# N-Dimensional Plane Gravitational Waves with Background Metric

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Abstract: In this paper, 
$$Z = \left(\frac{x^1 + x^2 + x^3 + \dots + x^{n-1}}{\sqrt{n-1}} - t\right)$$
 and  $Z = \left(\frac{\sqrt{n-1} t}{x^1 + x^2 + x^3 + \dots + x^{n-1}}\right)$  type n-

dimensional plane gravitational waveis studied for Vacuum space and Bulk viscous fluid in Rosen'sbimetric theory of relativity. It is shown that there is nil contributionfrom cosmological constant and Bulkviscousfluid inthis theory. Only vacuum model can be obtained.

Keywords: Plane gravitational waves, Vacuum space, Bulk viscous fluid, BimetricRelativity

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#### 1. Introduction

In recent years there have been a lot of research interests in modified theories of gravity because of their success in explaining a lot of observational data. In this context, Rosen's bimetric theory of gravitation [1] [2] has attracted much attention. The motivation behind this proposition of this bimetric theory is to avoid the problem of singularity occurring in Einstein's general relativity. The two metric tensors assumed in this theory are the Riemannian metric tensor  $g_{ij}$  which interacts with matter and a flat background metric  $\gamma_{ij}$  that describes the inertial forces. With the flat background metric  $\gamma_{ij}$  the physical content of the theory is the same as that of the general relativity.

Even though the large scale universe is homogeneous, some fluctuations are essential to trigger galaxy formation. At some stage small initial perturbations must have evolved into gravitationally bound systems. These fluctuations cause some local in homogeneity. If these metric fluctuations have a scale-dependent random phasecharacter, then amplitude is within the range  $10^{-4} - 10^{-5}$ . Most of the cosmological models are based on the assumption of high degree of isotropy and homogeneity of the universe which can be easily described by the Freidman-Robertson-Walker (FRW)models. In order to take account of local fluctuations leading to inhomogeneity in the space-time which may be responsible for galaxy formation, it is wise to think of some anisotropic and inhomogeneous models to understand the evolution of the early phase of universe. There are good many works on inhomogeneous models. P. K. Sahoo [16] has investigated inhomogeneous plane symmetric string cosmological models in biometric theory. In this present work, we have investigated the existence of plane gravitational wave with vacuum space and also with bulk viscous fluid in bimetric relativity. Therefore, we consider two line elements in modified theory of bimetric relativity are -

$$ds^2 = g_{ij} dx^i dx^j \tag{1.1}$$

And

$$d\sigma^2 = \gamma_{ii} dx^i dx^j \tag{1.2}$$

Where ds is the interval between two neighboring events as measured by means of a clock and a measuring rod. The interval  $d\sigma$  is an abstract or geometrical quantity not directly measurable. One can regard it as describing the geometry that would exist if no matter were present. H Takeno (1961) [3] propounded a rigorous discussion of plane gravitational waves, defined various terms by formulating a meaningful mathematical version and obtained numerous results. Using definition of plane wave, we will use here,

$$Z = \left(\frac{x^{1} + x^{2} + x^{3} + \dots + x^{n-1}}{\sqrt{n-1}} - t\right) \text{ and}$$
$$Z = \left(\frac{\sqrt{n-1} t}{x^{1} + x^{2} + x^{3} + \dots + x^{n-1}}\right) \text{ type plane}$$

gravitational waves by using the line elements,

$$ds^{2} = -A\left(dx^{1^{2}} + dx^{2^{2}} + dx^{3^{2}}\right) - C\sum_{i=4}^{n-1} dx^{i^{2}} + Adt^{2}(1.3)$$

