

$$\begin{aligned}
 & + \left(p - \frac{c_2}{q_2 x_2} \right) \sigma_1 \\
 & = p \left\{ r \left(1 - \frac{x_1}{k} \right) - \sigma_1 + \frac{\sigma_2 x_2}{x_1} \right\} + \\
 & \left(p - \frac{c_1}{q_1 x_1} \right) \left(-\frac{\sigma_2 x_2}{x_1} - \frac{r x_1}{k} \right) + \left(p - \frac{c_2}{q_2 x_2} \right) \sigma_1 \\
 & = p r - \frac{2 p r x_1}{k} + \frac{c_1 \sigma_2 x_2}{q_1 x_1^2} + \frac{c_1 r}{q_1 k} - \frac{c_2 \sigma_1}{q_2 x_2} \\
 \text{or, } & \frac{c_1 \sigma_2 x_2}{q_1 x_1^2} - \frac{2 p r x_1}{k} - \frac{c_2 \sigma_1}{q_2 x_2} + \frac{\delta c_1}{q_1 x_1} + \frac{c_1 r}{q_1 k} + p(r - \delta) = 0
 \end{aligned}
 \tag{25}$$

Again, from (18) and (24), we have

$$\begin{aligned}
 -\delta e^{-\delta t} \left(p - \frac{c_2}{q_2 x_2} \right) & = -e^{-\delta t} p q_2 E_2 - e^{-\delta t} \left(p - \frac{c_1}{q_1 x_1} \right) \sigma_2 \\
 & + e^{-\delta t} \left(p - \frac{c_2}{q_2 x_2} \right) (s + \sigma_2 + q_2 E_2) \\
 \text{or, } \delta \left(p - \frac{c_2}{q_2 x_2} \right) & = p \left(-s - \sigma_2 + \frac{\sigma_1 x_1}{x_2} \right) + \left(p - \frac{c_1}{q_1 x_1} \right) \sigma_2 \\
 & - \frac{\sigma_1 x_1}{x_2} \left(p - \frac{c_2}{q_2 x_2} \right) \\
 & = -p(s + \sigma_2) + \left(p - \frac{c_1}{q_1 x_1} \right) \sigma_2 + \frac{c_2 \sigma_1 x_1}{q_2 x_2^2} \\
 \text{or, } & \frac{c_2 \sigma_1 x_1}{q_2 x_2^2} + \frac{\delta c_2}{q_2 x_2} - \frac{c_1}{q_1 x_1} - p(\delta + s) = 0
 \end{aligned}
 \tag{26}$$

Solving the above non linear equations (25) and (26), we have the optimal equilibrium level of two subpopulations, $x_{1\delta}$ and $x_{2\delta}$ in inshore and offshore area respectively. Using these values in the state equations we have

$$E_{1\delta} = \frac{1}{q_1} \left\{ r \left(1 - \frac{x_{1\delta}}{k} \right) - \sigma_1 + \frac{\sigma_2 x_{2\delta}}{x_{1\delta}} \right\}
 \tag{27}$$

$$E_{2\delta} = \frac{1}{q_2} \left(-s - \sigma_2 + \frac{\sigma_2 x_{2\delta}}{x_{1\delta}} \right)
 \tag{28}$$

$$\tau_{1\delta} = p - \frac{c_1}{q_1 x_{1\delta}}
 \tag{29}$$

$$\tau_{2\delta} = p - \frac{c_2}{q_2 x_{2\delta}}
 \tag{30}$$

7. Numerical Example

Let $r = 5, k = 1000, \sigma_1 = 0.7, \sigma_2 = 0.3, s = 0.2, q_1 = 0.02, q_2 = 0.01, \lambda_1 = 1, \lambda_2 = 1, p = 10, c_1 = 50, c_2 = 60$ and $\delta = 0.4$.

Therefore, $p - \frac{r c_1}{k q_1 (r - \sigma_1)} = 7.093$,

$$m = \min \left\{ 1, \frac{\sigma_1 c_1 q_2}{c_2 q_1 (s + \sigma_2)} \right\} = 0.583$$

and $(1 - m)p = 4.17$.

Since $m \in (0, 1)$, for existence of the non-trivial steady state $P(x_1^*, x_2^*, E_1^*, E_2^*)$ the regulatory agencies have to determine the tax τ_1 per unit harvested biomass from the inshore fishery such that $4.17 < \tau_1 < 7.093$, by (8).

Suppose the agencies choose $\tau_1 = 7$.

