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Regulation of Combined Harvesting of an Inshore-Offshore Fishery by Taxation

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Abstract: ***This paper deals with the combined harvesting of an inshore-offshore fishery with taxation as a control instrument. Here fishing is permitted in both the inshore and offshore areas though the inshore area is the breeding place for the species. But in order to control the overexploitation of the species the regulatory agencies impose a higher tax for fishing in the inshore area compared to the tax for fishing in the offshore area. The fish subpopulation in inshore area obeys the logistic law of growth [1]. Only the non-trivial steady state is determined. The local and global stability of the non-trivial steady state are discussed. Taking the taxes as control variables, the optimal harvest policy is formulated and solved as a control problem. The results are illustrated with the help of a numerical example.***

Keywords: Inshore-offshore, local and global stability, optimal harvesting, Pontragin's maximum principle, taxation

1. Introduction

Before the twentieth century, nobody was involved in studying the problems related to the commercial marine fisheries. At that time there were no agencies or authorities to control exploitation of marine fisheries and as a result many marine fish species gradually declined due to overexploitation. For example, Antartic fin whales, Japanese sardines, California Sardines, North Sea herrings, etc., are very few in numbers in this century mainly due to overfishing. Scientists have been engaged in the study of such problems relating to exploitation of marine fisheries from the beginning of the twentieth century. Now-a-days many countries have their own agencies to monitor and regulate the overexploitation of the species.

Problems of law enforcement are intricately associated with the management of renewable resources. Various aspects of law enforcement in regulating fisheries have been discussed by Anderson and Lee [2], Sutinen and Anderson [3] and others. Taxation, license fees, lease of property rights, seasonal harvesting etc. are usually considered as possible governing instruments in fishery regulation. Among these, taxation is superior than the other control policies because of its flexibility. As described by Clark ([4], Art.4.6, p.116), "Economists are particularly attracted to taxation, partly because of its flexibility and partly because many of the advantages of a competitive economic system can be better maintained under taxation than under other regulatory methods."

A single species fishery model using taxation as a control measure was first discussed by Clark [4]. Chaudhuri and Johnson [5] extended that model using a catch-rate function which was more realistic than that in [4]. Ganguly and Chaudhuri [6] made a capital theoretic study of a single species fishery with taxation as control policy. Pradhan and Chaudhuri [7] developed a mathematical model for growth and exploitation of a schooling fish species, using a realistic catch-rate function and imposing a tax per unit biomass of landed fish to control harvesting. Pradhan and Chaudhuri [8] also studied a fully dynamic reaction model of fishery consisting two competing fish species with taxation as a control instrument. Pradhan and Chaudhuri [9] proposed a model to study the selective harvesting in an inshore-offshore fishery on the basis of CPUE (catch per unit effort) hypothesis [4]. Srinivas et. al [10] developed a prey-predator model with stage structure in two patchy marine aquatic environment. They studied the dynamic of a fishery resource system with stage structure in an aquatic environment that consists of two zones – unreserved zone where fishing is permitted and the reserved zone where fishing is prohibited.

This paper is an extension of the work of Ray and Pradhan [11]. They developed a dynamic reaction model of an inshore-offshore fishery with taxation as a control instrument. In their work, inshore area was considered as breeding place having a fixed carrying capacity where fishing was strictly prohibited. Fishing was permitted only in the offshore area after imposing a tax per unit harvested biomass by the regulatory agencies. But in reality it is very difficult to prevent the fishing in inshore area totally, because the low cost of fishing in inshore area attracted all the fishermen to fish in that area. Moreover, if the inshore fishing is totally stopped, then the revenue earned by the Government from fishery will decrease. Again, since inshore area is the breeding place of the species, so excess harvest from that area may cause overexploitation of the species. In such circumstances the regulatory agencies will have to impose a higher tax for fishing in inshore area than the tax for fishing in offshore area, in order to control overexploitation.

In this paper inshore area is considered as the breeding place, but fishing in inshore area is allowed by the regulatory agencies only after imposing a higher tax per unit harvested biomass from inshore area compared to the tax per unit harvested biomass from offshore area. The inshore area has a fixed carrying capacity where the fish sub population obeys the logistic law of growth [1]. Only the existence of non-trivial steady state is discussed. The local and global stability of the non-trivial steady state are also discussed. The optimal harvest policy is studied using Pontragin's maximum principle [12]. A numerical example is given as an illustration.

2. The Mathematical Model

Let at any time t, $x_i(t)$ $(i = 1,2)$ be the population density of a fish species in inshore area and offshore area respectively. Let the fish subpopulation of the inshore area migrate into offshore area at a rate σ_1 $(0 < \sigma_1 < 1)$ and the fish subpopulation of the offshore area migrate into inshore area at a rate σ_2 $(0 < \sigma_2 < 1)$. Also we assume that the fish subpopulation of the inshore area obeys the logistic law of growth [1]. Here we consider that fishing is allowed in both the areas after imposing different taxes for fishing in different areas by the regulatory agencies. Let $E_i(t)$ $(i = 1,2)$ be the efforts for harvesting in inshore and offshore areas respectively and the regulatory agencies impose the taxes $\tau_i(> 0)$ $(i = 1,2)$ per unit biomass of the harvested fish from the inshore and offshore area respectively.

