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Peaked Soliton (Peakon) in Degasperis Procesi Equation

Muzzammil Ahmad Bhat[1], Gautam Johri[2], Nand Kishore Pandey[3]

[1, 2, 3]Department of Postgraduate Studies and Research in Physics and Electronics, Rani Durgavati University Jabalpur-482001, India

Abstract: ***By using the dynamical system theory the Degasperis-Processi equation are studied .The bounded travelling wave solutions such as peakons are analytically described. The Degasperis-Procesi equation is converted to the ordinary differential equation which are solved for all possible soliton solutions of Degasperis-Procesi equation. We construct exact travelling wave solution for degasperis-Procesiequation,and the obtained solution agrees well with the previously known result.***

Keywords: Degasperis-ProcesiEquation,Travelling wave solution

1. Introduction

Degasperis and Procesi [1]showed, by the use of the method of asymptotic integrability, that the PDE

$$u_t - u_{xxxt} + (b+1)uu_x = bu_xu_{xx} + uu_{xxx} \quad \text{......... (1)}$$

cannot be completely integrable unless b = 2 or b = 3. The case b = 2 is the following Camassa- Holm (CH) shallow water equation (see [2])

$$u_t - u_{xxt} + 3uu_x = 2u_xu_{xx} + uu_x \quad \text{................. (2)}$$

which is well known to be integrable and to possess multi-peakon solutions. The case *b* = 3is the following Degasperis-Procesi (DP) shallow water equation

$$u_t - u_{xxt} + 4uu_x = 3u_xu_{xx} + uu_{xxx} \quad \text{.................... (3)}$$

Although, the DP equation (3) has a similar form to the CH equation (2), two equations are pretty different. For two equations, the different isospectral problem and the fact that there is no simple transformation of equation (3) into equation (2) imply that equation (3) is different from equation (2) in the integrable structures and the form of the conservation laws. The DP equation (3) is very interesting as it is an integrable shallow water equation and presents a quite rich structure. Degasperis, Holm and Hone [3-5] proved that equation (3) is integrable by constructing its lax pair, and admits multi- peakon solutions, and explained connection with a negative flow in the Kaup-kupershmidt hierarchy via a neciprocaltransformation. Landmark and Szmigielski [6] presented an inverse scattering approach for computing n-peakon solutions of the equation (3). The blow-up phenomenon of equation (3) was discussed and the global existence of the solution was proved in [7]. In [8, 9] the Cauchy problem for equation (3) was demonstrated. Much work on the DP equation (3) has been done [10-11].

It is well known that nonlinear phenomena exist everywhere. For example, they exist in fluid physics, condensed matter physics ,biophysics, plasma physics, quantum field theory ,particle physics and nonlinear optics etc. they also connect with our everyday's life. Generally speaking, nonlinear phenomena can be described by nonlinear partial differential equations (NLPDEs). In order to understand and study the physical mechanism of these nonlinear phenomena, one should know the exact solutions of the NLPDEs corresponding to the nonlinear phenomena. However, it is usually difficult to solve NLPDEs. So seeking exact solutions of NLPDEs have become one of the extremely active research areas and DegasperisProcesi is one of them. Degasperis-Procesi equation is a real nonlinear partial differential equation which models propagation of nonlinear dispersive waves. This PDE is not only of mathematical interest but it has also proved to be an approximate model of shallow water wave propagation in the small amplitude and long wavelength regime. On the mathematical side the DP equation is very special because it belongs to the class of integrable equations, or solitonic equations with infinitely many conservation laws.

In this paper, our main purpose is to focus on traveling wave solutions of Eq. (3). The rest of this paper is organized as follows: In Section 2, by using the dynamical system theory, the exact peaked wave solution for Eq. (3) was established, which is full agreement with the previously known result. Finally, some conclusions are given in Section 3.

2. The Degasperis-Procesi (DP) Equation

Next we consider exact traveling wave solutions of the DP equation (3). Substituting the transformation $u(x,t) = \phi(x-ct)$ where c is the speed of wave with $\xi = (x-ct)$into Eq. (3), we obtain the following ordinary differential equations.

$$-c\phi' + c\phi''' + 4\phi\phi' - 3\phi'\phi'' - \phi\phi''' = 0 \quad \text{.......... (4)}$$

Integrating (4) once, (4) becomes

$$(c-\phi)'' = \phi'^2 - (2\phi - c)\phi \quad \text{.... (5)}$$

then Eq. (5) is equivalent to the following two-dimensional autonomous system

$$\frac{d\phi}{d\xi} = y$$

$$\frac{dy}{d\xi} = \frac{y^2 - 2\phi^2 + \phi c}{(c-\phi)y}$$

yielding

$$\frac{dy}{d\phi} = \frac{y^2 - 2\phi^2 + \phi c}{(c-\phi)y}$$

which gives us

$$y^2 = \frac{\phi^2 - 2\phi^3 c + \phi^2 c^2 + k}{(\phi - c)^2}$$

where k is the constant of integration and k = 0 corresponds to the solution on separatrix where soliton like solutions exists as shown in the graph(a).

if k = 0 and $\frac{d\phi}{d\xi} = y$ we get

$\phi = e^{\pm(\xi + c)}$

when $\xi \to \pm\infty$ then

$$\phi = exp(-|\xi + c|) \text{ .... } (6)$$

The result is identical with one another in [3], the peakon expressed by (6) are showed in Fig. b (under someparameter conditions c = 1).

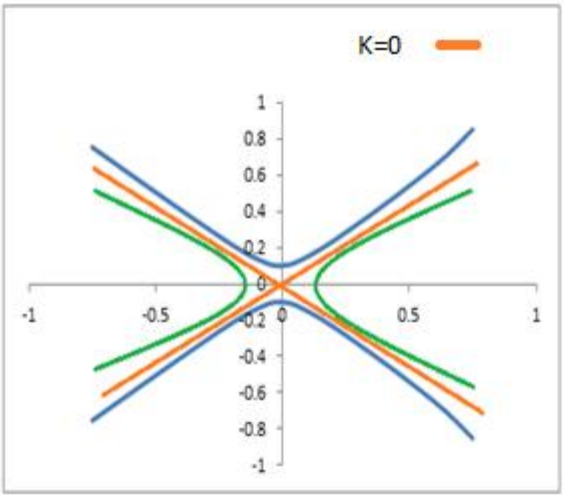

Figure (a)

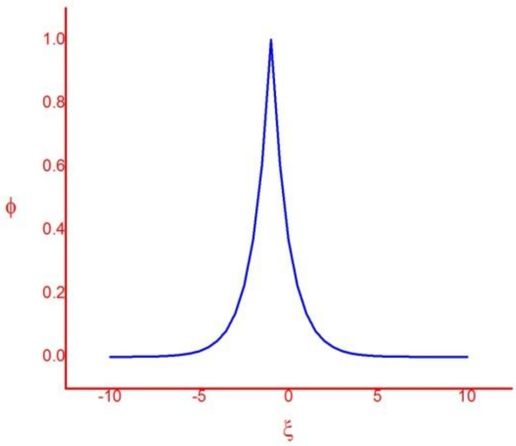

Figure (b): The Profile of the Peakon

3. Conclusion

In this paper we presented the process to find exact solutions for the DegasperisProcesi equation. An analytic method was applied for contructing the solutions of DP equation. The analytic solutions of peakons governing the travelling wave was obtained. By means of transformation of independent variables, a peaked soliton was obtained, which agreed well with exact solutions.

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