

Natural Convection Heat and Mass Transfer of a Chemically Reacting Micropolar Fluid in a Vertical Double Passage Channel

J. Prathap Kumar¹, J.C. Umavathi², Jagtap Sharadkumar³

^{1,2,3}Department of Mathematics, Gulbarga University, Kalaburagi -585 106, Karnataka, India

Abstract: *The problem of fully developed mixed convection for a laminar flow of a micropolar fluid in a vertical double passage channel has been investigated analytically. The channel is divided into two passages by means of a thin perfectly conductive plane baffle and the velocity, temperature and concentration will be individual in each streams. After inserting the baffle the fluid is concentrated in stream-I. The governing equations of the flow have been solved analytically subject to the relevant boundary and interface conditions. The closed form solutions are represented graphically. The effect of governing parameters namely, buoyancy ratio, vortex viscosity parameter and chemical reaction parameter on the velocity, microrotation velocity, temperature and concentration has been discussed. It is seen that increasing the vortex viscosity parameter tends to increase the magnitude of microrotation velocity and thus decreases the fluid velocity in the vertical channel. Moreover, the volume flow rate, the total heat rate added to the fluid and the total species rate added to the fluid for micropolar fluids are lower than those of Newtonian fluids.*

Keywords: Baffle, first order chemical reaction, micropolar fluid, vortex viscosity

1. Introduction

The analysis of the flow properties of non-Newtonian fluids is very much important in the field of fluid dynamics because of their technological applications. Mechanics of non-Newtonian fluids present challenges to engineers, physicist and mathematicians. Due to complex stress strain relationship of non-Newtonian fluids, not many investigators have studied the flow behavior of the fluids in various flow fields.

Studies of micropolar fluids have recently received considerable attention due to their applications in a number of processes that occur in industry. Such applications include the extrusion of polymer fluids, solidifications of liquid crystals, cooling of a metallic plate in a bath, animal bloods, exotic lubricants and colloidal and suspensions solutions, for which the classical Nernst Stokes theory is inadequate. The essence of the theory of micropolar fluid flow lies in the extension of the constitutive equations for Newtonian fluids so that more complex fluids can be described by this theory. In this theory, rigid particles contained in a small fluid volume element are limited to rotation about the center of the volume element described by the microrotation vector. This local rotation of the particles is in addition to the usual rigid body motion of the entire volume element. In the micropolar fluid theory, the laws of classical continuum mechanics are augmented with additional equations that account for the conservation of microinertia moments and the balance of first stress moments that arise due to consideration of the microstructure in a material and also additional local constitutive parameters are introduced.

The shear behavior of many real fluids used in the modern technology cannot be characterized by the Newtonian relationship and hence researchers have proposed diverse non-Newtonian fluid theories to explain the behavior of real fluids. The description of the motion of such fluids requires

a new approach, which is different from the classical concepts. Thus, additional balance laws are needed to describe such types of complex behaviors. Physically micropolar fluids may be described as the non-Newtonian fluids consisting of dumb bell molecules or short rigid cylindrical elements, polymer fluids, fluid suspensions, animal blood, etc. The presence of dust or smoke, particularly in a gas, may also be modeled using micropolar fluid dynamics. The theory of micropolar fluids, first proposed by Eringen [1, 2] is capable of describing such fluids. Extensive reviews of theory of micropolar fluids and its applications can be found in review articles by Ariman et al. [3,4] and recent books by Łukaszewicz [5] and Eringen [6]. Recently Chamkha et al. [7] have studied the fully developed free convection of a micropolar fluid in a vertical channel. Cheng [8] has presented the fully developed natural convective flow due to heat and mass transfer of a micropolar fluid in a vertical channel with asymmetric wall temperature and concentrations.

It is well known that many of the physiological fluids behave like suspensions of deformable or rigid particles in Newtonian fluid. In view of this, some researchers have used non-Newtonian fluid models for the biofluids. Dash et al. [9] estimated the increased flow resistance in a narrow catheterized artery using the Caisson fluid model. Banerjee et al. [10] investigated the changes in flow and mean pressure gradient across a coronary artery with stenosis in the presence of a translational catheter using the finite element method for the Carreau model, a shear rate dependent non-Newtonian fluid model. Eringen [1] proposed the theory of micropolar fluids to study fluids with suspension nature. That is, each material volume element contains micro volume elements that can translate and rotate independently of the motion of macro volume. In this model, two independent kinematic vector fields are introduced one representing the translation velocities of fluid particles; and the other representing angular (spin) velocities of the particles, called as microrotation vector

(Lukaszewicz, [5]). In many transport processes, both in nature and in industrial applications, heat and mass transfer is a consequence of the buoyancy effects caused by diffusion of heat and chemical species. The natural convective flow is often caused not entirely by temperature gradients, but also by the difference in the concentrations of dissimilar chemical species. Convection and transport processes are governed by buoyancy mechanisms arising from both thermal and species diffusion.

In particular process involving the mass transfer effects has been considered to be important precisely in chemical engineering equipments. The other applications include solidification of binary alloys and crystal growth dispersion of dissolved materials or particulate water in flows, drying and dehydration operations in chemical and food processing plants, evaporation at the surface of water body. The order of the chemical reaction depends on several factors. One of the simplest chemical reactions is the first order reaction in which rate of reaction is directly proportional to the species concentration. Das et al. [11] have studied the effect of mass transfer on the flow started impulsively past an infinite vertical plate in the presence of wall heat flux and chemical reaction. Muthucumaraswamy and Ganeshan [12, 13] have studied the impulsive motion of a vertical plate with heat flux/mass flux/suction and diffusion of chemically reactive species.