In general relativity, plane and plane fronted gravitational waves have been studied by number of authors Peres [6] (1959), Takeno H. [3][ (1961), Zakhanov [9](1973) etc. obtained the plane fronted wave solutions in Peres spacetime. The theory of plane gravitational waves have been studied by many investigators, H Takeno [4][5][7];Lal and Ali[8]; Pandey et.al [10][12]; Lal and Shafiullah [11]; Hogan, P.A.[13]; Lal K.B and Pandey S.N[14];Rane. R.S and Katore S. D (2009)[17];Bhoyar S.R and Deshmukh A.G [15][18][19] ;Deo and Ronghe[20],[21],;Deo and Suple [22][23]and they obtained the various solutions . In this paper, we will study

$$Z = \left(\frac{x^{1} + x^{2} + x^{3} + \dots + x^{n-1}}{\sqrt{n-1}} - t\right) \text{ and}$$
$$Z = \left(\frac{\sqrt{n-1} t}{x^{1} + x^{2} + x^{3} + \dots + x^{n-1}}\right) \text{ type plane}$$

gravitational waves for vacuum space and with Bulk viscous

Volume 4 Issue 4, April 2015 <u>www.ijsr.net</u> Licensed Under Creative Commons Attribution CC BY fluid and will observe the result in the context of Bimetric theory of relativity.

#### **Field Equations in Biometric Relativity**

Rosen N. has proposed the field equations of Bimetric Relativity from variation principle as

$$K_{i}^{j} = N_{i}^{j} - \frac{1}{2}Ng_{i}^{j} = -8\pi\kappa T_{i}^{j} \qquad (2.1)$$

Where 
$$N_i^{\ j} = \frac{1}{2} \gamma^{\alpha\beta} \left[ g^{\ hj} g_{\ hi|\alpha} \right]_{|\beta}$$
 (2.2)

$$N = N^{\alpha}_{\alpha}, \ \kappa = \sqrt{\frac{g}{\gamma}}$$
 (2.3)

and 
$$g = \det g_{ij}$$
,  $\gamma = \det \gamma_{ij}$  (2.4)

Where a vertical bar (|) denotes a covariant differentiation with respect to  $\gamma_{ii}$ .

3. 
$$Z = \left(\frac{x^1 + x^2 + x^3 + \dots + x^{n-1}}{\sqrt{n-1}} - t\right)$$
 type

## plane gravitational wave with

Vacuum space

ForZ = 
$$\left(\frac{x^1 + x^2 + x^3 + \dots + x^{n-1}}{\sqrt{n-1}} - t\right)$$
 plane

gravitational waves, we have the line element as

$$ds^{2} = -A\left(dx^{1^{2}} + dx^{2^{2}} + dx^{3^{2}}\right) - C\sum_{i=4}^{n-1} dx^{i^{2}} + Adt^{2}$$
(3.1)

where A = A(Z) C = C (Z) and

$$Z = \left(\frac{x^{1} + x^{2} + x^{3} + \dots + x^{n-1}}{\sqrt{n-1}} - t\right)$$

Corresponding to the equation (3.1), we consider the line element for background metric  $\gamma_{ii}$  as

$$d\sigma^{2} = -\left(dx^{1^{2}} + dx^{2^{2}} + dx^{3^{2}} + \dots + dx^{(n-1)^{2}}\right)$$
  
+  $dt^{2}$  (3.2).

Since  $\gamma_{ij}$  is Lorentz metric (-1,-1,-1,-1,-1,-1,1), hence

 $\gamma$  – covariant derivative becomes the ordinary partial derivative. For empty space, field equations in bimetric relativity as analogue to General relativity and we assume the form

$$N_i^j = 0 \tag{3.3}$$

where  $N_i^j$  is already defined in (2.2)

Using equations (2.1) to (2.4) with (3.1) and (3.3), We get the field equations as

$$N_1^1 = N_2^2 = N_3^3 = N_4^4 = \dots = N_n^n = 0$$
 (3.4)

