

Techniques Used in Hypothesis Testing in Research Methodology – A Review

Joginder Kaur

Lecturer (Mathematics), Punjab Technical University, Punjab, India

Abstract: This paper reviews the methods to select correct statistical tests for research projects or other investigations. Research is a scientific search on a particular topic including various steps in which formulating and testing of hypothesis is an important step. To test a hypothesis there are various tests like Student's t-test, F test, Chi square test, ANOVA etc. and the conditions and methods to apply these tests are explained here. Only the correct use of these tests gives valid results about hypothesis testing.

Keywords: Hypothesis, Testing Chi square, ANOVA, F test, t- test.

1. Introduction

Today less attention is paid to the Research Methodology and its tools in decision making by the students. Even though it is not possible to get 100% precision in decision making using these tools, but accuracy can be brought using the research methodology scientifically and systematically.[1]Research is an art of scientific investigation. It is a systematic method of enunciating the problem, formulating a hypothesis, collecting the facts or data, analyzing the facts and reaching certain conclusion either in the form of solutions towards the concerned problem or in certain generalizations for some theoretical formulations. The word research is composed of two syllables –‘re’ and ‘search’. ‘Re’ is a prefix meaning again, a new or over again ‘search’ is a verb meaning to examine closely and carefully, to test and try, or to probe. Together they form a noun describing a careful, systematic, study and investigation in some field of knowledge undertaken to establish facts or principles. [6]

1.1 Sampling

In this process, the researcher needs to know some sampling fundamentals. The theory of sampling is a study of relationship existing between a population and sample drawn from the population. The main object of sampling is to get as much information as possible about the whole universe by examining only a part of it. Sampling is often used in our day-to-day life.

For example: - When political polltakers need to predict the outcome of an upcoming election, they may use random sampling to figure out which politician a population favors most. Asking every member of a population would be very time consuming, so polltakers will use random sampling to randomly select subjects within that population to create a sample group and use the responses those subjects give them to predict who that population of people as a whole will select on voting day. [8] The methods of inference used to support or reject claims based on sample data are known as Tests of Significance. [5]

1.2 Tests of significance

Every test of significance begins with a *null hypothesis* H_0 .

1.3 Null Hypothesis

A null hypothesis is a specific baseline statement to be tested and it usually takes such forms as “no effect” or “no difference.” An alternative (research) hypothesis is denial of the null hypothesis. [7]. We always make null hypothesis which is of the form like “There is no significant difference between x and y”

Or

“There is no association between x and y”

{In case of Chi Square test as
independence of attributes

Or

“The attributes are independent”

{In case of Chi Square test as
independence of attributes

1.4 Alternative Hypothesis

An *Alternative Hypothesis* is denoted by H_1 or H_a , is the hypothesis that sample observations are influenced by some non-random cause. Rejection of null hypothesis leads to the acceptance of alternative hypothesis

e.g.

Null hypothesis: “ $x = y$.”

Alternative hypothesis: “ $x \neq y$ ” → (Two tailed)

“ $x < y$ ” (Left tailed) → (Single tailed)

“ $x > y$ ” (Right tailed) → (Single tailed)

1.5 Numerical Steps in Testing of Hypothesis

- 1) Establish the null hypothesis and alternative hypothesis.
- 2) set up a suitable significance level e.g. at 1%, 5%, 10% level of significance etc.
- 3) Determine a suitable test tool like t, Z, F, Chi Square, ANOVA etc.
- 4) Calculate the value of test statistic using any of test tools
- 5) Compare this calculated value with table value

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6) Draw conclusions

If calculated value < Table value then null hypothesis is accepted

If calculated value > Table value then null hypothesis is rejected

Table 1: Important *notations* or *symbols*

Parameter	Sample	Population
Mean	\bar{x}	μ
Standard Deviation	s	σ

2. Steps of Research Methodology

The main steps in research process are:

- Defining the research problem
- Review of literature
- Formulating the Hypothesis
- Preparing the research design
- Data collection
- Data analysis
- Interpretation and Report writing [6]

3. Tools available for testing Hypothesis and Decision Making

3.1 Large Sample tests

When the sample size is $n \geq 30$, then apply large sample tests.