The rate of heat transfer in a vertical channel could be enhanced by using special inserts. These inserts can be specially designed to increase the included angle between the velocity vector and the temperature gradient vector, rather than to promote turbulence. This increases the rate of heat transfer without a considerable drop in the pressure by Guo et al. [14]. A plane baffle may be used as an insert to enhance the rate of heat transfer in the channel. To avoid a considerable increase in the transverse thermal resistance into the channel, a thin and perfectly conductive baffle is used. The effect of such baffle on the laminar fully developed combined convection in a vertical channel with different uniform wall temperatures has been studied analytically by Salah El-Din [15]. Umavathi et al. [16], Umavathi and Prathap Kumar [17] and Umavathi and Jaweria Sultana [18] studied the flow and heat transfer of a micropolar fluid in a vertical channel.

In this paper, we aim to study the fully developed heat and mass transfer by natural convection of a micropolar fluid inside a plane vertical channel for asymmetric wall temperatures and concentrations by inserting a perfectly thin baffle. The closed form exact solutions are derived and the effects of the buoyancy ratio and the vortex viscosity parameter on the flow, heat transfer and mass transfer characteristics, such as the velocity, microrotation velocity, volume flow rates, total heat rate added to the fluid and the total species rate added to the fluid are analyzed.

2. Mathematical Formulation

Consider a steady fully developed laminar natural convection flow of a micropolar fluid between two vertical plates. The vertical plates are separated by a distance h . Further the vertical two plates are separated by inserting a

moving perfectly thin baffle. The inlet temperature is T_0 and inlet concentration is C_0 . The inner surface of the left plate ($y = -h/2$) is kept at a constant temperature T_1 while the inner surface of the right plate ($y = h/2$) is maintained at a constant temperature T_2 . In addition, the concentration of a certain constituent in the solution varies from C_1 on the inner surface of the left plate to C_2 on the inner surface of the right plate. Because the flow is fully developed, the transverse velocity is zero and the flow depends only on the transverse coordinate Y . The fluid properties are assumed to be constant except for density variations in the buoyancy force term.

The governing equations for velocity, temperature, microrotation velocity and concentrations are

$$\rho g \beta_T (T_1 - T_0) + \rho g \beta_c (C_1 - C_0) + (\mu + \kappa) \frac{d^2 U_1}{dY^2} + \kappa \frac{dv_1}{dY} = 0 \quad (1)$$

$$k \frac{d^2 T_1}{dY^2} = 0 \quad (2)$$

$$\gamma \frac{d^2 v_1}{dY^2} - 2\kappa v_1 - \kappa \frac{dU_1}{dY} = 0 \quad (3)$$

$$D \frac{d^2 C}{dY^2} - k C = 0 \quad (4)$$

Stream-II

$$\rho g \beta_T (T_2 - T_0) + \rho g \beta_c (C_2 - C_0) + (\mu + \kappa) \frac{d^2 U_2}{dY^2} + \kappa \frac{dv_2}{dY} = 0 \quad (5)$$

$$k \frac{d^2 T_2}{dY^2} = 0 \quad (6)$$

$$\gamma \frac{d^2 v_2}{dY^2} - 2\kappa v_2 - \kappa \frac{dU_2}{dY} = 0 \quad (7)$$

The boundary and interface conditions on velocity, temperature, microrotation velocity and concentration are

$$U_1 = 0, T = T_1, v_1 = 0, C = C_1 \quad \text{at} \quad Y = -h/2$$

$$U_2 = 0, T = T_2, v_2 = 0, \quad \text{at} \quad Y = h/2$$

$$U_1 = 0, U_2 = 0, T_1 = T_2, \frac{dT_1}{dY} = \frac{dT_2}{dY}, \quad \text{at} \quad Y = h^*$$

$$v_1 = 0, v_2 = 0, C = C_2 \quad \text{at} \quad Y = h^* \quad (8)$$

Here we introduce the following non-dimensional variables

$$U_i = \frac{u_i Gr \mu}{\rho}, v_i = \frac{H_i Gr \mu}{h^2 \rho}, v_2 = \frac{H_2 Gr \mu}{h^2 \rho}, v = (\mu + \frac{\kappa}{2}) j,$$

$$\theta_i = \frac{T_i - T_0}{T_1 - T_0}, Gr = \frac{g \beta_i \Delta T h^3}{\nu^2}, Gc = \frac{g \beta_c \Delta C h^3}{\nu^2},$$

$$\phi = \frac{C - C_0}{C_1 - C_0}$$

$$\Delta T = T_1 - T_0, \Delta C = C_1 - C_0, Y = \frac{y}{h}, Y^* = \frac{y^*}{h},$$

$$\alpha^2 = \frac{kh^2}{D}, N = \frac{\beta_c(C_1 - C_0)}{\beta_t(T_1 - T_0)} \quad (9)$$

where U_i are the velocity components in the stream wise direction. T and C are the fluid temperature and species concentration, respectively. v_i are the angular velocity of the micropolar fluid, κ is the vortex viscosity and j is the microinertia density. Here γ is the spin gradient viscosity and we assume that $v = \left(\mu + \frac{\kappa}{2}\right)j$. Property μ is the dynamic viscosity of the fluid and ρ is the fluid density. β_t is the coefficients for thermal expansion and β_c is the concentration expansion coefficient and g is the gravitational acceleration. Note that the boundary condition for the microrotation velocity at the fluid solid interface is $v_i = 0$, the condition of zero spin, as used by Takhar et al. [19]. The microstructure does not rotate relative to the surface. By using these parameters one obtains the momentum, energy, microrotation velocity and concentration equations corresponding to stream-I and stream-II as

