqrt{Sc}\right)\left\{\frac{-1}{b^2} + \frac{1}{b}\left(t + \frac{Y^2 Sc}{2}\right)\right\} - \frac{Y}{b} e^{\frac{Y^2 Sc}{4t}}\sqrt{\frac{Sc\,t}{\pi}}\right]
\end{aligned} \quad (19)$$

Where $C = \frac{-M'}{Pr-1}$, $b = \frac{-M'}{Sc-1}$, $\eta = \frac{Y}{2\sqrt{t}}$

4. Results and Discussions

The problem of the Heat and Mass transfer on the MHD flow past an oscillating infinite vertical plate with variable temperature through the porous media is formulated and solved analytically. The value of the velocity are obtained for the physical parameter such as Thermal Grashof Number (Gr), Modified Grashof Number (Gc), Prandtl Number (Pr), Schmidt Number (Sc), time (t) , Permeability Parameter (K), Magnetic Parameter (M′) and Phase angle (ωt) on the flow patterns and the computation of the flow fields are carried out. The value of the Prandtl Number Pr is chosen to represent air (Pr = 0.71). The value of the Schmidt Number is chosen to represent the water vapour (Sc=0.6).

The velocity profiles are studied and presented in the Fig 1 to 6.The effect of the velocity for the different values of the phase angle (ωt = 0, π/6, π/4, π/2) is presented in the Fig:1. It is observed that the velocity decreases with the increasing of the phase angle (ωt). The velocity profiles for the different values of (M′ = 5, 7, 10, 15) is shown in the Fig:2 and it is noted that the velocity decreases with the increasing of the Magnetic Parameter (M′). The Fig:3 shows that the velocity profiles for the different values of the Modified Grashof Number (Gc = -10, -5 , 3, 5) and it is observed that the velocity increases with the increasing of the Gc. The velocity profile for the different values of the Thermal Grashof Number (Gr = -5, -2, 2, 5) is seen in the Fig :4. It is clearly shown that the velocity increases with the increasing of the Gr . The effect of velocity for the different values of the permeability (k = 0.25, 0.5, 0.75, 1) is seen in the Fig:5.It is noted that the velocity increases with the increasing of the permeability (k). The velocity profile for the different values of time (t = 0.2, 0.4, 0.6, 0.8) is presented in the Fig:6 and is shown that the velocity decreases with increasing of the time (t).

5. Conclusions

In the paper the Heat and Mass transfer effect on the MHD flow past an oscillating plate embedded in the porous medium is presented. Results are presented graphically to illustrate the variation of the velocity with various parameters. In the study the following conclusions are set out.

(1) The velocity decreases with the increasing of the Phase angle (ωt).
(2) The velocity decreases with the increasing of the Magnetic Parameter (M′).
(3) The velocity increases with the increasing of the Modified Grashof Number (Gc).
(4) The velocity increases with the increasing of the Thermal Grashof Number (Gr).
(5) The velocity increases with the increasing of the Permeability (k)
(6) The velocity decreases with the increasing of time (t).

Appendix – Nomenclature

C'_∞ concentration in the fluid far away from the plate.
C'_W concentration of the plate
A constant
Y' coordinate axis normal to the plate
C dimensionless concentration
Y dimensionless coordinate axis normal to the Plate
U Dimensionless velocity
B_0 External magnetic field
Gc Modified Grashof Number
Pr Prandtl Number
Sc Schmidt Number
C' Species concentration in the fluid
C_P Specific heat at constant pressure
T'_∞ Temperature of the fluid far away from the Plate
T' Temperature of the fluid near the plate
T'_w Temperature of the plate
K Thermal conductivity of fluid
Gr Thermal Grashof Number
t′ Time
u' Velocity of the fluid in the x- direction
u_o Velocity of the plate
g Acceleration due to Gravity
k Permeability Parameter
M' Magnetic field parameter
μ Coefficient of viscosity
erfc Complementary error function
ρ Density of the fluid
Θ Dimensionless Temperature
σ Electric Conductivity
erf Error function
υ Kinematic Viscosity
β Volumetric coefficient of Thermal expansion
W Condition on the wall
∞ Free stream conditions

