Natural Convection Heat and Mass Transfer of a Chemically Reacting Micropolar Fluid in a Vertical Double Passage Channel

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Abstract: The problem of fully developed mixed convection for a laminar flow of a micropolar fluid in a vertical double passage channel has been investigated analytically. The channel is divided into two passages by means of a thin perfectly conductive plane baffle and the velocity, temperature and concentration will be individual in each streams. After inserting the baffle the fluid is concentrated in stream-I. The governing equations of the flow have been solved analytically subject to the relevant boundary and interface conditions. The closed form solutions are represented graphically. The effect of governing parameters namely, buoyancy ratio, vortex viscosity parameter and chemical reaction parameter on the velocity, microrotation velocity, temperature and concentration has been discussed. It is seen that increasing the vortex viscosity parameter tends to increase the magnitude of microrotation velocity and thus decreases the fluid velocity in the vertical channel. Moreover, the volume flow rate, the total heat rate added to the fluid and the total species rate added to the fluid for micropolar fluids are lower than those of Newtonian fluids.

Keywords: Baffle, first order chemical reaction, micropolar fluid, vortex viscosity

1. Introduction

The analysis of the flow properties of non-Newtonian fluids is very much important in the field of fluid dynamics because of their technological applications. Mechanics of non-Newtonian fluids present challenges to engineers, physicist and mathematicians. Due to complex stress strain relationship of non-Newtonian fluids, not many investigators have studied the flow behavior of the fluids in various flow fields.

Studies of micropolar fluids have recently received considerable attention due to their applications in a number of processes that occur in industry. Such applications include the extrusion of polymer fluids, solidifications of liquid crystals, cooling of a metallic plate in a bath, animal bloods, exotic lubricants and colloidal and suspensions solutions, for which the classical Navier stokes theory is inadequate. The essence of the theory of micropolar fluid flow lies in the extension of the constitutive equations for Newtonian fluids so that more complex fluids can be described by this theory. In this theory, rigid particles contained in a small fluid volume element are limited to rotation about the center of the volume element described by the microrotation vector. This local rotation of the particles is in addition to the usual rigid body motion of the entire volume element. In the micropolar fluid theory, the laws of classical continuum mechanics are augmented with additional equations that account for the conservation of microinertia moments and the balance of first stress moments that arise due to consideration of the microstructure in a material and also additional local constitutive parameters are introduced.

The shear behavior of many real fluids used in the modern technology cannot be characterized by the Newtonian relationship and hence researchers have proposed diverse non-Newtonian fluid theories to explain the behavior of real fluids. The description of the motion of such fluids requires a new approach, which is different from the classical concepts. Thus, additional balance laws are needed to describe such types of complex behaviors. Physically micropolar fluids may be described as the non-Newtonian fluids consisting of dumb bell molecules or short rigid cylindrical elements, polymer fluids, fluid suspensions, animal blood, etc. The presence of dust or smoke, particularly in a gas, may also be modeled using micropolar fluid dynamics. The theory of micropolar fluids, first proposed by Eringen[1, 2] is capable of describing such fluids. Extensive reviews of theory of micropolar fluids and its applications can be found in review articles by Ariman et al. [3,4] and recent books by Łukaszewicz [5] and Eringen [6]. Recently Chamkha et al. [7] have studied the fully developed free convection of a micropolar fluid in a vertical channel. Cheng [8] has presented the fully developed natural convective flow due to heat and mass transfer of a micropolar fluid in a vertical channel with asymmetric wall temperature and concentrations.

It is well known that many of the physiological fluids behave like suspensions of deformable or rigid particles in Newtonian fluid. In view of this, some researchers have used non-Newtonian fluid models for the biofluids. Dash et al. [9] estimated the increased flow resistance in a narrow catheterized artery using the Caisson fluid model. Banerjee et al. [10] investigated the changes in flow and mean pressure gradient across a coronary artery with stenos in the presence of a translational catheter using the finite element method for the Carreau model, a shear rate dependent non-Newtonian fluid model. Eringen [1] proposed the theory of micropolar fluids to study fluids with suspension nature. That is, each material volume element contains micro volume elements that can translate and rotate independently of the motion of macro volume. In this model, two independent kinematic vector fields are introduced one representing the translation velocities of fluid particles; and the other representing angular (spin) velocities of the particles, called as microrotation vector.