Since the harvest cost in inshore area is lower than that of offshore area, so all the fishermen would like to harvest in inshore area. Moreover, the inshore area being the breeding area for the species, the regulatory agencies always want to restrict the harvesting in inshore area in order to control the over exploitation. So the regulatory agencies impose a higher tax for harvesting in inshore area than the tax for harvesting in offshore area. So throughout the paper it is consider that $\tau_1 > \tau_2$.

The net economic revenue to the fishermen (*perceived rent*) is $\{q_i(p - \tau_i)x_i - c_i\}E_i$ $(i = 1,2)$ for harvesting in inshore and offshore area respectively, where p is the market price per unit biomass of the harvested fish, q_i $(i = 1,2)$ are the catchability coefficients of inshore and offshore subpopulation respectively, c_i $(i = 1,2)$ are the cost per unit effort for harvesting in inshore and offshore area respectively $(c_1 < c_2)$ and $\tau_i(> 0)$ $(i = 1,2)$ are the taxes per unit biomass of the harvested fish from inshore and offshore area respectively, imposed by the regulatory agencies.

Here we consider $E_i(t)$ $(i = 1,2)$ as the dynamic (time dependent) variables governed by the differential equations $\frac{dE_i}{dt} = \lambda_i\{q_i(p - \tau_i)x_i - c_i\}E_i$, $i = 1,2$, where $\lambda_i (i = 1,2)$ are the stiffness parameters measuring the effort and the perceived rent for the inshore and offshore fishery respectively.

Thus we have the following system of differential equations:

$$\left.\begin{aligned} \frac{dx_1}{dt} &= rx_1\left(1 - \frac{x_1}{k}\right) - \sigma_1 x_1 + \sigma_2 x_2 - q_1 E_1 x_1 \\ \frac{dx_2}{dt} &= -sx_2 + \sigma_1 x_1 - \sigma_2 x_2 - q_2 E_2 x_2 \\ \frac{dE_1}{dt} &= \lambda_1\{q_1(p - \tau_1)x_1 - c_1\}E_1 \\ \frac{dE_2}{dt} &= \lambda_2\{q_2(p - \tau_2)x_2 - c_2\}E_2 \end{aligned}\right\} \quad (1)$$

where $r, k, \sigma_1, \sigma_2, q_1, s, q_2, \tau_1, \tau_2, \lambda_1, \lambda_2, c_1, c_2$ are all positive constants.

Here r = intrinsic growth rate of the inshore subpopulation,
k = carrying capacity of the inshore area,
σ_1 = diffusion coefficient of inshore subpopulation,
σ_2 = diffusion coefficient of offshore subpopulation,
q_1 = catchability coefficient of inshore subpopulation,
q_2 = catchability coefficient of offshore subpopulation,
s = natural mortality rate of the offshore subpopulation,
λ_1 = stiffness parameter for inshore fishery,
λ_2 = stiffness parameter for offshore fishery,
τ_1 = tax per unit biomass of harvested fish from inshore area,
τ_2 = tax per unit biomass of harvested fish from offshore area,
c_1 = cost per unit effort of harvesting in inshore area,
c_2 = cost per unit effort of harvesting in offshore area.

3. The Steady States

Since the regulatory agencies are only interested in the nontrivial steady states, so we now find out only the nontrivial steady states. Here $P(x_1^*, x_2^*, E_1^*, E_2^*)$ is the only nontrivial steady state of the system of equations (1) where

$$x_1^* = \frac{c_1}{q_1(p-\tau_1)}, \quad (2)$$

$$x_2^* = \frac{c_2}{q_2(p-\tau_2)}, \quad (3)$$

$$E_1^* = \frac{1}{q_1}\left[r\left\{1 - \frac{c_1}{kq_1(p-\tau_1)}\right\} - \sigma_1 + \frac{\sigma_2 c_2 q_1 (p-\tau_1)}{c_1 q_2 (p-\tau_2)}\right] \quad (4)$$

and $E_2^* = \frac{1}{q_2}\left\{\frac{\sigma_1 c_1 q_2 (p-\tau_2)}{c_2 q_1 (p-\tau_1)} - (s + \sigma_2)\right\}$. (5)

Now, $\tau_1 > \tau_2$ implies $0 < \frac{p-\tau_1}{p-\tau_2} < 1$,
since $p > max\ (\tau_1, \tau_2) = \tau_1$.

We assume that $r > \sigma_1$ i.e. the growth rate is greater than the diffusion rate of inshore species. Otherwise, the inshore subpopulation gradually declines, which is discussed by Ray and Pradhan [11]. One of the sufficient condition for existence of $E_1^*(> 0)$ is $r - \sigma_1 > \frac{rc_1}{kq_1(p-\tau_1)}$, by (4).

i.e. $0 < \tau_1 < p - \frac{rc_1}{kq_1(r-\sigma_1)}$ (6)

Again, $E_2^* > 0$ iff $\frac{p-\tau_1}{p-\tau_2} < \frac{\sigma_1 c_1 q_2}{c_2 q_1 (s+\sigma_2)}$ by (5).

