$\sum X_i^{''}$ denote the cumulative amount of wastage caused to the grade two at K decision epochs.

$P[\sum X_i^{''} < Y_2]$ is the probability that the cumulative damage in grade two do not cross its threshold level.

If the cumulative damage in any one of the grades cross its threshold level, the transfer of personal is not possible. So the breakdown of the system occurs if either $P[\sum X_i^{'} > Y_1]$ or $P[\sum X_i^{''} > Y_2]$. Thus recruitment of personal is to be made only if $P[\sum X_i^{'} < Y_1]$ or $P[\sum X_i^{''} < Y_2]$.

The probability that the system survives even after the cumulative loss of manpower in the respective grades after K decision is given by

$$P\left[\sum_{i=1}^{k} X_i^{'} < Y_1 \cap \sum_{i=1}^{k} X_i^{''} < Y_2\right]$$

$$P\left(\sum_{i=1}^{k} X_i^{'} < Y_1\right) * P\left(\sum_{i=1}^{k} X_i^{''} < Y_2\right)$$

because of independence of $X_i^{'}$ and $X_i^{''}$.

Therefore $S(t) = P[T > t]$ is the survival function which gives the probability that the system will fail only after time t.

$$S(t) = \sum_{k=0}^{\infty} V_k(t)\left[P\left(\sum_{i=1}^{k} X_i^{'} < Y_1\right) * P\left(\sum_{i=1}^{k} X_i^{''} < Y_2\right)\right]$$

Let $K_1(\cdot) \sim$ Exponentiated exponential with parameter λ_1 and

$K_2(\cdot) \sim$ Exponentiated exponential with parameter λ_2

$$P\left(\sum_{i=1}^{k} X_i^{'} < Y_1\right) = \int_0^{\infty} g_k(x)\ K_1(x)dx$$

$$= \int_0^{\infty} g_k(x)\ \left(1 - e^{-\lambda_1 x}\right)^2 dx$$

Similarly

$$P\left(\sum_{i=1}^{k} X_i^{''} < Y_2\right) = \int_0^{\infty} h_k(x)\ K_2(x)dx$$

$$= \int_0^{\infty} h_k(x)\ \left(1 - e^{-\lambda_2 x}\right)^2 dx$$

$$S(t) = [F_k(t) - F_{k+1}(t)]\{[2g_k^*(\lambda_1) - g_k^*(2\lambda_1)] * [2h_k^*(\lambda_2) - h_k^*(2\lambda_2)]\}$$

$$= \sum_{k=0}^{\infty} V_k(t)\left\{P\left(\sum_{i=1}^{k} X_i^{'} < Y_1\right) * P\left(\sum_{i=1}^{k} X_i^{''} < Y_2\right)\right\}$$

$$= \sum_{k=0}^{\infty} V_k(t)\ \{[2g_k^*(\lambda_1) - g_k^*(2\lambda_1)] * [2h_k^*(\lambda_2) - h_k^*(2\lambda_2)]\}$$

$$= 4\sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\,[g^*(\lambda_1)h^*(\lambda_2)]^k - 2\sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\,[g^*(\lambda_1)h^*(2\lambda_2)]^k$$

$$-2\sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\,[g^*(2\lambda_1)h^*(\lambda_2)]^k + \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\,[g^*(2\lambda_1)h^*(2\lambda_2)]^k$$

$$S(t) = 4\{1 - [1 - g^*(\lambda_1)h^*(\lambda_2)]\}\sum_{k=1}^{\infty}[F_k(t)g^*(\lambda_1)h^*(\lambda_2)]^{k-1}$$

$$-2\{1 - [1 - g^*(\lambda_1)h^*(2\lambda_2)]\}\sum_{k=1}^{\infty}[F_k(t)g^*(\lambda_1)h^*(2\lambda_2)]^{k-1}$$

