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Independent of Two Grade System in Manpower System

S. Parthasarathy[1], M. Chitra[2]

[1] Associate Professor, Department of Statistics,Annamalai University, Chidambaram,Tamilnadu,India.

[2] Associate Professor, Department of Mathematics,Thiruvalluvar University,Vellore, TamilNadu,India.

Abstract: ***In any organization the required staff strength is maintained through new recruitments. The exit of personnel from an organization is a common phenomenon, which is known as wastage. Many stochastic models dealing with wastage are found in Bartholomew and Forbes (1979). In this paper the model it is assumed that the threshold level of the breakdown cannot be extended by means of transfer of personnel from one grade to other. In short, it is assumed that transfer of personnel is not possible so that the two grades are independent and the system fails if the depletion in any one of the two grades crosses the so called threshold level individually. Hence the minimum of the thresholds of two independent systems becomes the overall threshold..***

Keywords: Manpower planning, CDP, wastage, grading system, reliability**.**

1. Introduction

Let us assume that the threshold level of grade Y_1 and thershold level of grade two is Y_2 in terms of manhours and also Y_1 and Y_2 are independent. Sathiyamoorthi and Parthasarathy (2002) assumed Y_1 and Y_2 are expressed in terms of total depletion of manhours in grade one and grade two put together and so the threshold level of the organization is maximum (Y_1, Y_2) and obtained the expected time to recruitment in the organization. Here, the assumption that the threshold levels (Y_1, Y_2) at the two grades respectively in terms of the total loss of manpower put together is dropped.

In this model it is assumed that the threshold level of the breakdown cannot be extended by means of transfer of personnel from one grade to other. In short, it is assumed that transfer of personnel is not possible so that the two grades are independent and the system fails if the depletion in any one of the two grades crosses the so called threshold level individually. Hence the minimum of the thresholds of two independent systems becomes the overall threshold.

It is very interesting to observe that in the previous model the breakdown of the system occurs if only both the subsystem fails that is, if the thresholds in both the segments are crossed by depletion. Hence, it is similar to a parallel system of reliability. In the present model the breakdown occurs if the threshold is crossed in any one of the grades, it amounts to a series system in reliability.

2. Assumptions

1) The depletion of manpower is linear and cumulative.
2) Exit of person from an organization takes place whenever the policy decisions regarding targets, incentives and promotions are made.
3) The depletion of manpower occurs to the two grades are independent of their threshold levels.
4) The inter arrival times between successive occasions of wastage are i.i.d. random variables.

The breakdown of their organization occurs if the cumulative damage crosses a particular level which is assumed to be a random variable

3. Notations

X_i', X_i'' : a continuous random variable denoting the amount of loss of manpower caused to grade one and grade two on the i[th] occasion of policy announcement, 1,2….k and X_i', X_i'' are i.i.d

Y_1, Y_2 : continuous random variable denoting the threshold levels for the two grades

U_i : a continuous random variable denoting the inter arrival times between decision epochs and $U_i \sim \exp(c)$

$g(.)$: The probability density function of X_i'

$g^*(.)$: Laplace transform of $g(.)$

$g_k(.)$: The k- fold convolution of $g(.)$, i.e., p.d.f. of $\sum_{i=1}^{k} X_i'$

$h(.)$: The probability density function of X_i''

$h^*(.)$: Laplace transform of $h(.)$

$h_k(.)$: The k- fold convolution of $h(.)$, i.e., p.d.f. of $\sum_{i=1}^{k} X_i''$

$K_1(\cdot)$: c.d.f of random variable Y_1.

$K_2(\cdot)$: c.d.f of random variable Y_2.

$f(.)$: p.d.f. of random variable denoting inter-arrival times between successive policy announcement with the corresponding c.d.f $F(.)$.

$F_k(.)$: k-fold convolution of $F(.)$.