Therefore, the necessary and sufficient condition for existence of $E_2^*(> 0)$ is

$0 < \frac{p-\tau_1}{p-\tau_2} < min\left(1, \frac{\sigma_1 c_1 q_2}{c_2 q_1 (s+\sigma_2)}\right)$.

Let $m = min\left(1, \frac{\sigma_1 c_1 q_2}{c_2 q_1 (s+\sigma_2)}\right)$ (7)

Therefore, $0 < m \le 1$.

Here m is defined as the tax determination parameter for the regulatory agencies.

If $m = 1$, then $0 < \tau_2 < \tau_1 < p - \frac{rc_1}{kq_1(r-\sigma_1)}$ (8)

If $0 < m < 1$, then $\frac{p-\tau_1}{p-\tau_2} < m$,

i.e. $\tau_1 - m\tau_2 > (1 - m)p$ or, $\frac{\tau_1}{(1-m)p} + \frac{\tau_2}{\left(1-\frac{1}{m}\right)p} > 1$ (9)

If $\tau_1 < (1 - m)p$, then $\tau_2 < 0$.
i.e. if the regulatory agencies impose the tax τ_1 for inshore fishing below a certain level which is $(1 - m)p$, then the agencies have to pay the subsidy (negative tax) to the fishermen for offshore fishing in order to maintain the equilibrium level of the inshore and offshore species. But in the real situation no agencies want to pay the subsidy to the fishermen for fishing. So the agencies determine the tax τ_1 is such a way that $\tau_2 > 0$.
Therefore, $(1 - m)p$ is the minimum value of τ_1 for $\tau_2 > 0$.

i.e. $(1-m)p < \tau_1 < p - \frac{rc_1}{kq_1(r-\sigma_1)}$ (10)

Again, if $0 < m < 1$, then $\frac{p-\tau_1}{p-\tau_2} < m$ implies

$\tau_2 < \frac{\tau_1}{m} + \left(1 - \frac{1}{m}\right)p = n$ (say) (11)

But $\tau_2 < \tau_1$ implies $0 < \tau_2 < min(\tau_1, n)$ (12)

Therefore, for existence of the interior equilibrium $P(x_1^*, x_2^*, E_1^*, E_2^*)$ the regulatory agencies will have to determine the taxes τ_1 and τ_2 as follows:

If $m = 1$, then the agencies will have to determine the taxes τ_1 and τ_2 satisfying the condition (8). But if $0 < m < 1$, then the agencies have to determine the tax τ_1 satisfying the condition (6) and after fixing the tax τ_1 then the agencies will have to determine the tax τ_2 satisfying the condition (12).

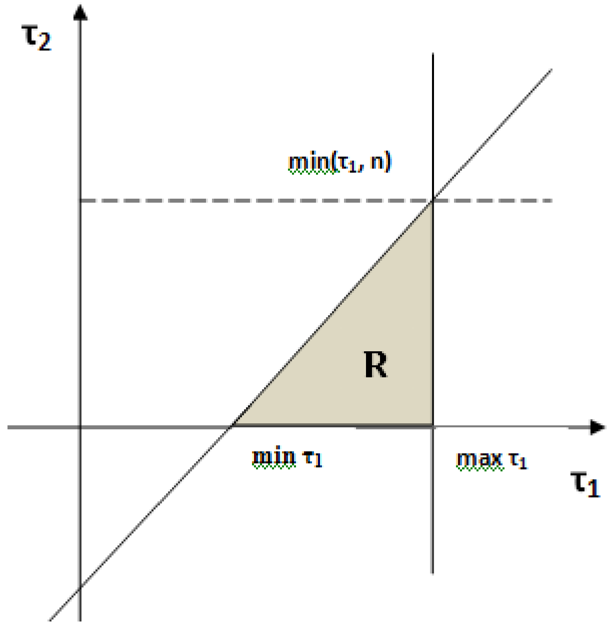

Figure 1: Feasible region (R) of the taxes τ_1 *and* τ_2

Here $R = \{(\tau_1, \tau_2): \tau_1 > 0,\ \tau_2 > 0,$ together with the conditions (10), (11) and (12) $\}$ is the bounded feasible region of the taxes.

4. Local Stability

The variation matrix of the system of equations (1) is $V = (M_{ij})_{4\times 4}$, where

$M_{11} = \frac{\partial}{\partial x_1}\left(\frac{dx_1}{dt}\right) = r - \frac{2rx_1}{k} - \sigma_1 - q_1E_1$,

$M_{12} = \frac{\partial}{\partial x_2}\left(\frac{dx_1}{dt}\right) = \sigma_2$, $M_{13} = \frac{\partial}{\partial E_1}\left(\frac{dx_1}{dt}\right) = -q_1x_1$,