Stream-I

$$(1+K)\frac{d^2u_1}{dy^2} + \theta_1 + N\phi + K\frac{dH_1}{dy} = 0 \quad (10)$$

$$\frac{d^2\theta_1}{dy^2} = 0 \quad (11)$$

$$\left(1 + \frac{K}{2}\right)\frac{d^2H_1}{dy^2} - BK\left(2H_1 + \frac{dU_1}{dy}\right) = 0 \quad (12)$$

$$\frac{d^2\phi}{dy^2} - \alpha^2\phi = 0 \quad (13)$$

Stream-II

$$(1+K)\frac{d^2u_2}{dy^2} + \theta_2 + K\frac{dH_2}{dy} = 0 \quad (14)$$

$$\frac{d^2\theta_2}{dy^2} = 0 \quad (15)$$

$$\left(1 + \frac{K}{2}\right)\frac{d^2H_2}{dy^2} - BK\left(2H_2 + \frac{dU_2}{dy}\right) = 0 \quad (16)$$

Subject to the boundary and interface conditions

$$u_1 = 0, \theta_1 = 1, H_1 = 0, \phi = 1 \quad \text{at} \quad y = -1/4$$

$$u_2 = 0, \theta_2 = 0, H_2 = 0 \quad \text{at} \quad y = 1/4$$

$$u_1 = 0, u_2 = 0, \theta_1 = \theta_2, \frac{d\theta_1}{dy} = \frac{d\theta_2}{dy} \quad \text{at} \quad y = y^*$$

$$H_1 = 0, H_2 = 0, \phi = n \quad \text{at} \quad y = y^* \quad (17)$$

where $n = \frac{C_2 - C_0}{C_1 - C_0}$ is the wall concentration ratio,

$B = \frac{h^2}{j}$ and $K = \frac{\kappa}{\mu}$ are the material parameters

3. Solutions

Solutions of equations (10) to (16) using the boundary and interface conditions (17) are

Stream-I

$$\theta_1 = c_1y + c_2 \quad (18)$$

$$\phi = c_3 \text{Cosh}(\alpha y) + c_4 \text{Sinh}(\alpha y) \quad (19)$$

$$H_1 = c_5 C \text{osh}(\sqrt{\tau}y) + c_6 S \text{inh}(\sqrt{\tau}y) + l_5 S \text{inh}(\alpha y) + l_6 C \text{osh}(\alpha y) + l_7 y^2 + l_8 y + l_9 \quad (20)$$

$$u_1 = \frac{-1}{K+1} \left(\frac{Kc_7}{\sqrt{\tau}} \text{Sinh}(\sqrt{\tau}y) + \frac{Kc_8}{\sqrt{\tau}} \text{Cosh}(\sqrt{\tau}y) + \frac{Kl_5}{\alpha} C \text{osh}(\alpha y) + \frac{Kl_6}{\alpha} S \text{inh}(\alpha y) + \frac{Kl_7 y^3}{3} + \frac{Kl_8 y^2}{2} + \left(\frac{c_1 y^3}{6} + \frac{c_2 y^2}{2}\right) + \frac{N c_5 C \text{osh}(\alpha y)}{\alpha^2} + \frac{N c_6 S \text{inh}(\alpha y)}{\alpha^2} + c_9 y + c_{10} \right) \quad (21)$$

Stream-II

$$\theta_2 = c_3 y + c_4 \quad (22)$$

$$H_2 = d_1 \text{Cosh}(\sqrt{\tau}y) + d_2 \text{Sinh}(\sqrt{\tau}y) + p_5 y^2 + p_6 y + p_7 \quad (23)$$

$$u_2 = \frac{-1}{K+1} \left(\frac{Kd_1}{\sqrt{\tau}} \text{Sinh}(\sqrt{\tau}y) + \frac{Kd_2}{\sqrt{\tau}} \text{Cosh}(\sqrt{\tau}y) + \frac{Kp_5 y^3}{3} + \frac{Kp_6 y^2}{2} + \left(\frac{c_3 y^3}{6} + \frac{c_4 y^2}{2}\right) + F_1 y + F_2 \right) \quad (24)$$

3.1 Special case for Newtonian fluid ($\kappa = 0$)

The governing equations for velocity, temperature and concentrations are

Stream-I

$$\rho g \beta_t (T_1 - T_0) + \rho g \beta_c (C_1 - C_0) + \mu \frac{d^2 U_1}{dY^2} = 0 \quad (25)$$

$$k \frac{d^2 T_1}{dY^2} = 0 \quad (26)$$

$$D \frac{d^2 C}{dY^2} - kC = 0 \quad (27)$$

Stream-II

$$\rho g \beta_t (T_2 - T_0) + \mu \frac{d^2 U_2}{dY^2} = 0 \quad (28)$$

$$k \frac{d^2 T_2}{dY^2} = 0 \quad (29)$$

Using the non-dimensional parameters in equations (25) to (29), we obtain the non-dimensionalised momentum, energy and concentration equations corresponding to stream-I and stream-II as