$V_k(t)$: Probability of exactly k policy announcements

$S(.)$: Survival function

$L(t)$: $1 - S(t)$

4. Results

$\sum X_i'$ denote the cumulative amount of damage caused to the grade one due to K decision epochs.

$P\left[\sum X_i' < Y_1\right]$ is the probability that the cumulative damage in grade one does not cross its threshold level.

$\sum X_i''$ denote the cumulative amount of wastage caused to the grade two at K decision epochs.
$P[\sum X_i'' < Y_2]$ is the probability that the cumulative damage in grade two do not cross its threshold level.

If the cumulative damage in any one of the grades cross its threshold level, the transfer of personal is not possible. So the breakdown of the system occurs if either $P[\sum X_i' > Y_1]$ or $P[\sum X_i'' > Y_2]$. Thus recruitment of personal is to be made only if $P[\sum X_i' < Y_1]$ or $P[\sum X_i'' < Y_2]$.

The probability that the system survives even after the cumulative loss of manpower in the respective grades after K decision is given by

$$P\left[\sum_{i=1}^{k} X_i' < Y_1 \cap \sum_{i=1}^{k} X_i'' < Y_2\right]$$

$$P\left(\sum_{i=1}^{k} X_i' < Y_1\right) * P\left(\sum_{i=1}^{k} X_i'' < Y_2\right)$$

because of independence of X_i' and X_i''.

Therefore $S(t) = P[T > t]$ is the survival function which gives the probability that the system will fail only after time t.

$$S(t) = \sum_{k=0}^{\infty} V_k(t)\left[P\left(\sum_{i=1}^{k} X_i' < Y_1\right) * P\left(\sum_{i=1}^{k} X_i'' < Y_2\right)\right]$$

Let $K_1(\cdot) \sim$ Exponentiated exponential with parameter λ_1 and
$K_2(\cdot) \sim$ Exponentiated exponential with parameter λ_2

$$P\left(\sum_{i=1}^{k} X_i' < Y_1\right) = \int_0^{\infty} g_k(x)\ K_1(x)dx$$

$$= \int_0^{\infty} g_k(x)\ \left(1 - e^{-\lambda_1 x}\right)^2 dx$$

Similarly

$$P\left(\sum_{i=1}^{k} X_i'' < Y_2\right) = \int_0^{\infty} h_k(x)\ K_2(x)dx$$

$$= \int_0^{\infty} h_k(x)\ \left(1 - e^{-\lambda_2 x}\right)^2 dx$$

$$S(t) = [F_k(t) - F_{k+1}(t)]\{[2g_k^*(\lambda_1) - g_k^*(2\lambda_1)] * [2h_k^*(\lambda_2) - h_k^*(2\lambda_2)]\}$$

$$= \sum_{k=0}^{\infty} V_k(t)\left\{P\left(\sum_{i=1}^{k} X_i' < Y_1\right) * P\left(\sum_{i=1}^{k} X_i'' < Y_2\right)\right\}$$

$$= \sum_{k=0}^{\infty} V_k(t)\ \{[2g_k^*(\lambda_1) - g_k^*(2\lambda_1)] * [2h_k^*(\lambda_2) - h_k^*(2\lambda_2)]\}$$

$$= 4\sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\,[g^*(\lambda_1)h^*(\lambda_2)]^k - 2\sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\,[g^*(\lambda_1)h^*(2\lambda_2)]^k$$

$$-2\sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\,[g^*(2\lambda_1)h^*(\lambda_2)]^k + \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\,[g^*(2\lambda_1)h^*(2\lambda_2)]^k$$