Then $n = \frac{\tau_1}{m} + \left(1 - \frac{1}{m} \right) p = 4.86$ and $\min(n, \tau_1) = 4.86$.

So the regulatory agencies choose the tax τ_2 such that $\tau_2 < 4.86$, by (12), for existence of the non-trivial steady state $P(x_1^*, x_2^*, E_1^*, E_2^*)$.

But the agencies are always interested for sufficient harvesting in the offshore area whereas the fishermen are interested in fishing in inshore area. In such a situation the agencies would like to impose the tax τ_2 moderately low compared to its applicable maximum level. Keeping in mind for such a situation to arise, suppose the agencies choose the tax $\tau_2 = 3$.

Considering the above parameter values together with $\tau_1 = 7$ and $\tau_2 = 3$, the non-trivial steady state becomes $P(833.33, 857.14, 22.10, 18.06)$ and this steady state is locally as well as globally asymptotically stable.

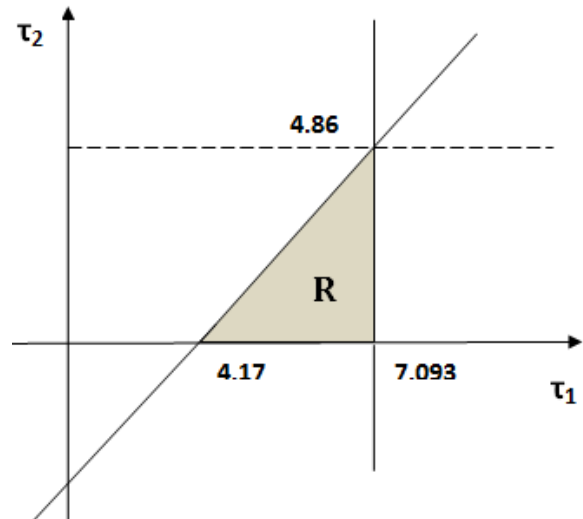


Figure 2: Feasible region (R) of the taxes τ_1 and τ_2 for $\min \tau_1 = 4.17$, $\max \tau_1 = 7.093$ and $\max \tau_2 = 4.86$

Figure 2 shows the feasible region of the taxes τ_1 and τ_2 for existence of the non-trivial steady state which is always locally and globally asymptotically stable.

Using these parameter values equations (25) and (26) become

$$\begin{aligned}
 750 x_1^{-2} x_2 - 0.1 x_1 - 4200 x_2^{-1} + 1000 x_1^{-1} + 58.5 & = 0 \\
 \text{and } 4200 x_1 x_2^{-1} + 2400 x_2^{-1} - 750 x_1^{-1} - 6 & = 0,
 \end{aligned}$$

respectively. Solving the above non-linear equations (using *Mathematica* software), we have the optimal equilibrium level of inshore and offshore subpopulations as $x_{1\delta} = 564.90$ and $x_{2\delta} = 755.88$ respectively. For these optimal values of populations, the optimal level of efforts and taxes are $E_{1\delta} = 93.85, E_{2\delta} = 2.31, \tau_{1\delta} = 5.57$ and $\tau_{2\delta} = 2.06$, obtained from (27), (28), (29) and (30) respectively.

Thus $P_\delta(564.90, 755.88, 93.85, 2.31)$ is the optimal equilibrium solution of the system (1) corresponding to the above parameter values and the optimal taxes are $\tau_{1\delta} = 5.57$ and $\tau_{2\delta} = 2.06$.

Comparing this optimal equilibrium solution with the biological equilibrium solution, we see that in view of economic consideration, the inshore fishing is more attractive than the offshore fishing and so the equilibrium level of inshore subpopulation decreases. Whenever the inshore subpopulation decreases then the offshore subpopulation automatically decreases, since the inshore area is the breeding area of the species.

8. Conclusion

In this paper, it has been studied that although the inshore area is the breeding place of the species, it is possible to allow the fishermen to harvest in that area also. But a higher tax is to be imposed for harvesting in inshore area compared to the tax for harvesting in the offshore area in order to control the over exploitation. Ray and Pradhan [11] considered the inshore area (being the breeding area) as the

restricted area where fishing is strictly prohibited. This paper is an extension of the work of Ray and Pradhan [11] without any restriction in inshore fishing. Only the non-trivial steady state is determined and its stability criterion is discussed here since the controlling agencies are uninterested in the existence of trivial or axial equilibrium points. Though the optimal equilibrium levels of two subpopulations could not be found out analytically, but their values can be found numerically.

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