Kantowski-Sachs String Cosmological Model with Bulk Viscosity and Time Dependent Λ term in General Relativity

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Abstract: *We have studied a new solution of Einstein's field equations for Kantowski-Sachs Model attached with cloud of string in the presence of bulk viscosity and time dependent cosmological term of the form* $\Lambda(t) \propto \dfrac{\dot{a}}{a} + \dfrac{2\dot{b}}{b}$.*To get deterministic model of the universe, here we have imposed the condition that the shear scalar is proportional to expansion scalar. Some physical and geometrical properties of the models are discussed. We conclude that the universe model does not approach isotropy through the evolution of the universe*.

Keywords: Kantowski-Sachs Universe, Cosmology, Variable cosmological term, String and Bulk viscosity.

1. Introduction

Cosmology is a science developed in the beginning of twentieth century rapidly. The aim of cosmology is to determine the large scale structure of the physical universe. Cosmology is one of the great estintellectual achievements of all time beginning from its origin. Cosmology, as a common man understands, is that branch of astronomy, which deals with the large scale structure of the universe. The basic problem in cosmology is to find the cosmological models of universe and to compare the resulting models with the present day universe using astronomical data. In most treatments of cosmology, cosmic fluid is considered as perfect fluid. However, bulk viscosity is expected to play an important role at certain stages of an expanding universe.

To consider more realistic models one must take into account viscosity mechanisms and indeed, viscosity mechanisms has attracted the attention of many researchers. At the early stages of evolution of the universe, when radiation is in the form of photons as well as neutrino decoupled, the matter behaved like a viscous fluid. Bulk viscosity could arise in many circumstances and could lead to an effective mechanism of galaxy formation. Shri Ram et.al. studied that Bianchi Type VI bulk viscous fluid models with variable Gravitational and Cosmological Constants [1]. Samdurkar and Sen investigated the effect of bulk viscosity on Bianchi Type V cosmological models with varying Λ in general relativity [2]. Dwivedi analysed that bulk viscous bianchi type –V cosmological models with stiff fluid and time dependent cosmological term [3].

In the last few years the study of cosmic strings has attracted considerable interest as they are believed to play an important role during early stages of the universe. The idea was that particles like the photon and the neutron could be regarded as waves on a string. The presence of strings in the early universe is a by product of Grant Unified Theories (GUT). Cosmic strings have stress energy and coupled in a simple way to the gravitational field. The general relativistic treatment of cosmic strings has been originally given by Letelier [4] and Stachel [5]. It appears that after the 'Big-Bang' the universe may have experienced a number of phase transitions. These phase transitions can produce vacuum domain structures such as domain walls, cosmic strings and monopoles. Letelier and Verdaguer [6] studied a new model of cloud formed by massive strings in the context of general relativity. They have considered the Bianchi type-I model as they are supported to be reasonable representation of the early universe. Recently Rao et al. [7] and Rao and Sireesha [8] have obtained Bianchi types II, VIII, and IX string cosmological models with bulk viscosity in Lyra [9] theory of gravitation, respectively.

Beside the Bianchi type metrics, the Kantowski-Sachs [10] models are also describing spatially homogeneous universes. For a review of Kantowski-Sachs of metrics one can refer to [11]. These metrics represent homogeneous but anisotropically expanding (or contacting) cosmologies and provide models where the effects of anisotropic can be estimated and compared with all well-known Friedmann-Robertson-Walker class of cosmologies. Wang [12] has obtained Kantowski-Sachs string cosmological model with bulk viscosity in general relativity. Kandalkar et al. [13] have discussed Kantowski-Sachs viscous fluid cosmological model with a varying Λ. Kandalkar et al. [14] have obtained string cosmology in Kantowski-Sachs space-time with bulk viscosity and magnetic field. Rao et al. [15] have studied various Bianchi type string cosmological models in the presence of bulk viscosity.

Kanika et al. [16] have investigated magnetized Kantowski-Sachs bulk viscous string cosmological models with decaying vacuum energy density. Rao [17] studied that Kantowski-Sachs bulk viscous string cosmological model in Lyra manifold.

Motivated by the above investigations, we study spatially homogeneous and anisotropic Kantowski-Sachs string cosmological model with bulk viscosity in the presence of

time dependent cosmological term of the form $\Lambda \propto \dfrac{\dot{a}}{a} + \dfrac{2\dot{b}}{b}$ or $\Lambda \propto H$ [18] in general theory of relativity. The paper is organized as follows: In section 2, Metric and Energy momentum tensor are mentioned. In section 3, Einstein field equations of Kantowski-Sachs cosmological model attached with string are presented. In section 4, we derive solutions in the presence of bulk viscosity and time varying cosmological term by imposing the condition that the shear scalar is proportional to expansion scalar. Some physical and geometrical features are observed in section 5. With the help of expressions of physical parameters; it is possible to draw some conclusions in the last section.