$M_{14} = \frac{\partial}{\partial E_2}\left(\frac{dx_1}{dt}\right) = 0$,

$M_{21} = \frac{\partial}{\partial x_1}\left(\frac{dx_2}{dt}\right) = \sigma_1$, $M_{22} = \frac{\partial}{\partial x_2}\left(\frac{dx_2}{dt}\right) = -(s + \sigma_2 + q_2E_2)$, $M_{23} = \frac{\partial}{\partial E_1}\left(\frac{dx_2}{dt}\right) = 0$, $M_{24} = \frac{\partial}{\partial E_2}\left(\frac{dx_2}{dt}\right) = -q_2x_2$,

$M_{31} = \frac{\partial}{\partial x_1}\left(\frac{dE_1}{dt}\right) = \lambda_1 q_1(p-\tau_1)E_1$, $M_{32} = \frac{\partial}{\partial x_2}\left(\frac{dE_1}{dt}\right) = 0$,

$M_{33} = \frac{\partial}{\partial E_1}\left(\frac{dE_1}{dt}\right) = \lambda_1\{q_1(p-\tau_1)x_1 - c_1\}$,

$M_{34} = \frac{\partial}{\partial E_2}\left(\frac{dE_1}{dt}\right) = 0$,

$M_{41} = \frac{\partial}{\partial x_1}\left(\frac{dE_2}{dt}\right) = 0$, $M_{42} = \frac{\partial}{\partial x_2}\left(\frac{dE_2}{dt}\right) = \lambda_2 q_2 E_2(p-\tau_2)$

$M_{43} = \frac{\partial}{\partial E_1}\left(\frac{dE_2}{dt}\right) = 0$ and

$M_{44} = \frac{\partial}{\partial E_2}\left(\frac{dE_2}{dt}\right) = \lambda_2\{q_2(p-\tau_2)x_2 - c_2\}$.

The variation matrix at the nontrivial steady state $P(x_1^*, x_2^*, E_1^*, E_2^*)$ is

$$V(x_1^*, x_2^*, E_1^*, E_2^*) = \begin{pmatrix} -A & \sigma_2 & -q_1x_1^* & 0 \\ \sigma_1 & -B & 0 & -q_2x_2^* \\ C & 0 & 0 & 0 \\ 0 & D & 0 & 0 \end{pmatrix}$$

Where $A = q_1E_1^* + \sigma_1 + \frac{2rx_1^*}{k} - r = \frac{rx_1^*}{k} + \frac{\sigma_2 x_2^*}{x_1^*} > 0$, by (2) and (4), $B = s + \sigma_2 + q_2E_2^* = \frac{\sigma_1 x_1^*}{x_2^*} > 0$ by (2) and (3),

$C = \lambda_1 q_1(p-\tau_1)E_1^* > 0$ and

$D = \lambda_2 q_2(p-\tau_2)E_2^* > 0$.

The characteristic equation of the matrix $V(x_1^*, x_2^*, E_1^*, E_2^*)$ is $\det(V(x_1^*, x_2^*, E_1^*, E_2^*) - \mu I_4) = 0$ which implies

$\mu^4 + (A+B)\mu^3 + (Dq_2x_2^* + Cq_1x_1^* + AB - \sigma_1\sigma_2)\mu^2 + (BCq_1x_1^* + ADq_2x_2^*)\mu + CDq_1q_2x_1^*x_2^* = 0.$

or, $\mu^4 + a_3\mu^3 + a_2\mu^2 + a_1\mu + a_0 = 0$

where $a_3 = A + B > 0$, since $A > 0, B > 0$,

$a_2 = Dq_2x_2^* + Cq_1x_1^* + AB - \sigma_1\sigma_2$

$= Dq_2x_2^* + Cq_1x_1^* + \left(\frac{rx_1^*}{k} + \frac{\sigma_2x_2^*}{x_1^*}\right)\frac{\sigma_1x_1^*}{x_2^*} - \sigma_1\sigma_2$

$= Dq_2x_2^* + Cq_1x_1^* + \frac{r\sigma_1(x_1^*)^2}{kx_2^*} > 0,$

$a_1 = BCq_1x_1^* + ADq_2x_2^* > 0$ and $a_0 = CDq_1q_2x_1^*x_2^* > 0.$

Now, $a_3a_2 - a_1 = (A+B)\left\{Dq_2x_2^* + Cq_1x_1^* + \frac{r\sigma_1(x_1^*)^2}{kx_2^*}\right\}$

$- (BCq_1x_1^* + ADq_2x_2^*)$

$= (A+B)\frac{r\sigma_1(x_1^*)^2}{kx_2^*} + ACq_1x_1^* + BDq_2x_2^* > 0$

and $a_3a_2a_1 - a_1^2 - a_3^2a_0$

$= (A+B)(Dq_2x_2^* + Cq_1x_1^* + R)(BCq_1x_1^* + ADq_2x_2^*)$

$-(BCq_1x_1^* + ADq_2x_2^*)^2 - (A+B)^2CDq_1q_2x_1^*x_2^*$

where $R = \frac{r\sigma_1(x_1^*)^2}{kx_2^*} > 0$

$= ABC^2q_1^2(x_1^*)^2 + ARBCq_1x_1^* + A^2DRq_2x_2^* + RB^2Cq_1x_1^* + ABD^2q_2^2(x_2^*)^2 + ABDRq_2x_2^* - 2ABCDq_1q_2x_1^*x_2^*$

$= AB(Cq_1x_1^* - Dq_2x_2^*)^2 + RBC(A+B)q_1x_1^* + ADR(A+B)q_2x_2^* > 0$, since all the parameters are real and positive.