$$S(t) = 4\{1 - [1 - g^*(\lambda_1)h^*(\lambda_2)]\}\sum_{k=1}^{\infty}[F_k(t)g^*(\lambda_1)h^*(\lambda_2)]^{k-1}$$
$$-2\{1 - [1 - g^*(\lambda_1)h^*(2\lambda_2)]\}\sum_{k=1}^{\infty}[F_k(t)g^*(\lambda_1)h^*(2\lambda_2)]^{k-1}$$
$$-2\{1 - [1 - g^*(2\lambda_1)h^*(\lambda_2)]\}\sum_{k=1}^{\infty}[F_k(t)g^*(2\lambda_1)h^*(\lambda_2)]^{k-1}$$
$$+\{1 - [1 - g^*(2\lambda_1)h^*(2\lambda_2)]\}\sum_{k=1}^{\infty}[F_k(t)g^*(2\lambda_1)h^*(2\lambda_2)]^{k-1}$$

$$L(t) = 1 - S(t)$$

Taking Laplace transform of $L(t)$, we get

$$L^*(s) = \frac{4[1 - g^*(\lambda_1)h^*(\lambda_2)]f^*(s)}{[1 - g^*(\lambda_1)h^*(\lambda_2)f^*(s)]} - \frac{2[1-g^*(\lambda_1)h^*(2\lambda_2)]f^*(s)}{[1-g^*(\lambda_1)h^*(2\lambda_2)f^*(s)]}$$
$$- \frac{2[1-g^*(2\lambda_1)h^*(\lambda_2)]f^*(s)}{[1-g^*(2\lambda_1)h^*(\lambda_2)f^*(s)]} + \frac{[1-g^*(2\lambda_1)h^*(2\lambda_2)]f^*(s)}{[1-g^*(2\lambda_1)h^*(2\lambda_2)f^*(s)]}$$

Let the random variable U denoting inter arrival time which follows exponential with parameter c. Now $f^*(s) = \left(\frac{c}{c+s}\right)$, substituting in the above equation (8), we get

$$= \frac{4[1 - g^*(\lambda_1)h^*(\lambda_2)]\frac{c}{c+s}}{\left[1 - g^*(\lambda_1)h^*(\lambda_2)\frac{c}{c+s}\right]} - \frac{2[1-g^*(\lambda_1)h^*(2\lambda_2)]\frac{c}{c+s}}{\left[1-g^*(\lambda_1)h^*(2\lambda_2)\frac{c}{c+s}\right]} - \frac{2[1-g^*(2\lambda_1)h^*(\lambda_2)]\frac{c}{c+s}}{\left[1-g^*(2\lambda_1)h^*(\lambda_2)\frac{c}{c+s}\right]}$$
$$+ \frac{[1 - g^*(2\lambda_1)h^*(2\lambda_2)]\frac{c}{c+s}}{\left[1 - g^*(2\lambda_1)h^*(2\lambda_2)\frac{c}{c+s}\right]}$$

$$L^*(s) = \frac{4c[1 - g^*(\lambda_1)h^*(\lambda_2)]}{[c + s - g^*(\lambda_1)h^*(\lambda_2)c]} - \frac{2c[1-g^*(\lambda_1)h^*(2\lambda_2)]}{[c+s-g^*(\lambda_1)h^*(2\lambda_2)c]} - \frac{2c[1-g^*(2\lambda_1)h^*(\lambda_2)]}{[c+s-g^*(2\lambda_1)h^*(\lambda_2)c]}$$
$$+ \frac{c[1-g^*(2\lambda_1)h^*(2\lambda_2)]}{[c+s-g^*(2\lambda_1)h^*(2\lambda_2)c]}$$

$$E(T) = -\frac{d}{ds}L^*(s) \quad \text{given} \quad s = 0$$

$$= \frac{4c[1-g^*(\lambda_1)h^*(\lambda_2)]}{c^2[1-g^*(\lambda_1)h^*(\lambda_2)]^2} - \frac{2c[1-g^*(\lambda_1)h^*(2\lambda_2)]}{c^2[1-g^*(\lambda_1)h^*(2\lambda_2)]^2} - \frac{2c[1-g^*(2\lambda_1)h^*(\lambda_2)]}{c^2[1-g^*(2\lambda_1)h^*(\lambda_2)]^2} + \frac{c[1-g^*(2\lambda_1)h^*(2\lambda_2)]}{c^2[1-g^*(2\lambda_1)h^*(2\lambda_2)]^2}$$