2. Metric and Energy Momentum Tensor

We consider a spatially homogeneous Kantowski-Sachs metric of the form

$$ds^2 = -dt^2 + a^2 dr^2 + b^2 d\Omega^2 \qquad (1)$$

where a and b are the functions of time t only.

The energy momentum tensor for a cloud of string along the x-direction is given by

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi\theta(u_i u_j + g_{ij}) \qquad (2)$$

Here ρ is the energy density for a cloud string with particles attached to them , λ is the string tension density, H is the Hubble parameter, ξ is the coefficient of bulk viscosity, u^i the four-velocity of the particles and x^i is a unit space-like vector representing the direction of string. In a co-moving coordinate system, we have

$$u_i u^i = -x_i x^i = -1, u^i x_i = 0$$

The particle density of the configuration is given by

$$\rho = \rho_p + \lambda \qquad (3)$$

3. Field Equations

To obtain Einstein field equations, consider the following equation

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} - \Lambda(t) g_{ij} \qquad (4)$$

Here R is Ricci scalar, R_{ij} is Ricci tensor, g_{ij} is metric element and Λ is time varying cosmological term.

For the metric (1) and energy momentum tensor (2) in co-moving system of co-ordinates the above field equation yields,

$$\frac{2\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} = \lambda + \xi\theta - \Lambda \qquad (5)$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} = \xi\theta - \Lambda \qquad (6)$$

$$\frac{2\dot{a}\dot{b}}{ab} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} = \rho - \Lambda \qquad (7)$$

An overdot indicates a derivative with respect to time t.
We define average scale factor $R(t)$ and generalized Hubble parameter H for Kantowski-Sachs universe as

$$V = R^3 = ab^2 \qquad (8)$$

$$H = \frac{\dot{R}}{R} = \frac{1}{3}(H_1 + H_2 + H_3) \qquad (9)$$

where are directional Hubble's factors in the directions of x,y,z respectively $H_1 = \dfrac{\dot{a}}{a}, H_2 = H_3 = \dfrac{\dot{b}}{b}$

Also expansion factor and shear scalar are

$$\theta = \frac{\dot{a}}{a} + \frac{2\dot{b}}{b} \qquad (10)$$

$$2\sigma^2 = H_1^2 + H_2^2 + H_3^2 - \frac{\theta^2}{3} \qquad (11)$$

The mean anisotropic parameter is given by

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \qquad (12)$$

In order to get a deterministic solution we take the following plausible physical condition, the shear scalar σ is proportional to scalar expansion θ. This condition leads to

$$a = b^n, n > 1 \qquad (13)$$

4. Solution of the field equations with $\Lambda \propto H$ and Nambu string

i.e. $\rho = \lambda$ (14)
Solving eq^n (5-7) with the help of (14), we get

$$\frac{\ddot{a}}{a} - \frac{\ddot{b}}{b} + \frac{3\dot{a}\dot{b}}{ab} = -\alpha H \qquad (15)$$

Using equation (13),

$$\frac{\ddot{b}}{b} + \frac{n(n+2)}{(n-1)}\frac{\dot{b}^2}{b^2} = -\frac{\alpha(n+2)}{3(n-1)}\frac{\dot{b}}{b} \qquad (16)$$

which on integration gives

$$b^{\frac{n(n+2)}{(n-1)}}\dot{b} = m e^{\frac{-\alpha(n+2)}{3(n-1)}t} \qquad (17)$$

Again integrating , we get

$$b = \left\{\frac{K}{n-1}\right\}^{\left(\frac{n-1}{K}\right)}\left[-m_1 e^{\frac{-\alpha(n+2)}{3(n-1)}t} + m_2\right]^{\left(\frac{n-1}{K}\right)} \qquad (18)$$

where $K = n^2 + 3n - 1$
Hence

$$a = \left\{\frac{K}{n-1}\right\}^{\left(\frac{n(n-1)}{K}\right)}\left[-m_1 e^{\frac{-\alpha(n+2)}{3(n-1)}t} + m_2\right]^{\left(\frac{n(n-1)}{K}\right)} \qquad (19)$$

Therefore the metric (1) becomes

$$ds^2 = -dt^2 + \left(\frac{K}{n-1}\right)^{\frac{2n(n-1)}{K}}\left[-m_1 e^{\frac{-\alpha(n+2)}{3(n-1)}t} + m_2\right]^{\frac{2n(n-1)}{K}} dr^2$$

$$+ \left(\frac{K}{n-1}\right)^{\frac{2(n-1)}{K}}\left[-m_1 e^{\frac{-\alpha(n+2)}{3(n-1)}t} + m_2\right]^{\frac{2(n-1)}{K}} d\Omega^2 \qquad (20)$$