Stream-I

$$\frac{d^2 u_1}{dy^2} + \theta_1 + N\phi = 0 \quad (30)$$

$$\frac{d^2 \theta_1}{dy^2} = 0 \quad (31)$$

$$\frac{d^2\phi_1}{dy^2} - \alpha^2\phi_1 = 0 \quad (32)$$

Stream-II

$$\frac{d^2u_2}{dy^2} + \theta_2 = 0 \quad (33)$$

$$\frac{d^2\theta_2}{dy^2} = 0 \quad (34)$$

Solving equations (30) to (34) with their corresponding boundary and interface conditions (17), we obtain the solutions corresponding to stream-I and stream-II as follows

$$\theta_1 = c_1y + c_2 \quad (35)$$

$$\phi = b_1 \text{Cosh}(\alpha y) + b_2 \text{Sinh}(\alpha y) \quad (36)$$

$$u_1 = c_5y + c_6 - y^2\left(\frac{c_2}{2}\right) - \frac{c_1}{6}y^3 - \frac{Nb_1 \text{Cosh}(\alpha y)}{\alpha^2} - \frac{Nb_2 \text{Sinh}(\alpha y)}{\alpha^2} \quad (37)$$

Stream-II

$$\theta_2 = c_3y + c_4 \quad (38)$$

$$u_2 = c_8 + c_7y + p_5y^2 + p_6y^3 \quad (39)$$

The dimensionless total volumetric flow rate is given by

$$Qv = Qv_1 + Qv_2 \quad (40)$$

$$\text{where } Qv_1 = \int_{-0.25}^0 u_1 dy, \quad Qv_2 = \int_0^{0.25} u_2 dy$$

The dimensionless total heat rate added to the fluid is given by

$$E = E_1 + E_2 \quad (41)$$

$$\text{where } E_1 = \int_{-0.25}^0 u_1 \theta_1 dy, \quad E_2 = \int_0^{0.25} u_2 \theta_2 dy$$

The dimensionless total species rate added to the fluid is given by

$$Cs = Cs_1 \quad (42)$$

$$\text{where } Cs_1 = \int_{-0.25}^0 u_1 \phi_1 dy$$

4. Results and Discussion

The heat and mass transfer by natural convection of chemically reacting micropolar fluid flowing in a vertical double passages channel has been analyzed. The analytical solutions obtained for fluid flow as well as heat and mass transfer have been obtained and the results are presented graphically.

Figures 1 a, b, c and 2a, b, c are the velocity and microrotation velocity profiles for various vortex viscosity parameters $K = 0, 0.5, 1$ and 1.5 , $n = 1$, $N = 2, \alpha = 2, h = 1$ and $B = 1$. Increasing in vortex viscosity parameter tends to decrease the fluid velocity in the double passage channel at all the baffle positions $y^* = -0.2, y^* = 0, y^* = 0.2$. The magnitude of microrotation velocity tends to increase as the vortex viscosity parameter is increased.

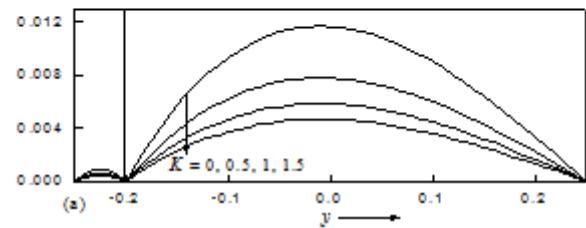
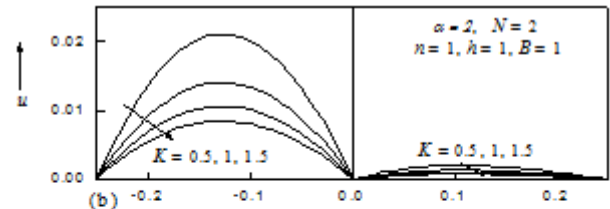
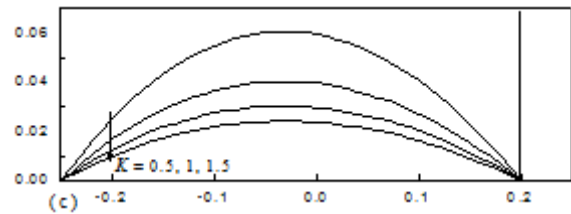


Figure 1: Velocity profiles for different values of vortex viscosity parameter K at (a) $y^* = -0.2$ (b) $y^* = 0$ (c) $y^* = 0.2$

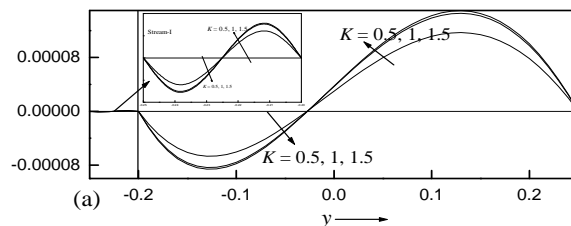
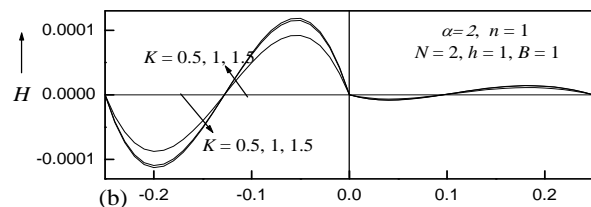
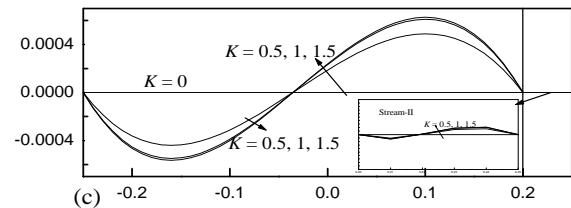