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(Lukaszewicz, [5]). In many transport processes, both in nature and in industrial applications, heat and mass transfer is a consequence of the buoyancy effects caused by diffusion of heat and chemical species. The natural convective flow is often caused not entirely by temperature gradients, but also by the difference in the concentrations of dissimilar chemical species. Convection and transport processes are governed by buoyancy mechanisms arising from both thermal and species diffusion.

In particular process involving the mass transfer effects has been considered to be important precisely in chemical engineering equipments. The other applications include solidification of binary alloys and crystal growth dispersion of dissolved materials or particulate water in flows, drying and dehydration operations in chemical and food processing plants, evaporation at the surface of water body. The order of the chemical reaction depends on several factors. One of the simplest chemical reactions is the first order reaction in which rate of reaction is directly proportional to the species concentration. Das et al. [11] have studied the effect of mass transfer on the flow started impulsively past an infinite vertical plate in the presence of wall heat flux and chemical reaction. Muthucumaraswamy and Ganeshan [12, 13] have studied the impulsive motion of a vertical plate with heat flux/mass flux/suction and diffusion of chemically reactive species.

The rate of heat transfer in a vertical channel could be enhanced by using special inserts. These inserts can be specially designed to increase the included angle between the velocity vector and the temperature gradient vector, rather than to promote turbulence. This increases the rate of heat transfer without a considerable drop in the pressure by Guo et al. [14]. A plane baffle may be used as an insert to enhance the rate of heat transfer in the channel. To avoid a considerable increase in the transverse thermal resistance into the channel, a thin and perfectly conductive baffle is used. The effect of such baffle on the laminar fully developed combined convection in a vertical channel with different uniform wall temperatures has been studied analytically by Salah El-Din [15]. Umavathi et al. [16]. Umavathi and Prathap Kumar [17] and Umavathi and Jaweria Sultana [18] studied the flow and heat transfer of a micropolar fluid in a vertical channel.

In this paper, we aim to study the fully developed heat and mass transfer by natural convection of a micropolar fluid inside a plane vertical channel for asymmetric wall temperatures and concentrations by inserting a perfectly thin baffle. The closed form exact solutions are derived and the effects of the buoyancy ratio and the vortex viscosity parameter on the flow, heat transfer and mass transfer characteristics, such as the velocity, microrotation velocity, volume flow rates, total heat rate added to the fluid and the total species rate added to the fluid are analyzed.

2. Mathematical Formulation

Consider a steady fully developed laminar natural convection flow of a micropolar fluid between two vertical plates. The vertical plates are separated by a distance h. Further the vertical two plates are separated by inserting a moving perfectly thin baffle. The inlet temperature is \( T_0 \) and inlet concentration is \( C_0 \). The inner surface of the left plate \( (y = -h/2) \) is kept at a constant temperature \( T_1 \) while the inner surface of the right plate \( (y = h/2) \) is maintained at a constant temperature \( T_2 \). In addition, the concentration of a certain constituent in the solution varies from \( C_i \) on the inner surface of the left plate to \( C_z \) on the inner surface of the right plate. Because the flow is fully developed, the transverse velocity is zero and the flow depends only on the transverse coordinate \( Y \). The fluid properties are assumed to be constant except for density variations in the buoyancy force term.

The governing equations for velocity, temperature, microrotation velocity and concentration and concentrations are

\[
\rho g \beta_y (T_1 - T_0) + \rho g \beta_y (C_1 - C_0) + (\mu + \kappa) \frac{d^2 U_1}{dY^2} + \frac{k}{dY} = 0
\]

\[
\gamma \frac{d^2 v_1}{dY^2} - 2\kappa v_1 - \frac{dU_1}{dY} = 0
\]

\[
D \frac{d^2 C}{dY^2} - kC = 0
\]

\[
\rho g \beta_y (T_2 - T_0) + \rho g \beta_y (C_2 - C_0) + (\mu + \kappa) \frac{d^2 U_2}{dY^2} + \frac{k}{dY} = 0
\]

\[
\gamma \frac{d^2 v_2}{dY^2} - 2\kappa v_2 - \frac{dU_2}{dY} = 0
\]