$$-2\{1 - [1 - g^*(2\lambda_1)h^*(\lambda_2)]\}\sum_{k=1}^{\infty}[F_k(t)g^*(2\lambda_1)h^*(\lambda_2)]^{k-1}$$

$$+\{1 - [1 - g^*(2\lambda_1)h^*(2\lambda_2)]\}\sum_{k=1}^{\infty}[F_k(t)g^*(2\lambda_1)h^*(2\lambda_2)]^{k-1}$$

$$L(t) = 1 - S(t)$$

Taking Laplace transform of $L(t)$, we get

$$L^*(s) = \frac{4[1 - g^*(\lambda_1)h^*(\lambda_2)]f^*(s)}{[1 - g^*(\lambda_1)h^*(\lambda_2)f^*(s)]} - \frac{2[1-g^*(\lambda_1)h^*(2\lambda_2)]f^*(s)}{[1-g^*(\lambda_1)h^*(2\lambda_2)f^*(s)]}$$

$$- \frac{2[1-g^*(2\lambda_1)h^*(\lambda_2)]f^*(s)}{[1-g^*(2\lambda_1)h^*(\lambda_2)f^*(s)]} + \frac{[1-g^*(2\lambda_1)h^*(2\lambda_2)]f^*(s)}{[1-g^*(2\lambda_1)h^*(2\lambda_2)f^*(s)]}$$

Let the random variable U denoting inter arrival time which follows exponential with parameter c. Now $f^*(s) = \left(\frac{c}{c+s}\right)$, substituting in the above equation (8), we get

$$= \frac{4[1 - g^*(\lambda_1)h^*(\lambda_2)]\frac{c}{c+s}}{\left[1 - g^*(\lambda_1)h^*(\lambda_2)\frac{c}{c+s}\right]} - \frac{2[1-g^*(\lambda_1)h^*(2\lambda_2)]\frac{c}{c+s}}{\left[1-g^*(\lambda_1)h^*(2\lambda_2)\frac{c}{c+s}\right]} - \frac{2[1-g^*(2\lambda_1)h^*(\lambda_2)]\frac{c}{c+s}}{\left[1-g^*(2\lambda_1)h^*(\lambda_2)\frac{c}{c+s}\right]}$$

$$+ \frac{[1 - g^*(2\lambda_1)h^*(2\lambda_2)]\frac{c}{c+s}}{\left[1 - g^*(2\lambda_1)h^*(2\lambda_2)\frac{c}{c+s}\right]}$$

$$L^*(s) = \frac{4c[1 - g^*(\lambda_1)h^*(\lambda_2)]}{[c + s - g^*(\lambda_1)h^*(\lambda_2)c]}$$

$$- \frac{2c[1-g^*(\lambda_1)h^*(2\lambda_2)]}{[c+s-g^*(\lambda_1)h^*(2\lambda_2)c]} - \frac{2c[1-g^*(2\lambda_1)h^*(\lambda_2)]}{[c+s-g^*(2\lambda_1)h^*(\lambda_2)c]}$$

$$+ \frac{c[1-g^*(2\lambda_1)h^*(2\lambda_2)]}{[c+s-g^*(2\lambda_1)h^*(2\lambda_2)c]}$$

$$E(T) = -\frac{d}{ds}L^*(s) \quad \text{given} \quad s = 0$$

$$= \frac{4c[1-g^*(\lambda_1)h^*(\lambda_2)]}{c^2[1-g^*(\lambda_1)h^*(\lambda_2)]^2} - \frac{2c[1-g^*(\lambda_1)h^*(2\lambda_2)]}{c^2[1-g^*(\lambda_1)h^*(2\lambda_2)]^2}$$

$$- \frac{2c[1-g^*(2\lambda_1)h^*(\lambda_2)]}{c^2[1-g^*(2\lambda_1)h^*(\lambda_2)]^2} + \frac{c[1-g^*(2\lambda_1)h^*(2\lambda_2)]}{c^2[1-g^*(2\lambda_1)h^*(2\lambda_2)]^2}$$