Figure 2: Microrotation velocity profiles for different values of vortex viscosity parameter K at (a) $y^* = -0.2$ (b) $y^* = 0$ (c) $y^* = 0.2$

The effect of buoyancy ratios N on the velocity and microrotation velocity are shown in figures 3 a, b, c and 4 a, b, c respectively. It is seen that the magnitude of microrotation velocity is enhanced with an increase in the buoyancy ratio. Moreover, increasing the buoyancy ratio tends to accelerate the fluid flow in the vertical double

passage channel at all the baffle positions, which is the similar result found by Cheng [8].

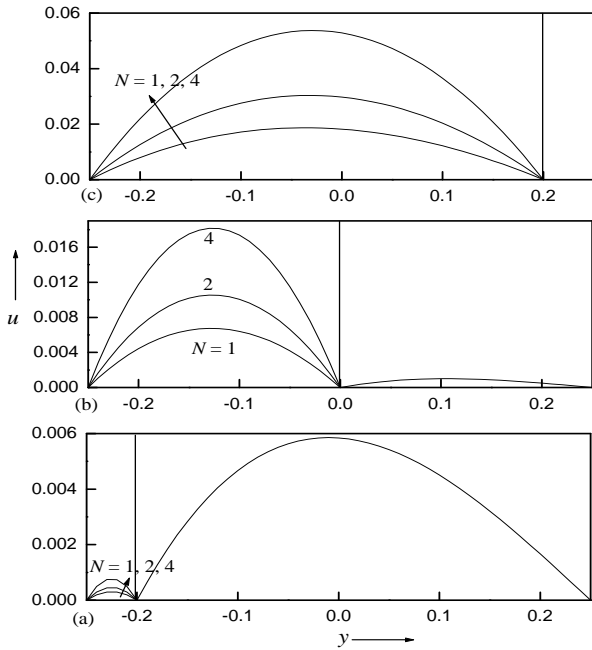


Figure 3: Velocity profiles for different values of buoyancy ratio N at (a) $y^* = -0.2$ (b) $y^* = 0$ (c) $y^* = 0.2$

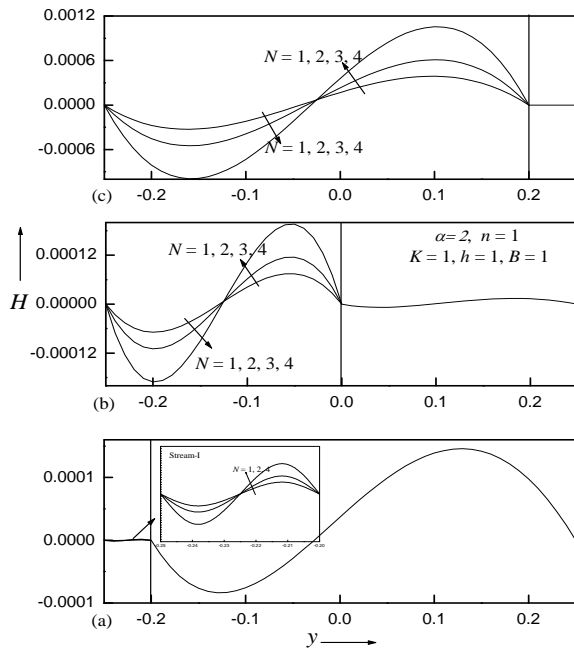


Figure 4: Microrotation velocity profiles for different values of buoyancy ratio N at (a) $y^* = -0.2$ (b) $y^* = 0$ (c) $y^* = 0.2$

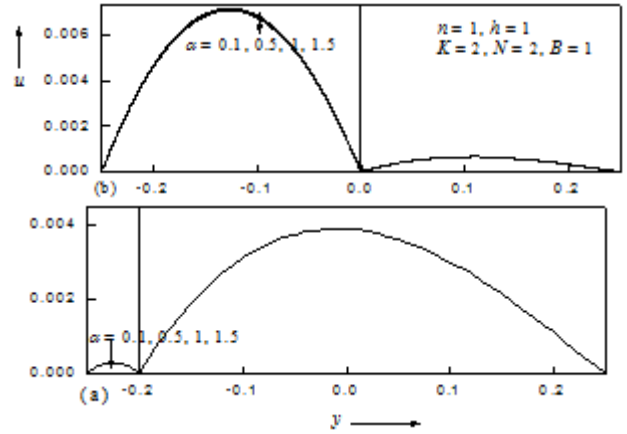
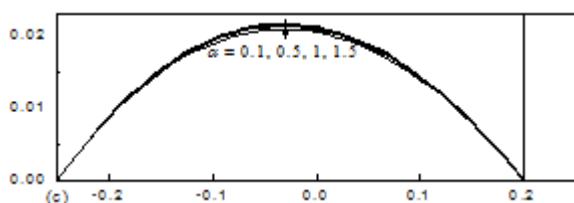


Figure 5: Velocity profiles for different values of chemical reaction parameter α at (a) $y^* = -0.2$ (b) $y^* = 0$ (c) $y^* = 0.2$.