Therefore, the characteristic equation of the variational matrix $V(x_1^*, x_2^*, E_1^*, E_2^*)$ is $\mu^4 + a_3\mu^3 + a_2\mu^2 + a_1\mu + a_0 = 0$ such that $a_n > 0 \quad \forall\, n = 0,1,2,3$, $a_3a_2 - a_1 > 0$ and $a_3a_2a_1 - a_1^2 - a_3^2a_0 > 0$.

Therefore, by Routh-Hurwitz criterion [13], the nontrivial steady state $P(x_1^*, x_2^*, E_1^*, E_2^*)$ is locally asymptotically stable. So, whenever the nontrivial steady state exists for the exploited system (1), it is always locally asymptotically stable steady state.

5. Global stability

Now, we prove whether the nontrivial steady state $P(x_1^*, x_2^*, E_1^*, E_2^*)$ is globally asymptotically stable or not. For this let us consider the following Lypunov function [14]:

$L(x_1, x_2, E_1, E_2) = \left\{x_1 - x_1^* - x_1^* ln\left(\frac{x_1}{x_1^*}\right)\right\}$

$$+L_1\left\{x_2-x_2^*-x_2^*ln\left(\frac{x_2}{x_2^*}\right)\right\}+L_2\left\{E_1-E_1^*-E_1^*ln\left(\frac{E_1}{E_1^*}\right)\right\}$$
$$+L_3\left\{E_2-E_2^*-E_2^*ln\left(\frac{E_2}{E_2^*}\right)\right\},$$

where L_1, L_2, L_3 are positive constants to be determined.
Therefore,

$$\frac{dL}{dt}=\frac{x_1-x_1^*}{x_1}\frac{dx_1}{dt}+L_1\frac{x_2-x_2^*}{x_2}\frac{dx_2}{dt}+L_2\frac{E_1-E_1^*}{E_1}\frac{dE_1}{dt}+L_3\frac{E_2-E_2^*}{E_2}\frac{dx_1}{dt}$$
$$=(x_1-x_1^*)\left\{r-\frac{rx_1}{k}-\sigma_1+\frac{\sigma_2x_2}{x_1}-q_1E_1\right\}$$
$$+L_1(x_2-x_2^*)\left\{-s+\frac{\sigma_1x_1}{x_2}-\sigma_2-q_2E_2\right\}$$
$$+L_2(E_1-E_1^*)\lambda_1\{q_1(p-\tau_1)x_1-c_1\}$$
$$+L_3(E_2-E_2^*)\lambda_2\{q_2(p-\tau_2)x_2-c_2\} \text{ by (1)}$$
$$=(x_1-x_1^*)\left\{-\frac{r}{k}(x_1-x_1^*)+\sigma_2\left(\frac{x_2}{x_1}-\frac{x_2^*}{x_1^*}\right)-\right.$$
$q1E1-E1*$

$$+L_1(x_2-x_2^*)\left\{\sigma_1\left(\frac{x_1}{x_2}-\frac{x_1^*}{x_2^*}\right)-q_2(E_2-E_2^*)\right\}$$
$$+L_2(E_1-E_1^*)\lambda_1q_1(p-\tau_1)(x_1-x_1^*)$$
$$+L_3(E_2-E_2^*)\lambda_2q_2(p-\tau_2)(x_2-x_2^*)$$
$$=-\frac{r}{k}(x_1-x_1^*)^2+\frac{\sigma_2}{x_1x_1^*}(x_1-x_1^*)(x_2x_1^*-x_1x_2^*)$$
$$-q_1(x_1-x_1^*)(E_1-E_1^*)+\frac{L_1\sigma_1}{x_2x_2^*}(x_2-x_2^*)(x_1x_2^*-x_2x_1^*)$$
$$-L_1q_2(x_2-x_2^*)(E_2-E_2^*)$$
$$+L_2\lambda_1q_1(p-\tau_1)(x_1-x_1^*)(E_1-E_1^*)$$
$$+L_3\lambda_2q_2(p-\tau_2)(x_2-x_2^*)(E_2-E_2^*).$$

Let $L_1=\frac{\sigma_2x_2^*}{\sigma_1x_1^*}>0$, $L_2=\frac{1}{\lambda_1(p-\tau_1)}>0$ and
$L_3=\frac{\sigma_2x_2^*}{(p-\tau_2)\lambda_2\sigma_1x_1^*}>0.$
Therefore,

$$\frac{dL}{dt}=-\frac{r}{k}(x_1-x_1^*)^2+\frac{\sigma_2}{x_1x_1^*}(x_1-x_1^*)(x_2x_1^*-x_1x_2^*)$$
$$+\frac{\sigma_2}{x_2x_1^*}(x_2-x_2^*)(x_1x_2^*-x_2x_1^*)$$
$$=-\frac{r}{k}(x_1-x_1^*)^2+\frac{\sigma_2}{x_1^*}\left\{(x_2x_1^*-x_1x_2^*)\left(\frac{x_1-x_1^*}{x_1}-\right.\right.$$
$x2-x2*x2$

$$=-\frac{r}{k}(x_1-x_1^*)^2-\frac{\sigma_2}{x_1x_2x_1^*}(x_2x_1^*-x_1x_2^*)^2<0.$$