$$E(T) = \frac{4}{c[1 - g^*(\lambda_1)h^*(\lambda_2)]} - \frac{2}{c[1 - g^*(\lambda_1)h^*(2\lambda_2)]}$$

$$- \frac{2}{c[1-g^*(2\lambda_1)h^*(\lambda_2)]} + \frac{1}{c[1-g^*(2\lambda_1)h^*(2\lambda_2)]}$$

on simplification

$$E(T^2) = \frac{d^2L^*(s)}{ds^2}$$

$$= \frac{4}{c^2[1-g^*(\lambda_1)h^*(\lambda_2)]^2} - \frac{2}{c^2[1-g^*(\lambda_1)h^*(2\lambda_2)]^2}$$

$$- \frac{2}{c^2[1-g^*(2\lambda_1)h^*(\lambda_2)]^2} + \frac{1}{c^2[1-g^*(2\lambda_1)h^*(2\lambda_2)]^2}$$

From which $V(T)$ can be obtained.

$$V(\mathrm{T}) = E(T^2) - [\mathrm{E(T)}]^2$$

$$= \frac{4}{c^2[1-g^*(\lambda_1)h^*(\lambda_2)]^2} - \frac{2}{c^2[1-g^*(\lambda_1)h^*(2\lambda_2)]^2}$$

$$- \frac{2}{c^2[1-g^*(2\lambda_1)h^*(\lambda_2)]^2} + \frac{1}{c^2[1-g^*(2\lambda_1)h^*(2\lambda_2)]^2}$$

$$-\left[\frac{4}{c[1-g^*(\lambda_1)h^*(\lambda_2)]} - \frac{2}{c[1-g^*(\lambda_1)h^*(2\lambda_2)]} - \frac{2}{c[1-g^*(2\lambda_1)h^*(\lambda_2)]} + \frac{1}{c[1-g^*(2\lambda_1)h^*(2\lambda_2)]}\right]^2$$

$$g(\cdot) \sim (\alpha_1);\ g(\lambda_1) = \frac{\alpha_1}{\alpha_1 + \lambda_1};$$

$$g(2\lambda_1) = \frac{\alpha_1}{\alpha_1 + 2\lambda_1};\ g(\cdot) \sim (\alpha_2); h(\lambda_2) = \frac{\alpha_2}{\alpha_2 + \lambda_2};$$

$$h(2\lambda_2) = \frac{\alpha_2}{\alpha_2 + 2\lambda_2}$$

$$E(T) = \frac{4}{c\left[1 - \frac{\alpha_1}{\alpha_1 + \lambda_1}\frac{\alpha_2}{\alpha_2 + \lambda_2}\right]} - \frac{2}{c\left[1 - \frac{\alpha_1}{\alpha_1 + \lambda_1}\frac{\alpha_2}{\alpha_2 + 2\lambda_2}\right]}$$

$$- \frac{2}{c\left[1 - \frac{\alpha_1}{\alpha_1 + 2\lambda_1}\frac{\alpha_2}{\alpha_2 + \lambda_2}\right]} + \frac{1}{c\left[1 - \frac{\alpha_1}{\alpha_1 + 2\lambda_1}\frac{\alpha_2}{\alpha_2 + 2\lambda_2}\right]}$$

$$\mathrm{E(T)} = \frac{1}{c}\left[\frac{4(\alpha_1+\lambda_1)(\alpha_2+\lambda_2)}{(\alpha_1+\lambda_1)(\alpha_2+\lambda_2)-\alpha_1\alpha_2} - \frac{2(\alpha_1+\lambda_1)(\alpha_2+2\lambda_2)}{(\alpha_1+\lambda_1)(\alpha_2+2\lambda_2)-\alpha_1\alpha_2}\right]$$