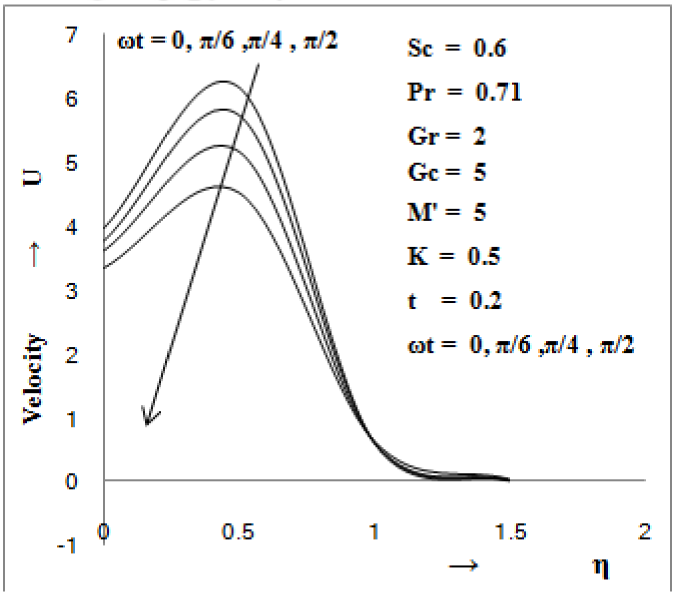

Figure 1: Velocity profile for different values of (ωt)