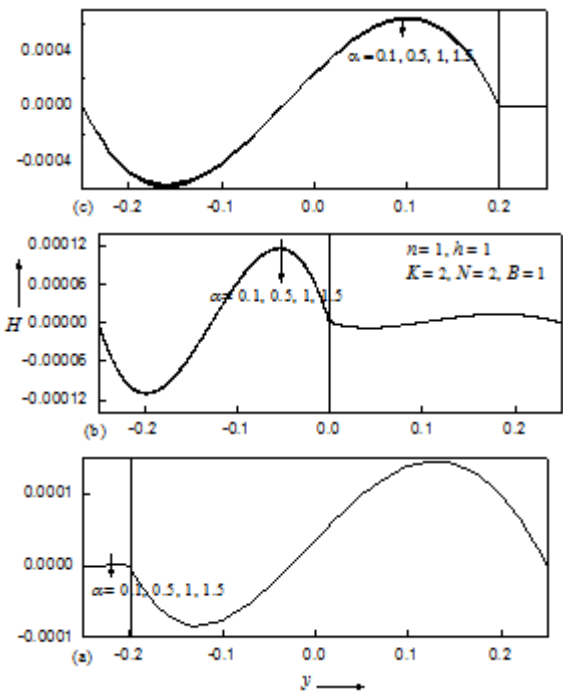


Figure 6: Microrotation velocity profiles for different values of chemical reaction parameter α at (a) $y^* = -0.2$ (b) $y^* = 0$ (c) $y^* = 0.2$.

The effect of first order chemical reaction parameter α , on velocity, microrotation velocity and concentration fields are seen in figures 5 a, b, c, 6 a, b, c and 7 a, b, c respectively. As α increases the velocity, microrotation velocity and concentration decreases in stream-I and remains constant in stream-II at all the baffle positions. The similar result was also obtained by Srinivas and Mutturajan [20] for mixed convective flow in a vertical channel. This is due to the fact that the fluid in stream-I is concentrated. The maximum value of velocity and microrotation velocity is seen in stream-II for the baffle position at $y^* = -0.2$ and in stream-I at baffle position at $y^* = 0$ and 0.2 .

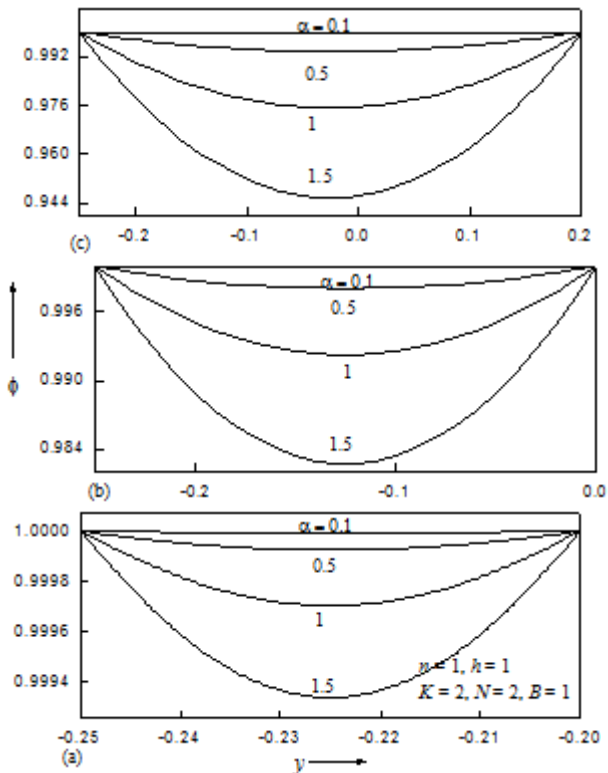


Figure 7: Concentration profiles for different values of chemical reaction parameter α at (a) $y^* = -0.2$ (b) $y^* = 0$ (c) $y^* = 0.2$.

Figure 8a, b, c are the plots for the variation of the dimensionless volumetric flow rate Q_v with the buoyancy ratio N for various vortex viscosity parameters $K = 0, 0.5, 1$ and 1.5 , $\alpha = 2, n = 1, h = 1$ and $B = 1$. Increasing the buoyancy ratio tends to accelerate the fluid flow, thus raising the volume flow rate of the fluid flowing through the vertical double passage channel. Moreover, the dimensionless volume flow rate flowing through the vertical channel tends to decrease as the vortex viscosity parameter is increased at any position of the baffle. Figure 9a, b, c shows the variation of the dimensionless total species rate added to the fluid C_s with the buoyancy ratio N for various vortex viscosity parameters $K = 0, 0.5, 1$ and 1.5 , $\alpha = 2, n = 1, h = 1$ and $B = 1$. Increasing the buoyancy ratio accelerates the fluid flow, thus enhancing the mass transfer rate between the wall and the fluid flowing through the vertical double passage channel at all the baffle positions. Moreover, increasing the vortex viscosity parameter tends to decrease the dimensionless total species rate added to the fluid in the vertical double passage channel at all the baffle positions.