$$E(T) = \frac{4}{c[1 - g^*(\lambda_1)h^*(\lambda_2)]} - \frac{2}{c[1 - g^*(\lambda_1)h^*(2\lambda_2)]} - \frac{2}{c[1-g^*(2\lambda_1)h^*(\lambda_2)]} + \frac{1}{c[1-g^*(2\lambda_1)h^*(2\lambda_2)]}$$

on simplification

$$E(T^2) = \frac{d^2L^*(s)}{ds^2}$$

$$= \frac{4}{c^2[1-g^*(\lambda_1)h^*(\lambda_2)]^2} - \frac{2}{c^2[1-g^*(\lambda_1)h^*(2\lambda_2)]^2} - \frac{2}{c^2[1-g^*(2\lambda_1)h^*(\lambda_2)]^2} + \frac{1}{c^2[1-g^*(2\lambda_1)h^*(2\lambda_2)]^2}$$

From which $V(T)$ can be obtained.

$$V(T) = E(T^2) - [E(T)]^2$$

$$= \frac{4}{c^2[1-g^*(\lambda_1)h^*(\lambda_2)]^2} - \frac{2}{c^2[1-g^*(\lambda_1)h^*(2\lambda_2)]^2} - \frac{2}{c^2[1-g^*(2\lambda_1)h^*(\lambda_2)]^2} + \frac{1}{c^2[1-g^*(2\lambda_1)h^*(2\lambda_2)]^2}$$
$$-\left[\frac{4}{c[1-g^*(\lambda_1)h^*(\lambda_2)]} - \frac{2}{c[1-g^*(\lambda_1)h^*(2\lambda_2)]} - \frac{2}{c[1-g^*(2\lambda_1)h^*(\lambda_2)]} + \frac{1}{c[1-g^*(2\lambda_1)h^*(2\lambda_2)]}\right]^2$$

$$g(\cdot) \sim (\alpha_1);\ g(\lambda_1) = \frac{\alpha_1}{\alpha_1 + \lambda_1};$$

$$g(2\lambda_1) = \frac{\alpha_1}{\alpha_1 + 2\lambda_1};\ g(\cdot) \sim (\alpha_2); h(\lambda_2) = \frac{\alpha_2}{\alpha_2 + \lambda_2};$$

$$h(2\lambda_2) = \frac{\alpha_2}{\alpha_2 + 2\lambda_2}$$

$$E(T) = \frac{4}{c\left[1 - \frac{\alpha_1}{\alpha_1 + \lambda_1}\frac{\alpha_2}{\alpha_2 + \lambda_2}\right]} - \frac{2}{c\left[1 - \frac{\alpha_1}{\alpha_1 + \lambda_1}\frac{\alpha_2}{\alpha_2 + 2\lambda_2}\right]} - \frac{2}{c\left[1 - \frac{\alpha_1}{\alpha_1 + 2\lambda_1}\frac{\alpha_2}{\alpha_2 + \lambda_2}\right]} + \frac{1}{c\left[1 - \frac{\alpha_1}{\alpha_1 + 2\lambda_1}\frac{\alpha_2}{\alpha_2 + 2\lambda_2}\right]}$$

$$E(T) = \frac{1}{c}\left[\frac{4(\alpha_1+\lambda_1)(\alpha_2+\lambda_2)}{(\alpha_1+\lambda_1)(\alpha_2+\lambda_2)-\alpha_1\alpha_2} - \frac{2(\alpha_1+\lambda_1)(\alpha_2+2\lambda_2)}{(\alpha_1+\lambda_1)(\alpha_2+2\lambda_2)-\alpha_1\alpha_2}\right]$$
$$-\frac{1}{c}\left[\frac{2(\alpha_1+2\lambda_1)(\alpha_2+\lambda_2)}{(\alpha_1+2\lambda_1)(\alpha_2+\lambda_2)-\alpha_1\alpha_2} + \frac{(\alpha_1+2\lambda_1)(\alpha_2+2\lambda_2)}{(\alpha_1+2\lambda_1)(\alpha_2+2\lambda_2)-\alpha_1\alpha_2}\right]$$