$$-\frac{1}{c}\left[\frac{2(\alpha_1+2\lambda_1)(\alpha_2+\lambda_2)}{(\alpha_1+2\lambda_1)(\alpha_2+\lambda_2)-\alpha_1\alpha_2} + \frac{(\alpha_1+2\lambda_1)(\alpha_2+2\lambda_2)}{(\alpha_1+2\lambda_1)(\alpha_2+2\lambda_2)-\alpha_1\alpha_2}\right]$$

$$V(T)=\frac{4}{c^2[1-g^*(\lambda_1)h^*(\lambda_2)]^2}-\frac{2}{c^2[1-g^*(\lambda_1)h^*(2\lambda_2)]^2}-\frac{2}{c^2[1-g^*(2\lambda_1)h^*(\lambda_2)]^2}+\frac{1}{c^2[1-g^*(2\lambda_1)h^*(2\lambda_2)]^2}-\left[\frac{4}{c[1-g^*(\lambda_1)h^*(\lambda_2)]}-\frac{2}{c[1-g^*(\lambda_1)h^*(2\lambda_2)]}-\frac{2}{c[1-g^*(2\lambda_1)h^*(\lambda_2)]}+\frac{1}{c[1-g^*(2\lambda_1)h^*(2\lambda_2)]}\right]^2$$

$$V(T)=\frac{2}{c^2}\left[\left[\frac{4(\alpha_1+\lambda_1)(\alpha_2+\lambda_2)}{(\alpha_1+\lambda_1)(\alpha_2+\lambda_2)-\alpha_1\alpha_2}\right]^2-\left[\frac{2(\alpha_1+\lambda_1)(\alpha_2+2\lambda_2)}{(\alpha_1+\lambda_1)(\alpha_2+2\lambda_2)-\alpha_1\alpha_2}\right]^2-\left[\frac{2(\alpha_1+2\lambda_1)(\alpha_2+\lambda_2)}{(\alpha_1+2\lambda_1)(\alpha_2+\lambda_2)-\alpha_1\alpha_2}\right]^2+\left[\frac{(\alpha_1+2\lambda_1)(\alpha_2+2\lambda_2)}{(\alpha_1+2\lambda_1)(\alpha_2+2\lambda_2)-\alpha_1\alpha_2}\right]^2\right]-\left[\frac{1}{c}\left[\frac{4(\alpha_1+\lambda_1)(\alpha_2+\lambda_2)}{(\alpha_1+\lambda_1)(\alpha_2+\lambda_2)-\alpha_1\alpha_2}-\frac{2(\alpha_1+\lambda_1)(\alpha_2+2\lambda_2)}{(\alpha_1+\lambda_1)(\alpha_2+2\lambda_2)-\alpha_1\alpha_2}-\frac{2(\alpha_1+2\lambda_1)(\alpha_2+\lambda_2)}{(\alpha_1+2\lambda_1)(\alpha_2+\lambda_2)-\alpha_1\alpha_2}+\frac{(\alpha_1+2\lambda_1)(\alpha_2+2\lambda_2)}{(\alpha_1+2\lambda_1)(\alpha_2+2\lambda_2)-\alpha_1\alpha_2}\right]\right]^2$$

5. Numerical Illustration

Table 1

$\alpha_1=0.2, \alpha_2=0.4, \lambda_1=0.3$					
C	Λ_2=0.4	Λ_2=0.7	Λ_2=1	Λ_2=1.3	Λ_2=1.6
1	1.50	1.34	1.26	1.21	1.18
2	0.75	0.67	0.63	0.61	0.59
3	0.50	0.45	0.42	0.40	0.39
4	0.37	0.34	0.32	0.30	0.29
5	0.30	0.27	0.25	0.24	0.24
6	0.25	0.22	0.21	0.20	0.20
7	0.21	0.19	0.18	0.17	0.17
8	0.19	0.17	0.16	0.15	0.15
9	0.17	0.15	0.14	0.13	0.13
10	0.15	0.13	0.13	0.12	0.12