3.1.1. Testing of significance for single proportion:-

Applications: To find significant difference between *proportion* of sample and population

$$Z = \frac{p - P}{\sqrt{PQ/n}}$$

3.1.2 Testing of significance for difference of proportions:-

Applications: To find significant difference between two sample proportions p_1 and p_2

$$Z = \frac{p_1 - p_2}{\sqrt{PQ(\frac{1}{n_1} + \frac{1}{n_2})}}$$

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}, Q = 1 - P$$

3.1.3 Testing of significance for single mean:-

Applications: To find significant difference between *mean* of sample and population

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \text{ When population S.D. is known}$$

$$Z = \frac{\bar{x} - \mu}{s / \sqrt{n}} \text{ When population S.D. is not known}$$

3.1.4 Testing of significance for difference of means:-

Applications: To find significant difference between two sample means \bar{x}_1 and \bar{x}_2

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \text{ When population S.D. is known}$$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \text{ When population S.D. is not known}$$

3.1.5. Testing of significance for difference of Standard Deviations:-

Applications: To find significant difference between two sample S. D. s_1 and s_2

$$Z = \frac{s_1 - s_2}{\sqrt{\frac{\sigma_1^2}{2n_1} + \frac{\sigma_2^2}{2n_2}}} \text{ When population S.D. is known.}$$

$$Z = \frac{s_1 - s_2}{\sqrt{\frac{s_1^2}{2n_1} + \frac{s_2^2}{2n_2}}} \text{ When population S.D. is not known.}$$

Table 2: Large Sample Tests

Unknown Parameter	One Sample	Two Samples
Proportion	$z = \frac{p - P}{\sqrt{PQ/n}}$	$z = \frac{p_1 - p_2}{\sqrt{PQ(\frac{1}{n_1} + \frac{1}{n_2})}}$, $P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}, Q = 1 - P$
Mean	$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$ Or $z = \frac{\bar{x} - \mu}{s / \sqrt{n}}$	$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$ Or $z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$
S.D.	—	$z = \frac{s_1 - s_2}{\sqrt{\frac{\sigma_1^2}{2n_1} + \frac{\sigma_2^2}{2n_2}}}$ Or $z = \frac{s_1 - s_2}{\sqrt{\frac{s_1^2}{2n_1} + \frac{s_2^2}{2n_2}}}$

3.1.6 Chi Square Test

It is an important test amongst various tests of significance and was developed by Karl Pearson in 1900. It is based on frequencies and not on the parameters like mean, S.D. etc.

Applications: Chi Square test is used to compare observed and expected frequencies objectively. It can be used (i) as a test of goodness of fit and (ii) as a test of independence of attributes.

Conditions for applying χ^2 Test:-

- The total number of items N must be at least 50.
- No expected cell frequency should be smaller than 10. If this type of problem occurs then difficulty is overcome by grouping two or more classes before calculating (O-E).

3.1.6. (a) Chi Square Test As a test of goodness of fit:-

Chi square test enables us to see how well does the assumed theoretical distribution (such as Binomial distribution, Poisson distribution or Normal distribution) fit to the observed data.

Formula: $\chi^2 = \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$

O_{ij} = observed frequency of the cell in ith row and jth column.

E_{ij} = expected frequency of the cell in ith row and jth column.

Degree of freedom=
 n-1 (For Binomial Distribution)
 n-2 (For Poisson Distribution)
 n-3 (For Normal Distribution)

Where n= total no. of terms in a series

3.1.6. (b) Chi Square Test As a test of independence of attributes:-

χ^2 test enables us to explain whether two attributes are associated or not.

e.g. It may help in finding whether a new drug is effective in curing a disease or not

Formula: $\chi^2 = \sum (O - E)^2 / E$

Where 'O' represents the observed frequency. E is the expected frequency under the null hypothesis and computed by

$$E = \frac{\text{row total} \times \text{column total}}{\text{sample size}}$$

Here degree of freedom= (r-1) (s-1)

3.1.6.(c) Difference in test of goodness of fit and independence of attributes

For the goodness-of-fit test, a theoretical relationship is used to calculate the expected frequencies. For the test of independence, only the observed frequencies are used to calculate the expected frequencies. [4]

3.2 Small Sample tests

When the sample size is $n < 30$ then apply small sample tests.

3.2.1. Student's t-test

t statistic was developed by William S. Gossett and was published under the pseudonym Student.

Applications: t-test is used to test the significance of sample mean, difference of two sample means or two related sample means in case of small samples when population variance is unknown.

3.2.1.(a) t-test for the mean of a random sample:-

Applications:- It is used to test whether the mean of a sample deviates significantly from a stated value when variance of population is unknown.

Formula: $t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$ when s.d. is given

When S.D is not given then find s by using the formula $s^2 =$

$\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$

Degree of freedom= n-1

3.2.1.(b) t-test for difference of means of two samples:-

Applications: It is used to compare the mean of two samples of size n_1 and n_2 when population variances are equal. [9].