$$V(T) = \frac{4}{c^2[1-g^*(\lambda_1)h^*(\lambda_2)]^2} - \frac{2}{c^2[1-g^*(\lambda_1)h^*(2\lambda_2)]^2} - \frac{2}{c^2[1-g^*(2\lambda_1)h^*(\lambda_2)]^2} + \frac{1}{c^2[1-g^*(2\lambda_1)h^*(2\lambda_2)]^2} - \left[\frac{4}{c[1-g^*(\lambda_1)h^*(\lambda_2)]} - \frac{2}{c[1-g^*(\lambda_1)h^*(2\lambda_2)]} - \frac{2}{c[1-g^*(2\lambda_1)h^*(\lambda_2)]} + \frac{1}{c[1-g^*(2\lambda_1)h^*(2\lambda_2)]}\right]^2$$

$$V(T) = \frac{2}{c^2}\left[\left[\frac{4(\alpha_1+\lambda_1)(\alpha_2+\lambda_2)}{(\alpha_1+\lambda_1)(\alpha_2+\lambda_2)-\alpha_1\alpha_2}\right]^2 - \left[\frac{2(\alpha_1+\lambda_1)(\alpha_2+2\lambda_2)}{(\alpha_1+\lambda_1)(\alpha_2+2\lambda_2)-\alpha_1\alpha_2}\right]^2 - \left[\frac{2(\alpha_1+2\lambda_1)(\alpha_2+\lambda_2)}{(\alpha_1+2\lambda_1)(\alpha_2+\lambda_2)-\alpha_1\alpha_2}\right]^2 + \left[\frac{(\alpha_1+2\lambda_1)(\alpha_2+2\lambda_2)}{(\alpha_1+2\lambda_1)(\alpha_2+2\lambda_2)-\alpha_1\alpha_2}\right]^2\right] - \left[\frac{1}{c}\left[\frac{4(\alpha_1+\lambda_1)(\alpha_2+\lambda_2)}{(\alpha_1+\lambda_1)(\alpha_2+\lambda_2)-\alpha_1\alpha_2} - \frac{2(\alpha_1+\lambda_1)(\alpha_2+2\lambda_2)}{(\alpha_1+\lambda_1)(\alpha_2+2\lambda_2)-\alpha_1\alpha_2} - \frac{2(\alpha_1+2\lambda_1)(\alpha_2+\lambda_2)}{(\alpha_1+2\lambda_1)(\alpha_2+\lambda_2)-\alpha_1\alpha_2} + \frac{(\alpha_1+2\lambda_1)(\alpha_2+2\lambda_2)}{(\alpha_1+2\lambda_1)(\alpha_2+2\lambda_2)-\alpha_1\alpha_2}\right]\right]^2$$

5. Numerical Illustration

Table 1

$\alpha_1 = 0.2, \alpha_2 = 0.4, \lambda_1 = 0.3$					
C	Λ_2=0.4	Λ_2=0.7	Λ_2=1	Λ_2=1.3	Λ_2=1.6
1	1.50	1.34	1.26	1.21	1.18
2	0.75	0.67	0.63	0.61	0.59
3	0.50	0.45	0.42	0.40	0.39
4	0.37	0.34	0.32	0.30	0.29
5	0.30	0.27	0.25	0.24	0.24
6	0.25	0.22	0.21	0.20	0.20
7	0.21	0.19	0.18	0.17	0.17
8	0.19	0.17	0.16	0.15	0.15
9	0.17	0.15	0.14	0.13	0.13
10	0.15	0.13	0.13	0.12	0.12