The boundary and interface conditions on velocity, temperature, microrotation velocity and concentration are

\[
U_1 = 0, \ T = T_1, \ v_1 = 0, \ C = C_1 \quad \text{at} \quad Y = -h/2
\]

\[
U_2 = 0, \ T = T_2, \ v_2 = 0, \ C = C_2 \quad \text{at} \quad Y = h/2
\]

The governing equations for velocity, temperature, microrotation velocity and concentration and concentrations are

\[
\rho g \beta_y (T_1 - T_0) + \rho g \beta_y (C_1 - C_0) + (\mu + \kappa) \frac{d^2 U_1}{dY^2} + \frac{k}{dY} = 0
\]

\[
\gamma \frac{d^2 v_1}{dY^2} - 2\kappa v_1 - \frac{dU_1}{dY} = 0
\]

\[
D \frac{d^2 C}{dY^2} - kC = 0
\]

\[
\rho g \beta_y (T_2 - T_0) + \rho g \beta_y (C_2 - C_0) + (\mu + \kappa) \frac{d^2 U_2}{dY^2} + \frac{k}{dY} = 0
\]

\[
\gamma \frac{d^2 v_2}{dY^2} - 2\kappa v_2 - \frac{dU_2}{dY} = 0
\]

Here we introduce the following non-dimensional variables

\[
U_j = \frac{u_j}{Gr \mu}, \ v_i = \frac{h^2 Gr \mu}{h^2 \rho}, \ v_2 = \frac{h^2 Gr \mu}{h^2 \rho}, \ v = (\mu + \kappa) j
\]

\[
\theta_i = \frac{T_i - T_0}{T_i - T_0}, \ Gr = \frac{g \beta_i \Delta T h^3}{\nu^2}, \ Gc = \frac{g \beta_i \Delta C h^3}{\nu^2}
\]

\[
\phi = \frac{C - C_0}{C_1 - C_0}
\]
\[ \Delta T = T_1 - T_0, \Delta C = C_i - C_0, Y = \frac{Y}{h}, Y^* = \frac{Y}{h} \]

\[ \alpha^2 = \frac{kh^2}{D}, N = \frac{\beta(C_i - C_0)}{\beta(T_1 - T_0)} \]

where \( U_i \) are the velocity components in the stream wise direction. \( T \) and \( C \) are the fluid temperature and species concentration, respectively. \( \nu \) is the angular velocity of the micropolar fluid, \( \kappa \) is the vortex viscosity and \( j \) is the microinertia density. \( \gamma \) is the spin gradient viscosity and we assume that \( \nu = \left( \mu + \frac{\kappa}{2} \right) j \). Property \( \mu \) is the dynamic viscosity of the fluid and \( \rho \) is the fluid density. \( \beta \) is the coefficients for thermal expansion and \( \beta \) is the concentration expansion coefficient and \( g \) is the gravitational acceleration. Note that the boundary condition for the microrotation velocity at the fluid solid interface is \( \nu = 0 \), the condition of zero spin, as used by Takhar et al. [19]. The microstructure does not rotate relative to the surface. By using these parameters one obtains the momentum, energy, microrotation velocity and concentration equations corresponding to stream-I and stream-II as

Stream-I

\[ (1 + K) \frac{d^2 u_i}{dy^2} + \phi_i + N \phi_i + K \frac{dH_i}{dy} = 0 \]

\[ \frac{d^2 \phi_i}{dy^2} = 0 \]

Stream-II

\[ (1 + K) \frac{d^2 u_i}{dy^2} + \phi_i + K \frac{dH_i}{dy} = 0 \]

\[ \frac{d^2 \phi_i}{dy^2} = 0 \]