Therefore, $\frac{dL}{dt}<0\ \forall(x_1,x_2,E_1,E_2)\in E^4$ where
$E^4=R_4^+\backslash\{(x_1^*,x_2^*,E_1^*,E_2^*)\}$ and $\frac{dL}{dt}=0$ at $P(x_1^*,x_2^*,E_1^*,E_2^*)$.
This shows that $\frac{dL}{dt}$ is negative definite in the region E^4 and hence the nontrivial steady state $P(x_1^*,x_2^*,E_1^*,E_2^*)$ is globally asymptotically stable in the Lyapunov sense [14].

6. Optimal Harvest Policy

In this section an optimal harvesting policy is determined to maximize the total discounted net revenue from the harvesting biomass using taxes as a control parameters.
The objective of the regulatory agencies is to maximize $J=\int_0^\infty \pi(x_1,x_2,E_1,E_2,t)e^{-\delta t}\,dt$ where δ denotes the instantaneous annual rate of discount and $\pi(x_1,x_2,E_1,E_2,t)$ is the net revenue i.e. $\pi(x_1,x_2,E_1,E_2,t)=$ net revenue of the fishermen + net revenue of the regulatory agencies

$$=(p-\tau_1)q_1E_1x_1-c_1E_1+(p-\tau_2)q_2E_2x_2-c_2E_2$$
$$+\tau_1q_1E_1x_1+\tau_2q_2E_2x_2$$
$$=(pq_1x_1-c_1)E_1+(pq_2x_2-c_2)E_2.$$

Here the objective of the regulatory agencies is to determine the optimal values of the taxes τ_1 and τ_2 in order to maximize J subject to the state equations

$$\frac{dx_1}{dt}=\frac{dx_2}{dt}=\frac{dE_1}{dt}=\frac{dE_2}{dt}=0 \quad (13)$$

and the constrains are $\tau_{1(min)}<\tau_1<\tau_{1(max)}$ and $\tau_{2(min)}<\tau_2<\tau_{2(max)}$.
Negative taxes mean that the subsidy is given by the regulatory agencies to the loser fishermen. But in reality, it is assume that the agencies are not in position to give such subsidy. So we assume that $0<\tau_1<\tau_{1(max)}$
and $0<\tau_2<\tau_{2(max)}$ (14)
Here Pontryagin Maximum Principle [12] is applied to obtain the optimal equilibrium solution.
The Hamiltonian of this control problem is

$$H=e^{-\delta t}\{(pq_1x_1-c_1)E_1+(pq_2x_2-c_2)E_2\}+$$
$$\mu_1(t)\left\{rx_1\left(1-\frac{x_1}{k}\right)-\sigma_1x_1+\sigma_2x_2-q_1E_1x_1\right\}$$
$$+\mu_2(t)\{-sx_2+\sigma_1x_1-\sigma_2x_2-q_2E_2x_2\}$$
$$+\mu_3(t)[\lambda_1\{q_1(p-\tau_1)x_1-c_1\}E_1]$$
$$+\mu_4(t)[\lambda_2\{q_2(p-\tau_2)x_2-c_2\}E_2] \quad (15)$$

where $\mu_i(t)(i=1,2,3,4)$are adjoint variables.
Since H is the linear function of τ_1 and τ_2, the conditions that the Hamiltonian H be maximum for τ_1 and τ_2 satisfying the conditions (13) are $\frac{\partial H}{\partial\tau_1}=\frac{\partial H}{\partial\tau_2}=0$.
This implies $\mu_3(t)=\mu_4(t)=0$ (16)
The adjoint equations are

$$\frac{d\mu_1}{dt}=-\frac{\partial H}{\partial x_1},\ \frac{d\mu_2}{dt}=-\frac{\partial H}{\partial x_2},\ \frac{d\mu_3}{dt}=-\frac{\partial H}{\partial E_1},\ \frac{d\mu_4}{dt}=-\frac{\partial H}{\partial E_2}$$

Therefore,

$$\frac{d\mu_1}{dt}=-\left\{e^{-\delta t}pq_1E_1+\mu_1\left(r-\frac{2rx_1}{k}-\sigma_1-q_1E_1\right)+\mu_2\sigma_1\right.$$
$$\left.+\mu_3\lambda_1q_1(p-\tau_1)\right\}$$
$$=-e^{-\delta t}pq_1E_1-\mu_1\left(r-\frac{2rx_1}{k}-\sigma_1-q_1E_1\right)-\mu_2\sigma_1$$