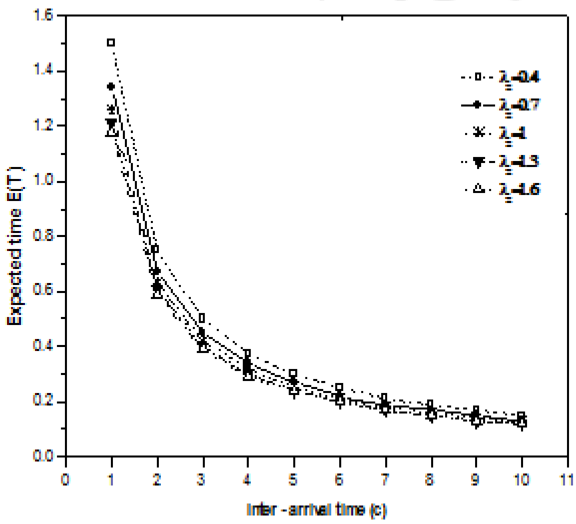

Figure 1

Table 2

$\alpha_1 = 0.2, \alpha_2 = 0.4, \lambda_2 = 0.3$					
C	Λ_1=0.4	Λ_1=0.7	Λ_1=1	Λ_1=1.3	Λ_1=1.6
1	1.46	1.29	1.21	1.16	1.13
2	0.73	0.64	0.60	0.58	0.57
3	0.49	0.43	0.40	0.39	0.38
4	0.37	0.32	0.30	0.29	0.28
5	0.29	0.26	0.24	0.23	0.23
6	0.24	0.21	0.20	0.19	0.19
7	0.21	0.18	0.17	0.17	0.16
8	0.18	0.16	0.15	0.15	0.14
9	0.16	0.14	0.13	0.13	0.13
10	0.15	0.13	0.12	0.12	0.11

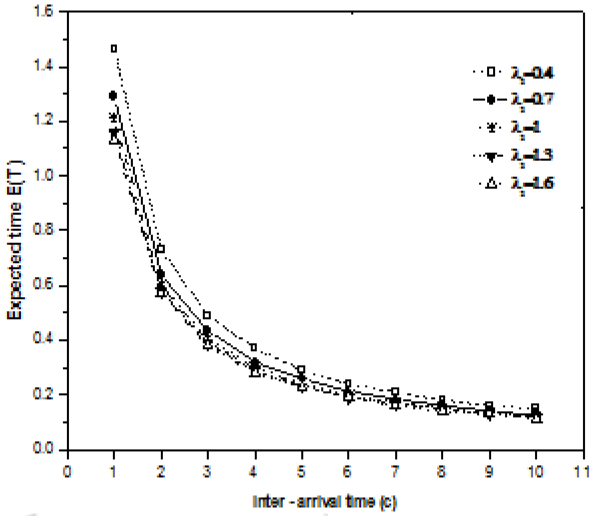

Figure 2

Table 3

$\alpha_1 = 0.2, \alpha_2 = 0.4, \lambda_1 = 0.3$					
C	Λ_2=0.4	Λ_2=0.7	Λ_2=1	Λ_2=1.3	Λ_2=1.6
1	29.04	24.94	22.91	21.71	20.91
2	7.26	6.23	5.73	5.43	5.23
3	3.23	2.77	2.55	2.41	2.32
4	1.81	1.56	1.43	1.36	1.31
5	1.16	1.00	0.92	0.87	0.84
6	0.81	0.69	0.64	0.60	0.58
7	0.59	0.51	0.47	0.44	0.43
8	0.45	0.39	0.36	0.34	0.33
9	0.36	0.31	0.28	0.27	0.26
10	0.29	0.25	0.23	0.22	0.21