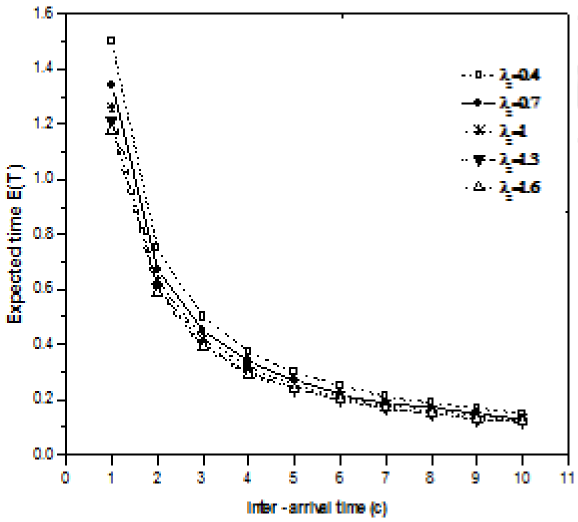

Figure 1

Table 2

$\alpha_1=0.2, \alpha_2=0.4, \lambda_2=0.3$					
C	Λ_1=0.4	Λ_1=0.7	Λ_1=1	Λ_1=1.3	Λ_1=1.6
1	1.46	1.29	1.21	1.16	1.13
2	0.73	0.64	0.60	0.58	0.57
3	0.49	0.43	0.40	0.39	0.38
4	0.37	0.32	0.30	0.29	0.28
5	0.29	0.26	0.24	0.23	0.23
6	0.24	0.21	0.20	0.19	0.19
7	0.21	0.18	0.17	0.17	0.16
8	0.18	0.16	0.15	0.15	0.14
9	0.16	0.14	0.13	0.13	0.13
10	0.15	0.13	0.12	0.12	0.11

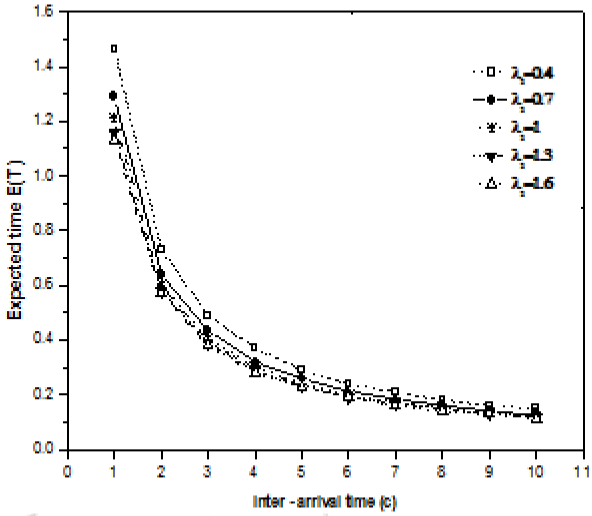

Figure 2

Table 3

$\alpha_1=0.2, \alpha_2=0.4, \lambda_1=0.3$					
C	Λ_2=0.4	Λ_2=0.7	Λ_2=1	Λ_2=1.3	Λ_2=1.6
1	29.04	24.94	22.91	21.71	20.91
2	7.26	6.23	5.73	5.43	5.23
3	3.23	2.77	2.55	2.41	2.32
4	1.81	1.56	1.43	1.36	1.31
5	1.16	1.00	0.92	0.87	0.84
6	0.81	0.69	0.64	0.60	0.58
7	0.59	0.51	0.47	0.44	0.43
8	0.45	0.39	0.36	0.34	0.33
9	0.36	0.31	0.28	0.27	0.26
10	0.29	0.25	0.23	0.22	0.21