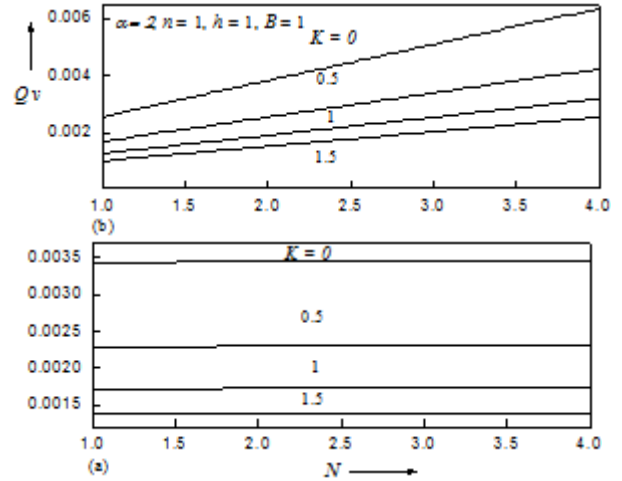
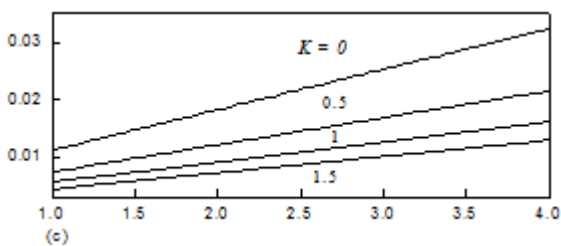


Figure 8: Effect of buoyancy ratio on volumetric flow rate for different values of vortex viscosity parameter at (a) $y^* = -0.2$ (b) $y^* = 0$ (c) $y^* = 0.2$.

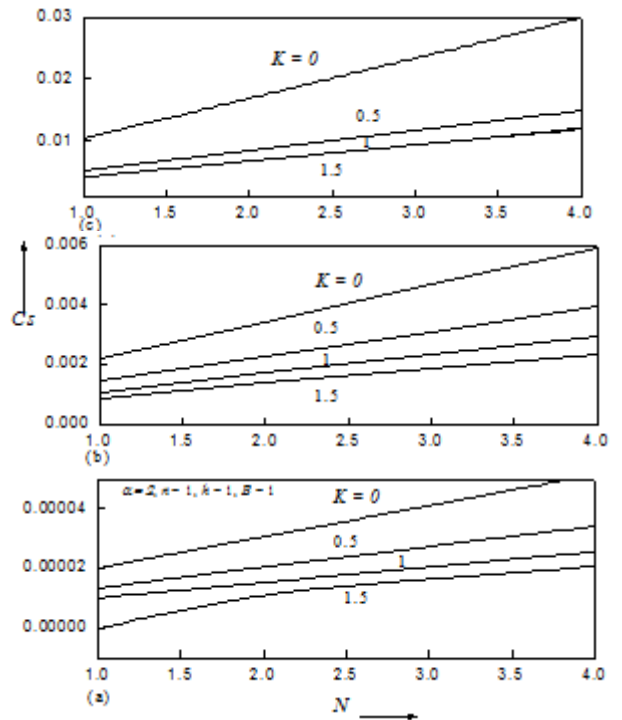
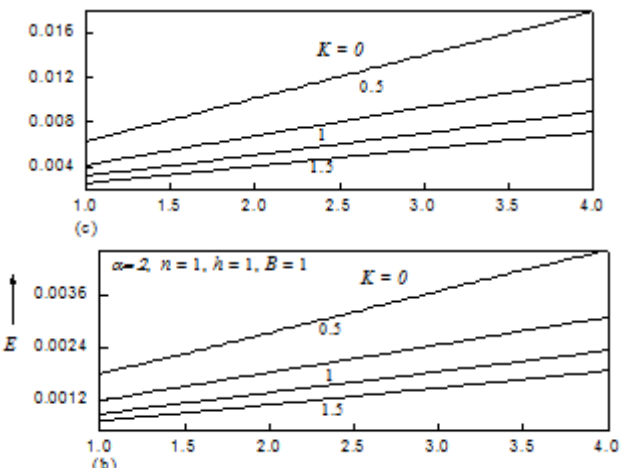


Figure 9: Effect of buoyancy ratio on species concentration rate for different values of vortex viscosity parameter at (a) $y^* = -0.2$ (b) $y^* = 0$ (c) $y^* = 0.2$.



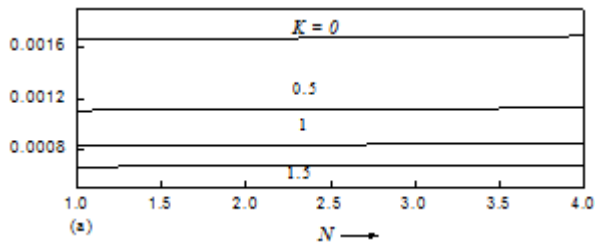


Figure 10: Effect of buoyancy ratio on total energy flow rate for different values of vortex viscosity parameter at (a) $y^* = -0.2$ (b) $y^* = 0$ (c) $y^* = 0.2$.

The dimensionless total heat rate added to the fluid E is plotted as functions of the buoyancy ratio N for various vortex viscosity parameters $K = 0, 0.5, 1$ and 1.5 , $n = 1$, $h = 1$ and $B = 1$, as shown in figure 10a, b, c. Increasing the buoyancy ratio tends to accelerate the fluid flow, raising the heat transfer rate between the wall and the fluid and thus increasing the total heat rate added to the fluid in the vertical double passage channel. However, higher vortex viscosity parameter leads to a decrease in the dimensionless total heat rate added to the fluid in the vertical double passage channel at all the baffle positions.