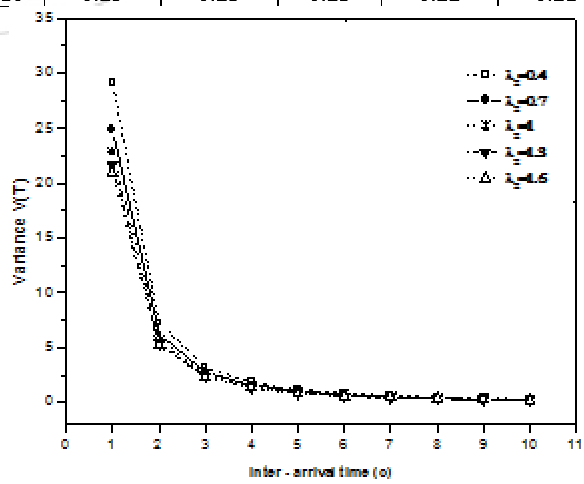

Figure 3

Table 4

$\alpha_1 = 0.2, \alpha_2 = 0.4, \lambda_2 = 0.3$					
c	Λ_1=0.4	Λ_1=0.7	Λ_1=1	Λ_1=1.3	Λ_1=1.6
1	28.21	23.63	21.71	20.64	19.97
2	7.05	5.91	5.43	5.16	4.99
3	3.13	2.63	2.41	2.29	2.22
4	1.76	1.48	1.36	1.29	1.25
5	1.13	0.95	0.87	0.83	0.80
6	0.78	0.66	0.60	0.57	0.55
7	0.58	0.48	0.44	0.42	0.41
8	0.44	0.37	0.34	0.32	0.31
9	0.35	0.29	0.27	0.25	0.25
10	0.28	0.24	0.22	0.21	0.20

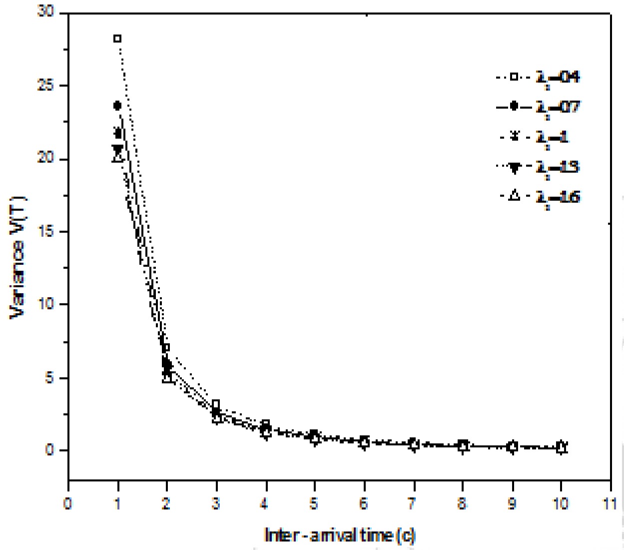

Figure 4

6. Conclusions

1. If c which is the parameter of the distribution of the inter arrival times between decision epochs is increasing then the expected time $E(T)$ decreases when the parameters $\alpha_1, \alpha_2, \lambda_1$ and λ_2 are kept fixed, This is due to the fact that since U_i follows exponential distribution $E(U_i) = 1/c$ and the mean inter arrival time decreases as c increases. Therefore the depletion will be frequent because the decision epochs are at shorter intervals, hence $E(T)$ decreases. This is same for different values of λ_2, it is indicated in Table 1 and the corresponding Figure 1.

2. It can be observed that if c is fixed and λ_2 increases expected time $E(T)$ decreases, when all the other parameters are kept fixed. Similarly if c is fixed and λ_1 is increasing then expected time $E(T)$ decreases as in the previous case. This is indicated in Table 2 and the corresponding Figure 2.

3. If c increases when all the parameters are kept fixed variance $V(T)$ decreases. If c is fixed and λ_2 increases $V(T)$ decreases, and it is uniformly so if λ_1 increases for a fixed c . This is indicated in Table 3 and Table 4 with the corresponding Figure 3 and Figure 4.

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