(17)
Similarly,

$$\frac{d\mu_2}{dt}=-e^{-\delta t}pq_2E_2-\mu_1\sigma_2+\mu_2(s+\sigma_2+q_2E_2) \quad (18)$$
$$\frac{d\mu_3}{dt}=-e^{-\delta t}(pq_1x_1-c_1)+\mu_1q_1x_1 \quad (19)$$
$$\frac{d\mu_4}{dt}=-e^{-\delta t}(pq_2x_2-c_2)+\mu_2q_2x_2 \quad (20)$$

Now, from (16), (19) and (20), we have

$$\mu_1(t)=e^{-\delta t}\left(p-\frac{c_1}{q_1x_1}\right) \quad (21)$$

and $\mu_2(t)=e^{-\delta t}\left(p-\frac{c_2}{q_2x_2}\right)$ (22)
From (21) and (22), using the state equations $\frac{dx_1}{dt}=\frac{dx_2}{dt}=0$, we have

$$\frac{d\mu_1}{dt}=-\delta e^{-\delta t}\left(p-\frac{c_1}{q_1x_1}\right)$$

(23)
and $\frac{d\mu_2}{dt}=-\delta e^{-\delta t}\left(p-\frac{c_2}{q_2x_2}\right)$

(24)
Now, from (17) and (23), we have

$$-\delta e^{-\delta t}\left(p-\frac{c_1}{q_1x_1}\right)=-e^{-\delta t}pq_1E_1-$$
$$e^{-\delta t}\left(p-\frac{c_1}{q_1x_1}\right)\left(r-\frac{2rx_1}{k}-\sigma_1-q_1E_1\right)$$
$$-e^{-\delta t}\left(p-\frac{c_2}{q_2x_2}\right)\sigma_1 \text{ by (21) and (22)}$$

or,$\delta\left(p-\frac{c_1}{q_1x_1}\right)=pq_1E_1+\left(p-\frac{c_1}{q_1x_1}\right)\left(r-\frac{2rx_1}{k}-\sigma_1-\right.$
$q1E1$

$$+\left(p-\frac{c_2}{q_2x_2}\right)\sigma_1$$
$$=p\left\{r\left(1-\frac{x_1}{k}\right)-\sigma_1+\frac{\sigma_2x_2}{x_1}\right\}+\left(p-\frac{c_1}{q_1x_1}\right)\left(-\frac{\sigma_2x_2}{x_1}-\frac{rx_1}{k}\right)+\left(p-\frac{c_2}{q_2x_2}\right)\sigma_1$$
$$=pr-\frac{2prx_1}{k}+\frac{c_1\sigma_2x_2}{q_1x_1^2}+\frac{c_1r}{q_1k}-\frac{c_2\sigma_1}{q_2x_2}$$
$$\text{or, }\frac{c_1\sigma_2x_2}{q_1x_1^2}-\frac{2prx_1}{k}-\frac{c_2\sigma_1}{q_2x_2}+\frac{\delta c_1}{q_1x_1}+\frac{c_1r}{q_1k}+p(r-\delta)=0 \tag{25}$$

Again, from (18) and (24), we have

$$-\delta e^{-\delta t}\left(p-\frac{c_2}{q_2x_2}\right)=-e^{-\delta t}pq_2E_2-e^{-\delta t}\left(p-\frac{c_1}{q_1x_1}\right)\sigma_2+e^{-\delta t}\left(p-\frac{c_2}{q_2x_2}\right)(s+\sigma_2+q_2E_2)$$
$$\text{or, }\delta\left(p-\frac{c_2}{q_2x_2}\right)=p\left(-s-\sigma_2+\frac{\sigma_1x_1}{x_2}\right)+\left(p-\frac{c_1}{q_1x_1}\right)\sigma_2-\frac{\sigma_1x_1}{x_2}\left(p-\frac{c_2}{q_2x_2}\right)$$
$$=-p(s+\sigma_2)+\left(p-\frac{c_1}{q_1x_1}\right)\sigma_2+\frac{c_2\sigma_1x_1}{q_2x_2^2}$$
$$\text{or, }\frac{c_2\sigma_1x_1}{q_2x_2^2}+\frac{\delta c_2}{q_2x_2}-\frac{c_1}{q_1x_1}-p(\delta+s)=0 \tag{26}$$

Solving the above non linear equations (25) and (26), we have the optimal equilibrium level of two subpopulations, $x_{1\delta}$ and $x_{2\delta}$ in inshore and offshore area respectively. Using these values in the state equations we have

$$E_{1\delta}=\frac{1}{q_1}\left\{r\left(1-\frac{x_{1\delta}}{k}\right)-\sigma_1+\frac{\sigma_2x_{2\sigma}}{x_{1\delta}}\right\} \tag{27}$$
$$E_{2\delta}=\frac{1}{q_2}\left(-s-\sigma_2+\frac{\sigma_2x_{2\sigma}}{x_{1\delta}}\right) \tag{28}$$
$$\tau_{1\delta}=p-\frac{c_1}{q_1x_{1\delta}} \tag{29}$$
$$\tau_{2\delta}=p-\frac{c_2}{q_2x_{2\delta}} \tag{30}$$

7. Numerical Example

Let $r=5, k=1000, \sigma_1=0.7, \sigma_2=0.3, s=0.2, q_1=0.02, q_2=0.01, \lambda_1=1, \lambda_2=1, p=10, c_1=50, c_2=60$ and $\delta=0.4$.