5. Some Physical and Geometrical Properties of the Models

For the model of equation (18), the other physical and geometrical parameters can be easily obtained. The expression for density is given by

$$\rho = \lambda = \left(\frac{L_4 e^{\frac{-\alpha(n+2)}{3(n-1)}t}}{\left[-m_1 e^{\frac{-\alpha(n+2)}{3(n-1)}t} + m_2 \right]} + 1 \right) \frac{L_3 e^{\frac{-2\alpha(n+2)}{3(n-1)}t}}{\left[-m_1 e^{\frac{-\alpha(n+2)}{3(n-1)}t} + m_2 \right]}$$

$$+ \frac{1}{L_2 \left[-m_1 e^{\frac{-\alpha(n+2)}{3(n-1)}t} + m_2 \right]^{\frac{2(n-1)}{K}}} \tag{21}$$

The coefficient of bulk viscosity is given by

$$\xi = \left(\frac{L_5 e^{\frac{-\alpha(n+2)}{3(n-1)}t}}{\left[-m_1 e^{\frac{-\alpha(n+2)}{3(n-1)}t} + m_2 \right]} + 1 \right) \frac{L_6}{(n+2)L_1} \tag{22}$$

The cosmological term is

$$\Lambda = \left(\frac{\alpha^2 m_1 (n+2)^2}{3K} \right) \cdot \frac{e^{\frac{-\alpha(n+2)}{3(n-1)}}}{\left[-m_1 e^{\frac{-\alpha(n+2)}{3(n-1)}} + m_2 \right]} \tag{23}$$

Expression for average scale factor R and generalized Hubble parameter H are as follows

$$V = R^3 = \left(\frac{K}{n-1} \right)^{\frac{(n-1)(n+2)}{K}} \left[-m_1 e^{\frac{-\alpha(n+2)}{3(n-1)}t} + m_2 \right]^{\frac{(n-1)(n+2)}{K}} \tag{24}$$

and

$$H = \left(\frac{\alpha^2 m_1 (n+2)}{3K} \right) \cdot \frac{e^{\frac{-\alpha(n+2)}{3(n-1)}}}{\left[-m_1 e^{\frac{-\alpha(n+2)}{3(n-1)}} + m_2 \right]} \tag{25}$$

Expression for expansion factor can be found as

$$\theta = \alpha \left(\frac{\alpha m_1 (n+2)^2}{3K} \right) \cdot \frac{e^{\frac{-\alpha(n+2)}{3(n-1)}}}{\left[-m_1 e^{\frac{-\alpha(n+2)}{3(n-1)}} + m_2 \right]} \tag{26}$$

Expression for shear scalar can be found as

$$\sigma^2 = \alpha\beta \left(\frac{m_1 (n+2)^3}{54K} \right) \frac{e^{\frac{-2\alpha(n+2)}{3(n-1)}t}}{\left[-m_1 e^{\frac{-\alpha(n+2)}{3(n-1)}t} + m_2 \right]^2} \tag{27}$$

Expression for mean anisotropic parameter is

$$\Delta = \frac{2(n-1)^2}{(n+2)^2} \tag{28}$$

Where

$$L_1 = \frac{\alpha m_1 (n+2)}{3(n^2+3n-1)}, \quad L_2 = \left(\frac{n^2+3n-1}{n-1} \right)^{\frac{2(n-1)}{n^2+3n-1}}$$

$$L_3 = \frac{\alpha(n+2)}{3} L_1$$

$$L_4 = \frac{3(2n+1)}{\alpha(n+2)} L_1$$

$$L_5 = \frac{\alpha^2 (n+2)^2 m_1}{3K} \left[1 - \frac{(n+1)K^2}{3(n-1)^3} \right]$$

$$L_6 = \frac{K}{3} \left(\frac{n^2}{K^2} + \frac{K(K-n+1)}{(n-1)^3} \right) \left[1 - \frac{(n+1)K^2}{3(n-1)^3} \right]^{-1}$$

$$\beta = 4 + \frac{2(n-1)^2}{(n+2)^2}$$

6. Conclusion

In this paper we have analysed Kantowski-Sachs String cosmological model with bulk viscosity and varying cosmological term of the form $\Lambda \propto H$. For suitable values of constants, it is observed that the cosmological term vanishes as t tends to infinity and infinite at t tends to zero. Hence the cosmological term is a decreasing function of time and it approaches a small positive value at late time.

The scale factors and spatial volume also vanishes as time increases and as t tends to infinity, scale factors and volume become infinite whereas ρ, σ, ξ and Λ tends to zero. Therefore the model has a point-type singularity at initial epoch. As time increases, the rate of expansion decreases, thus the rate of expansion slows down with increase in time. The model represents shearing , non-rotating and expanding model of the universe. We also observe that $\frac{\sigma}{\theta} \neq 0$ therefore model does not approach isotropy. It is clear from expression (28) that mean anisotropic parameter vanishes for n=1.

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425