Subject to the boundary and interface conditions

\[ u_i = 0, \phi_i = 1 \quad \text{at} \quad y = -1/4 \]

\[ u_i = 0, \phi_i = 0 \quad \text{at} \quad y = 1/4 \]

\[ u_i = 0, u_j = 0, \phi_i = 0, \phi_j = \frac{\partial \phi_i}{\partial y} = \frac{\partial \phi_j}{\partial y} \quad \text{at} \quad y = y^* \]

\[ H_i = 0, H_j = 0, \phi_i = n \quad \text{at} \quad y = y^* \]

where \( n = \frac{C_i - C_0}{C_i - C_0} \) is the wall concentration ratio, \( \frac{h^2}{j} \) and \( K = \frac{\kappa}{\mu} \) are the material parameters

3. Solutions

Solutions of equations (10) to (16) using the boundary and interface conditions (17) are

Stream-I

\[ \phi = c_y Cosh(\sqrt{\gamma y}) + c_y Sinh(\sqrt{\gamma y}) \]

\[ H_i = c_y Cosh(\sqrt{\gamma y}) + c_y Sinh(\sqrt{\gamma y}) + I_3 Sinh(\sqrt{\gamma y}) \]

Stream-II

\[ \phi = c_y Cosh(\sqrt{\gamma y}) + c_y Sinh(\sqrt{\gamma y}) \]

\[ H_i = c_y Cosh(\sqrt{\gamma y}) + c_y Sinh(\sqrt{\gamma y}) + I_3 Sinh(\sqrt{\gamma y}) \]

3.1 Special case for Newtonian fluid \((\kappa = 0)\)

The governing equations for velocity, temperature and concentrations are

Stream-I

\[ \rho g \beta_i (T_i - T_0) + \rho g \beta_i (C_i - C_0) + \mu \frac{d^2 U_i}{dy^2} = 0 \]

Stream-II

\[ \rho g \beta_i (T_i - T_0) + \mu \frac{d^2 U_i}{dy^2} = 0 \]

Using the non-dimensional parameters in equations (25) to (29), we obtain the non-dimensionalised momentum, energy and concentration equations corresponding to stream-I and stream-II as

Stream-I

\[ \frac{d^2 u_i}{dy^2} + \phi_i + N \phi_i = 0 \]

Stream-II

\[ \frac{d^2 u_i}{dy^2} = 0 \]
\[ \frac{d^2 \phi}{dy^2} - \alpha^2 \phi = 0 \]  
Stream-I
\[ \frac{d^2 u}{dy^2} + \theta_2 = 0 \]  
(33)
\[ \frac{d^2 \theta_2}{dy^2} = 0 \]  
(34)

Solving equations (30) to (34) with their corresponding boundary and interface conditions (17), we obtain the solutions corresponding to stream-I and stream-II as follows:

Stream-I
\[ \theta_1 = c_3 y + c_4 \]  
(35)
\[ \phi = b_1 \cosh(\alpha y) + b_2 \sinh(\alpha y) \]  
(36)
\[ u_1 = c_3 y + c_6 + \gamma^2 \left( \frac{c_5}{2} - \frac{c_6}{6} \right) y^3 - \frac{N b_1 \cosh(\alpha y)}{\alpha^2} \]  
(37)

Stream-II
\[ \theta_2 = c_3 y + c_4 \]  
(38)
\[ u_2 = c_3 y + p_3 y^2 + p_4 y^3 \]  
(39)
The dimensionless total volumetric flow rate is given by
\[ Q_v = Q_{v1} + Q_{v2} \]  
(40)
where \[ Q_{v1} = \int_{-0.25}^{0} u_1 dy, \quad Q_{v2} = \int_{0}^{0.25} u_2 dy \]
The dimensionless total heat rate added to the fluid is given by
\[ E = E_1 + E_2 \]  
(41)
where \[ E_1 = \int_{-0.25}^{0} u_1 \theta_1 dy, \quad E_2 = \int_{0}^{0.25} u_2 \theta_2 dy \]
The dimensionless total species rate added to the fluid is given by
\[ C_s = C_{s1} \]  
(42)
where \[ C_{s1} = \int_{-0.25}^{0} u_1 \phi dy \]

4. Results and Discussion

The heat and mass transfer by natural convection of chemically reacting micropolar fluid flowing in a vertical double passages channel has been analyzed. The analytical solutions obtained for fluid flow as well as heat and mass transfer have been obtained and the results are presented graphically.