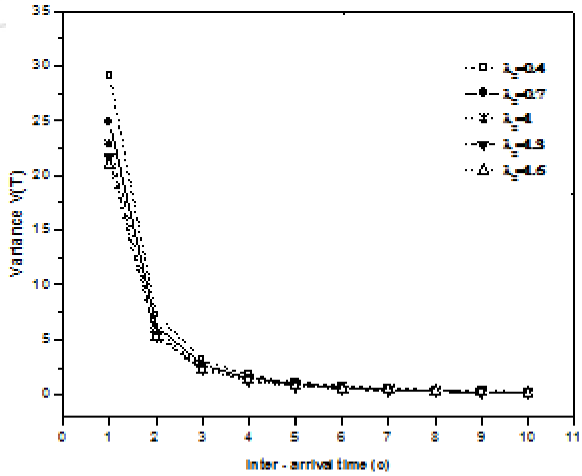

Figure 3

Table 4

$\alpha_1 = 0.2, \alpha_2 = 0.4, \lambda_2 = 0.3$					
C	Λ_1=0.4	Λ_1=0.7	Λ_1=1	Λ_1=1.3	Λ_1=1.6
1	28.21	23.63	21.71	20.64	19.97
2	7.05	5.91	5.43	5.16	4.99
3	3.13	2.63	2.41	2.29	2.22
4	1.76	1.48	1.36	1.29	1.25
5	1.13	0.95	0.87	0.83	0.80
6	0.78	0.66	0.60	0.57	0.55
7	0.58	0.48	0.44	0.42	0.41
8	0.44	0.37	0.34	0.32	0.31
9	0.35	0.29	0.27	0.25	0.25
10	0.28	0.24	0.22	0.21	0.20

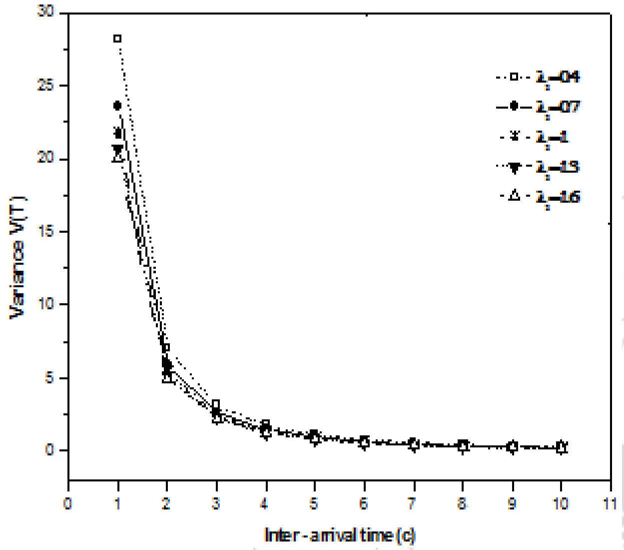

Figure 4

6. Conclusions

1. If c which is the parameter of the distribution of the inter arrival times between decision epochs is increasing then the expected time $E(T)$ decreases when the parameters $\alpha_1, \alpha_2, \lambda_1$ **and** λ_2 are kept fixed, This is due to the fact that since U_i follows exponential distribution $E(U_i) = {}^1\!/_c$ and the mean inter arrival time decreases as c increases. Therefore the depletion will be frequent because the decision epochs are at shorter intervals, hence $E(T)$ decreases. This is same for different values of λ_2, it is indicated in Table 1 and the corresponding Figure 1.

2. It can be observed that if c is fixed and λ_2 increases expected time $E(T)$ decreases, when all the other parameters are kept fixed. Similarly if c is fixed and λ_1 is increasing then expected time $E(T)$ decreases as in the previous case. This is indicated in Table 2 and the corresponding Figure 2.

3. If c increases when all the parameters are kept fixed variance $V(T)$ decreases. If c is fixed and λ_2 increases $V(T)$ decreases, and it is uniformly so if λ_1 increases for a fixed c . This is indicated in Table 3 and Table 4 with the corresponding Figure 3 and Figure 4.

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