Therefore, $p-\frac{rc_1}{kq_1(r-\sigma_1)}=7.093$,

$$m=min\left\{1,\frac{\sigma_1c_1q_2}{c_2q_1(s+\sigma_2)}\right\}=0.583$$

and $(1-m)p=4.17$.

Since $m\in(0,1)$, for existence of the non-trivial steady state $P(x_1^*,x_2^*,E_1^*,E_2^*)$ the regulatory agencies have to determine the tax τ_1 per unit harvested biomass from the inshore fishery such that $4.17<\tau_1<7.093$, by (8).

Suppose the agencies choose $\tau_1=7$.

Then $n=\frac{\tau_1}{m}+\left(1-\frac{1}{m}\right)p=4.86$ and $min(n,\tau_1)=4.86$.

So the regulatory agencies choose the tax τ_2 such that $\tau_2<4.86$, by (12), for existence of the non-trivial steady state $P(x_1^*,x_2^*,E_1^*,E_2^*)$.

But the agencies are always interested for sufficient harvesting in the offshore area whereas the fishermen are interested in fishing in inshore area. In such a situation the agencies would like to impose the tax τ_2 moderately low compared to its applicable maximum level. Keeping in mind for such a situation to arise, suppose the agencies choose the tax $\tau_2=3$.

Considering the above parameter values together with $\tau_1=7$ and $\tau_2=3$, the non-trivial steady state becomes $P(833.33, 857.14, 22.10, 18.06)$ and this steady state is locally as well as globally asymptotically stable.

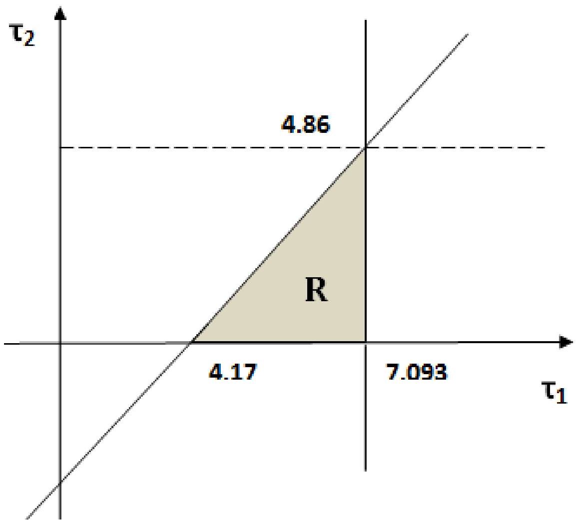

Figure 2: Feasible region (R) of the taxes τ_1 *and* τ_2 for $min\,\tau_1=4.17$, $max\,\tau_1=7.093$ and $max\,\tau_2=4.86$

Figure 2 shows the feasible region of the taxes τ_1 and τ_2 for existence of the non-trivial steady state which is always locally and globally asymptotically stable.

Using these parameter values equations (25) and (26) become

$750x_1^{-2}x_2-0.1x_1-4200x_2^{-1}+1000x_1^{-1}+58.5=0$

and $4200x_1x_2^{-1}+2400x_2^{-1}-750x_1^{-1}-6=0$,

respectively.

Solving the above non-linear equations (using *Mathematica* software), we have the optimal equilibrium level of inshore and offshore subpopulations as $x_{1\delta}=564.90$ and $x_{2\delta}=755.88$ respectively. For these optimal values of populations, the optimal level of efforts and taxes are $E_{1\delta}=93.85, E_{2\delta}=2.31, \tau_{1\delta}=5.57$ and $\tau_{2\delta}=2.06$, obtained from (27), (28), (29) and (30) respectively.

Thus $P_\delta(564.90, 755.88, 93.85, 2.31)$ is the optimal equilibrium solution of the system (1) corresponding to the above parameter values and the optimal taxes are $\tau_{1\delta}=5.57$ and $\tau_{2\delta}=2.06$.

Comparing this optimal equilibrium solution with the biological equilibrium solution, we see that in view of economic consideration, the inshore fishing is more attractive than the offshore fishing and so the equilibrium level of inshore subpopulation decreases. Whenever the inshore subpopulation decreases then the offshore subpopulation automatically decreases, since the inshore area is the breeding area of the species.

8. Conclusion

In this paper, it has been studied that although the inshore area is the breeding place of the species, it is possible to allow the fishermen to harvest in that area also. But a higher tax is to be imposed for harvesting in inshore area compared to the tax for harvesting in the offshore area in order to control the over exploitation. Ray and Pradhan [11] considered the inshore area (being the breeding area) as the restricted area where fishing is strictly prohibited. This paper is an extension of the work of Ray and Pradhan [11] without any restriction in inshore fishing. Only the non-trivial steady state is determined and its stability criterion is discussed here since the controlling agencies are uninterested in the existence of trivial or axial equilibrium points. Though the optimal equilibrium levels of two subpopulations could not be found out analytically, but their values can be found numerically.

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