Formula: $t = \frac{\bar{x} - \bar{y}}{s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$ when s.d. of two samples is known

And $s^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$

When S.D is not given then find s by using the formula $s^2 =$

$\frac{1}{n_1 + n_2 - 2} \sum_{i=1}^n (x_i - \bar{x})^2$

Degree of freedom= $n_1 + n_2 - 2$

3.2.1.(c) Paired t-test:-

Applications:- A paired samples t-test is used when size of two samples is equal and is used to compare two related means. [2]

Formula:- $t = \frac{\bar{d}}{s / \sqrt{n-1}}$

Where \bar{d} = Mean of paired differences

and $s^2 = \frac{\sum d^2}{n} - (\bar{d})^2$ where $d = x_i - y_i$

3.2.2. Snedecor's Variance Ratio Test or F- Test

The name was coined by George W. Snedecor, in honour of Sir Ronald A. Fisher. Fisher initially developed the statistic as the variance ratio in the 1920s.

Applications:

i) F-test is used to test whether the two samples are from the same normal population with equal variance or from two normal populations with equal variances.

ii) To test whether the two independent samples have been drawn from the same population, we test:

- A) Equality of means by t-test
- B) Equality of population variance by F-test

But as t-test assumes that the sample variances are equal, we first apply F-test and then t-test. [10].

Formula:- $F = \frac{\sigma_1^2}{\sigma_2^2}$ where numerator is greater than denominator

When variance is known

$\sigma_1^2 = \frac{n_1 s_1^2}{n_1 - 1}$, s_1^2 = Variance of sample 1

$\sigma_2^2 = \frac{n_2 s_2^2}{n_2 - 1}$, s_2^2 = Variance of sample 2

When variance is not known

$$\sigma_1^2 = \frac{\sum(x_1 - \bar{x}_1)^2}{n_1 - 1} \quad \sigma_2^2 = \frac{\sum(x_2 - \bar{x}_2)^2}{n_2 - 1}$$

3.3 Analysis of Variance (ANOVA)

ANOVA was developed in the 1920's by R.A. Fisher.

Applications: This technique is used when multiple sample cases are there. The significant difference between the means of two samples can be tested through t-test, but the difficulty arises when we are to find the significant difference amongst more than two sample means at the same time. [3]

Table 3: One way classification ANOVA

Source of variation	Degree of freedom(d.f.)	Sum of squares (s.s.)	Mean squares (m.s.)	F
Between rows	$v_1 = h - 1$	Calculate as in (i) below	s.s./d.f.	$F_1 = \frac{\text{m.s. between rows}}{\text{Error m.s.}}$
Between columns	$v_2 = k - 1$	Calculate as in (ii) below	s.s./d.f.	$F_2 = \frac{\text{m.s. between columns}}{\text{Error m.s.}}$
Error	$v = (h-1)(k-1)$	Calculate as in (iii) below	s.s./d.f.	—————

Here (i) Sum of squares between samples = $\frac{(\sum X_1)^2}{N} + \frac{(\sum X_2)^2}{N} + \frac{(\sum X_3)^2}{N} + \frac{(\sum X_4)^2}{N} - \frac{T^2}{N}$,

If X_1, X_2, X_3, X_4 are samples.

And correction factor = $\frac{T^2}{N} - \frac{(\sum X_1 + \sum X_2 + \sum X_3 + \sum X_4)^2}{N}$

(ii) Sum of squares with in samples = Total sum of squares - Sum of squares between samples

Where total sum of squares = $\sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

Table 4: Two way classification ANOVA

Source of variation	Degree of freedom (d.f.)	Sum of squares (s.s.)	Mean squares (m.s.)	F
Between Samples	$v_1 = h - 1$	Calculate as in (i) below	s.s./d.f.	$F = \frac{\text{m.s. between samples}}{\text{m.s. with in samples}}$
Within samples	$v_2 = N - h$	Calculate as in (ii) below	s.s./d.f.	Where Num. > Denom.

(i) Sum of squares between rows = $\frac{1}{k} \sum T_i^2 - \frac{T^2}{N}$ where $\sum T_i^2 =$ sum of squares of rows total

(ii) Sum of squares between columns = $\frac{1}{h} \sum T_j^2 - \frac{T^2}{N}$ where $\sum T_j^2 =$ sum of squares of columns total

(iii) Error sum of squares = Total sum of squares - (Sum of squares between rows) - (Sum of squares between columns)

Where total sum of squares = $\sum X_1^2 + \sum X_2^2 + \sum X_3^2 + \sum X_4^2 - \frac{T^2}{N}$

4. Conclusion

With the correct use of above discussed tests, valid results can be found. So precaution should be taken while selecting the tests of hypothesis for large and small sample tests otherwise one get invalid results. That is why selection of a correct statistical test is much important.

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Author Profile



Joginder Kaur is working as a Lecturer (Mathematics) in a college affiliated to Punjab Technical University, Punjab. She received her M.Sc. in Mathematics in 2007 from Punjab University, Chandigarh. Her interested areas of research are Probability and Statistics, Complex Numbers and Numerical Methods.