5. Conclusions

Heat and mass transfer of a chemically reacting micropolar fluid in a vertical double passage channel has been studied analytically and the main findings are:

1. Increase in the vortex viscosity parameter increases the magnitude of microrotation velocity and decelerates the fluid flow in the vertical double passage channel at all the baffle positions.
2. Increasing in the thermal buoyancy ratio enhance the flow in both the streams at different baffle positions.
3. Increase in the chemical reaction parameter suppresses the velocity and temperature in stream-I and remains invariant in stream-II. The chemical reaction parameter suppresses the concentration in stream-I.
4. The use of baffle in the flow channel resulted in the heat transfer enhancement as high as compared to the heat transfer in a straight channel.
5. Increase in the vortex viscosity parameter on the volume flow rate, the total heat rate added to the fluid, and species rate added to the fluid for micropolar fluids are found to be lower than those of Newtonian fluids, which agrees with the results obtained by Cheng [8].

6. Acknowledgement

One of the author J. Prathap Kumar would like to thank UGC-New Delhi for the financial support under UGC-Major Research Project (Project No. 41-774/2012(SR))

References

- [1] A. C. Eringen, "Theory of Micropolar Fluids", *J. Math. Mech.*, 16, pp. 1-18, 1966.
- [2] A. C. Eringen, "Theory of Thermomicropolar Fluids", *J. Math. Anal. App.*, 38, pp. 480-496, 1972.

- [3] T. Ariman, M. A. Turk and N. D. Sylvester, "Microcontinuum Fluid Mechanics a Review", *Int. J. Eng. Sci.*, 11, pp. 905-930, 1973.
- [4] T. Ariman, M. A. Turk and N. D. Sylvester, "Applications of Micro Continuum Fluid Mechanics", *Int. J. Eng. Sci.*, 12, pp. 273-293, 1974.
- [5] G. Lukaszewicz, "Micropolar Fluids - Theory and Applications", Birkhuser, Basel, 1999.
- [6] A. C. Eringen, "Microcontinuum field theories II: Fluent Media", Springer, New York, 2001.
- [7] A. J. Chamkha, T. Grosan and I. Pop, "Fully Developed Free Convection of a Micropolar Fluid in a Vertical Channel", *Int. Comm. Heat Mass Trans.*, 29, pp. 1119-1127, 2002.
- [8] Ching-Yang Cheng, "Fully Developed Natural Convection Heat and Mass Transfer of a Micropolar Fluid in a Vertical Channel with Asymmetric Wall Temperature and Concentrations", *Int. Comm. Heat Mass Trans.*, 33: 627-635, 2006.
- [9] R. K. Dash, G. Jayaraman and K. N. Mehta, "Estimation of Increased Flow Resistance in a Narrow Catheterized Artery - a Theoretical Model", *J. Biomech.*, 29, pp. 917-930, 1996.
- [10] R. K. Banerjee, L. H. Back, M. R. Back and Y. I. Cho, "Catheter Obstruction Effect on Pulsatile Flow Rate-Pressure Drop During Coronary Angioplasty", *Trans ASME, J. Biomech. Eng.*, 121, pp. 281-289, 1999.
- [11] U. N. Das, R. Deka and V. M. Soundalgekar, "Effects of Mass Transfer on Flow Past an Impulsively Started Infinite Vertical Plate with Constant Heat Flux and Chemical Reaction", *J. Forschung Im Ingenieurwesen-Eng. Res.*, Bd, 60, pp. 284-287, 1994.
- [12] R. Muthucumaraswamy and P. Ganesan, "First Order Chemical Reaction on Flow Past Impulsively Strated Vertical Plate with Uniform Heat and Mass Flux", *Acta Mech.*, 147, pp. 45-57, 2001.
- [13] R. Muthucumaraswamy and P. Ganesan, "On Impulsive Motion of a Vertical Plate with Heat Flux and Diffusion of Chemically Reactive Species", *Forsch Ingenieurwes*, 66, pp. 17-23, 2000.
- [14] Z. Y. Guo and B. X. Wang, "A Novel Concept for Convective Heat Transfer Enhancement", *Int. J. Heat Mass Trans.*, 41, pp. 2221-2225, 1998.
- [15] M. M. Salah El-Din, "Fully Developed Laminar Convection in a Vertical Double Passage Channel", *Appl. Energy*, 47, pp. 69-75, 1994.
- [16] J. C. Umavathi, J. Prathap Kumar and A. J. Chamkha, "Flow and Heat Transfer of a Micropolar Fluid Sandwiched Between Viscous Fluid Layers", *Can. J. Phys.*, 86, pp. 961-973, 2008.
- [17] J. C. Umavathi, and J. Prathap Kumar, "Mixed Convection Flow of Micropolar Fluid in a Vertical Channel with Symmetric and Asymmetric Wall Heating Conditions", *Int. J. Mech. Eng.*, 16, pp. 141-159, 2011.
- [18] J. C. Umavathi and Jaweriya Sultana, "Mixed Convection Flow of a Micropolar Fluid with Concentration in a Vertical Channel in the Presence of Heat Source or Sink", *Int. J. Math. Arch.*, 3, pp. 35556-3569, 2012.
- [19] H. S. Takhar, R. S. Agarwal, R. Bhargava and S. Jain, "Mixed Convection Flow of a Micropolar Fluid Over a Stretching Sheet", *Heat Mass Trans.*, 34, pp. 213-219, 1998.
- [20] S. Srinivas and R. Muthuraj, "Effect of Chemical Reaction and Space Porosity on MHD Mixed Convective Flow in a Vertical Asymmetric Channel with Peristalsis", *Math. Comp. Mod.*, 54, pp. 1213-1227, 2011.