Figures 1 a, b, c and 2a, b, c are the velocity and microrotation velocity profiles for various vortex viscosity parameters \( K = 0, 0.5, 1 \) and \( 1.5, \quad n = 1, \quad N = 2, \alpha = 2, \quad h = 1 \) and \( B = 1 \). Increasing in vortex viscosity parameter tends to decrease the fluid velocity in the double passage channel at all the baffle positions \( y^* = -0.2, y^* = 0, y^* = 0.2 \). The magnitude of microrotation velocity tends to increase as the vortex viscosity parameter is increased.
The effect of first order chemical reaction parameter $\alpha$, on velocity, microrotation velocity and concentration fields are seen in figures 5 a, b, c, 6 a, b, c and 7 a, b, c respectively. As $\alpha$ increases the velocity, microrotation velocity and concentration decreases in stream-I and remains constant in stream-II at all the baffle positions. The similar result was also obtained by Srinivas and Maturajan [20] for mixed convective flow in a vertical channel. This is due to the fact that the fluid in stream-I is concentrated. The maximum value of velocity and microrotation velocity is seen in stream-II for the baffle position at $y^* = -0.2$ and in stream-I at baffle position at $y^* = 0$ and 0.2.
Figure 7: Concentration profiles for different values of chemical reaction parameter $\alpha$ at (a) $y' = -0.2$ (b) $y' = 0$ (c) $y' = 0.2$.

Figure 8a, b, c are the plots for the variation of the dimensionless volumetric flow rate $Q_v$ with the buoyancy ratio $N$ for various vortex viscosity parameters $K = 0, 0.5, 1, and 1.5, \alpha = 2, n = 1, h = 1, B = 1$. Increasing the buoyancy ratio tends to accelerate the fluid flow, thus raising the volume flow rate of the fluid flowing through the vertical double passage channel. Moreover, the dimensionless volume flow rate flowing through the vertical channel tends to decrease as the vortex viscosity parameter is increased at any position of the baffle.

Figure 9a, b, c shows the variation of the dimensionless total species rate added to the fluid $C_s$ with the buoyancy ratio $N$ for various vortex viscosity parameters $K = 0, 0.5, 1, and 1.5, \alpha = 2, n = 1, h = 1, B = 1$. Increasing the buoyancy ratio accelerates the fluid flow, thus enhancing the mass transfer rate between the wall and the fluid flowing through the vertical double passage channel at all the baffle positions. Moreover, increasing the vortex viscosity parameter tends to decrease the dimensionless total species rate added to the fluid in the vertical double passage channel at all the baffle positions.
The dimensionless total heat rate added to the fluid $E$ is plotted as functions of the buoyancy ratio $N$ for various vortex viscosity parameters $K = 0, 0.5, 1$, and $1.5$ with $n = 1$, $h = 0.2$, and $B = 1$, as shown in figure 10a, b, c. Increasing the buoyancy ratio tends to accelerate the fluid flow, raising the heat transfer rate between the wall and the fluid and thus increasing the total heat rate added to the fluid in the vertical double passage channel. However, higher vortex viscosity parameter leads to a decrease in the dimensionless total heat rate added to the fluid in the vertical double passage channel at all the baffle positions.

5. Conclusions

Heat and mass transfer of a chemically reacting micropolar fluid in a vertical double passage channel has been studied analytically and the main findings are:

1. Increase in the vortex viscosity parameter increases the magnitude of microrotation velocity and decelerates the fluid flow in the vertical double passage channel at all the baffle positions.

2. Increasing in the thermal buoyancy ratio enhance the flow in both the streams at different baffle positions.

3. Increase in the chemical reaction parameter suppresses the velocity and temperate in stream-I and remains invariant in stream-II. The chemical reaction parameter suppresses the concentration in stream-I.

4. The use of baffle in the flow channel resulted in the heat transfer enhancement as high as compared to the heat transfer in a straight channel.

5. Increase in the vortex viscosity parameter on the volume flow rate, the total heat rate added to the fluid, and species rate added to the fluid for micropolar fluids are found to be lower than those of Newtonian fluids, which agrees with the results obtained by Cheng [8].

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