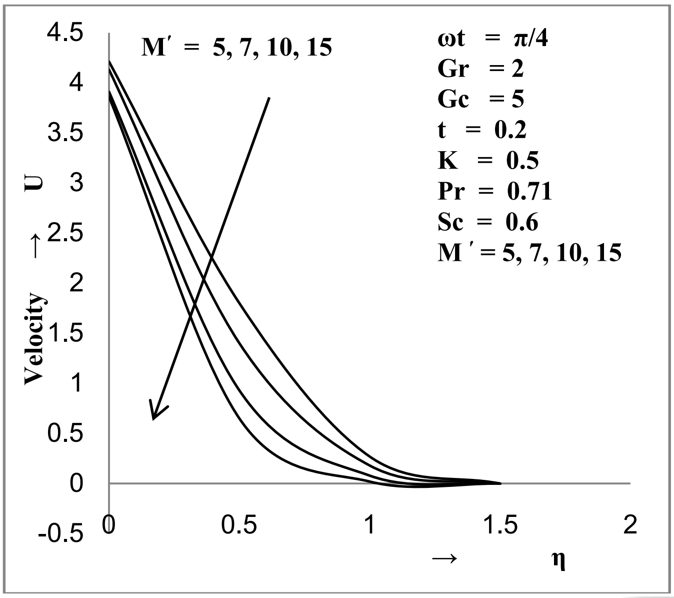

Figure 2: Velocity profile for different values of M′

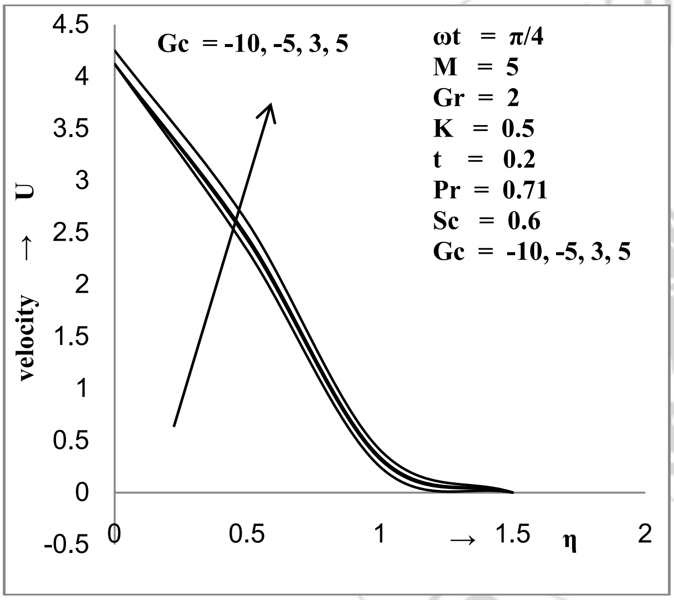

Figure 3: Velocity profile for different values of Gc

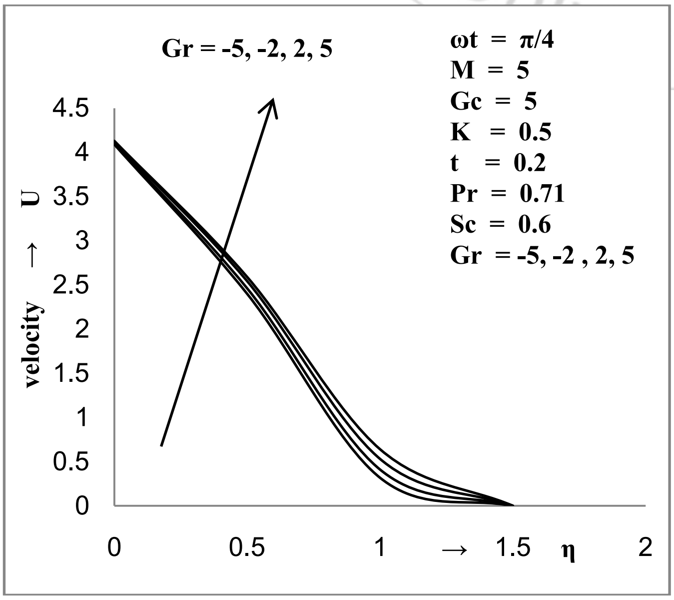

Figure 4: Velocity profile for different values of Gr

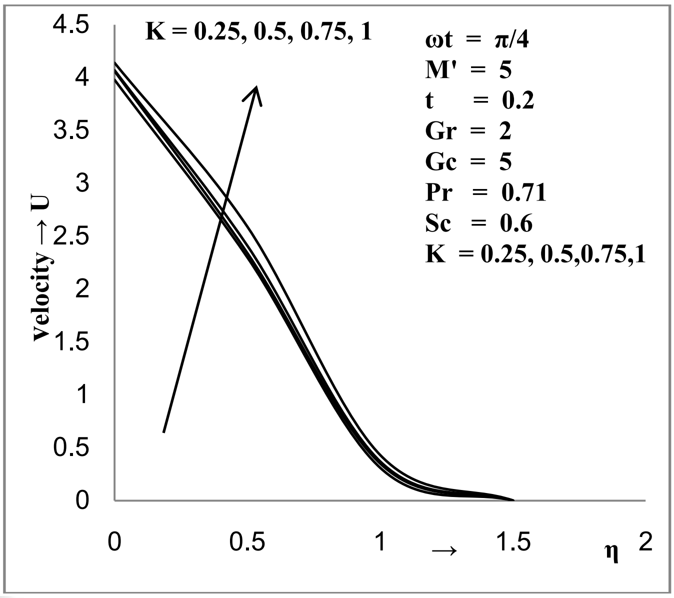

Figure 5: Velocity profile for different values of K

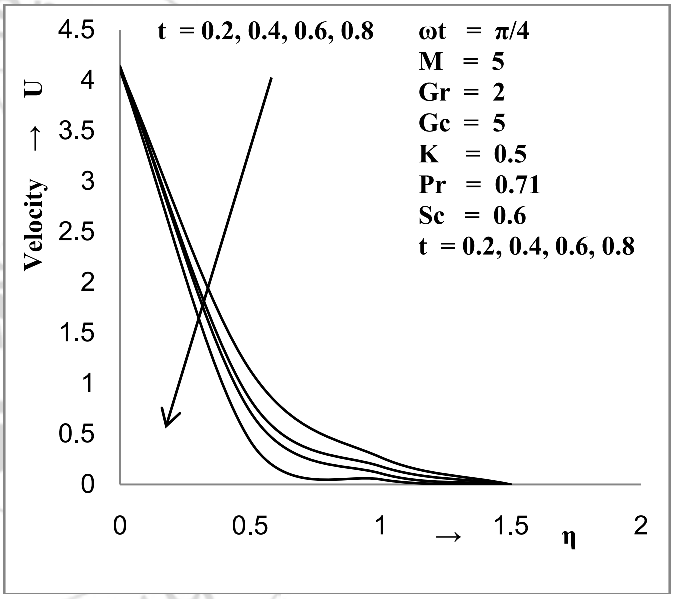

Figure 6: Velocity profile for different values of t

References

[1] Soundalgekar .V.M, "Free convection Effects on the Flow past a vertical Oscillating plate''. Astrophy. Space Sci vol 64 pp. 165 – 172, 1979.

[2] Soundalgekar .V.M and S.P. Akolkar, '' Effect of free convection currents and mass transfer on the flow past a vertical oscillating plate''. Astrophys Space Sci. vol 89 pp. 241 – 254, 1983.

[3] Chen.T.S Yuh and Moutsoglou.A. ,''Combined Heat and Mass transfer in mixed convection along a vertical and inclined plate''. Int .J. Heat and Mass transfer vol 23 pp. 527 – 537, 1980.

[4] Lin .H.T and Wu.C.M , ''Combined Heat and Mass transfer by Laminar Natural Convection from a vertical plate'' . Int .J. Heat and Mass transfer Vol 30 pp. 369 – 376, 1995.

[5] N.P.Sing and Atul. K.R Sing, '' MHD Effect on Heat and Mass transfer in a Flow of viscous fluid Induced

Magnetic Field''. Indian Journal of Pure and Applied Physics vol 38, pp. 182 – 189, 2000.

[6] R Muthucumaraswamy K. Manivanna,'' Mass transfer Effect on vertical Oscillating plate with Heat Flux''. Theoret Appl Mech Vol 34 no 4 pp. 309- 322, 2007.

[7] R. Muthucumaraswamy, Sathappan .K. E and Nataragan.R, "Heat transfer effect on Flow past an Exponentially Accelerated vertical plate with variable Temperature''. Theoretical Applied Mechanics vol 35 (4), pp. 323 – 333, 2008.

[8] Ch. V.Ramana Murthy and Sayad Parveen ''Behavior of skin Friction in a case of Heat and Mass transfer in an Oscillating plate in porous media''. (IJEST) Vol - 3 No: 6 2011.

[9] B.Rushi Kumar, K.Gangadhar,'' MHD Free convection Flow between two parallel porous walls with varying Temperature''. ANNALS – IJOE. pp. 67 – 72, 2012.

[10] Saraswat Amit and Srinivastara.R.K,"Heat and Mass Transfer Effect on Flow an Oscillating infinite vertical plate with variable Temperature through porous media''. Res .J. Recent. Sci Vol 2 ISC pp. 316 – 321, 2013.

[11] V .Rajesh , " Heat Source and Mass Transfer Effect on MHD Flow of an Elasto – Viscous Fluid through a Porous Medium ". ANNALS Tome IX. pp. 205 – 212, 2011.

[12] Revankar. S.T, "Free Convection Effect on flow past an impulsively started or Oscillating infinite plate". Mechanics Research Comm., vol 27, pp. 241- 246, 2000.

[13] Chowdhury, M. K, and Islam .M. N, "MHD Free convection Flow Visco – Elastic fluid past an infinite vertical porous plate". Heat and Mass Transfer, vol 36, pp 439 – 447, 2000.

[14] Soundalgekar. V. M Lahurikar R.M, Pohanerkar.S.G and Birajdar. N. S ," Effect of Mass Transfer on Flow past an Oscillating infinite vertical plate with constant Heat Flux" Thermo Physics and Aeromechanics, Vol 1, PP 119 – 124, 1994 .

[15] G .V. Ramana Reddy, CH. V .Ramana Murth, N. Bhaskar Reddy , "Unsteady MHD Flow past a vertical Oscillating plate with Thermal Radiation and variable Mass Diffusion" , J. Comp & Math Sci Vol 1(5) pp 528 – 536, 2010.

[16] Srihari.K, Anand Rao.J, and Krishan. N, "MHD Free Convection Flow of an incompressible Viscous Dissipative fluid in infinite vertical Oscillating plate with constant Heat Flux", JI , Energy, Heat and Mass Transfer Vol 28, PP 19 – 28 , 2006