Thus the field equations (3.3) are identically satisfied. It implies that

$$Z = \left(\frac{x^{1} + x^{2} + x^{3} + \dots + x^{n-1}}{\sqrt{n-1}} - t\right)$$

type N-dimentional plane gravitational wave exist in bimetric relativity. With the introduction of cosmological constant  $\lambda$ , the field equations of empty space-time in bimetric relativity assume the form

$$N_i^j = \lambda g_i^j \tag{3.5}$$

Using field equation (3.4) in (3.5) We get

Equation (3.6) immediately implies that there is no contribution of cosmological constant  $\lambda$  in

 $\lambda = 0$ 

Z = 
$$\left(\frac{x^{1} + x^{2} + x^{3} + \dots + x^{n-1}}{\sqrt{n-1}} - t\right)$$
type N-

dimensional plane gravitational wave in Rosen's Bimetric theory of relativity.

4. Z = 
$$\left(\frac{\sqrt{n-1} t}{x^1 + x^2 + x^3 + \dots + x^{n-1}}\right)$$
 type plane

gravitational wave with Vacuum space and Bulk viscous fluid.

#### **Case I: Vacuum Space**

For Z = 
$$\left(\frac{\sqrt{n-1} t}{x^1 + x^2 + x^3 + \dots + x^{n-1}}\right)$$

plane gravitational waves, we have the line element as defined in (3.1) and (3.2).

For empty space time, field equations in biometric relativity assume the form (3.3) and

We get the field equations as

$$N_{1}^{1} = N_{2}^{2} = N_{3}^{3} = \left(\frac{n-1}{2}\right) D\left(\frac{\overline{A}^{2}}{A^{2}} - \frac{\overline{A}}{A}\right) = 0$$

$$N_{4}^{4} = N_{5}^{5} = \dots = N_{n-1}^{n-1}$$
(4.1)

$$= \left(\frac{n-1}{2}\right) D\left(\frac{\overline{C}^2}{C^2} - \frac{\overline{C}}{C}\right) = 0$$
(4.2)

$$N_n^n = \left(\frac{n-1}{2}\right) D\left(\frac{\overline{A}^2}{A^2} - \frac{\overline{A}}{A}\right) = 0$$
(4.3)

Where

$$D = \left[\frac{(n-1)t^2 - \left(x^1 + x^2 + \dots + x^{n-1}\right)^2}{\left(x^1 + x^2 + \dots + x^{n-1}\right)^4}\right]$$
(4.4)

And

$$\overline{A} = \frac{\partial A}{\partial Z} \quad , \quad \overline{\overline{A}} = \frac{\partial^2 A}{\partial Z^2} \quad ,$$
$$\overline{C} = \frac{\partial C}{\partial Z} \quad , \quad \overline{C} = \frac{\partial^2 C}{\partial Z^2}$$

on solving (4.1)-(4.3), we obtain

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$$\left(\frac{\overline{A}^2}{A^2} - \frac{\overline{A}}{A}\right) = 0$$

and

$$\left(\frac{\overline{C}^2}{C^2} - \frac{\overline{C}}{C}\right) = 0 \tag{4.6}$$

Solving equations (4.5) and (4.6),

we get 
$$A = R_1 e^{S_1 Z}$$
 (4.7)

And

$$C = R_2 e^{S_2 Z} \tag{4.8}$$

where  $R_1$ ,  $S_1$  and  $R_2$ ,  $S_2$  are the constants of integration.

Thus substituting the value of (4.7) and (4.8) in (3.1), we get the vacuum line element as

$$ds^{2} = -R_{1}e^{S_{1}Z}\left(dx^{1^{2}} + dx^{2^{2}} + dx^{3^{2}}\right)$$
$$-R_{2}e^{S_{2}Z}\sum_{i=4}^{n-1}dx^{i^{2}} + R_{1}e^{S_{1}Z}dt^{2}$$
(4.9)

Case II: Bulk viscous fluid

For Z = 
$$\left(\frac{\sqrt{n-1} t}{x^1 + x^2 + x^3 + \dots + x^{n-1}}\right)$$

plane gravitational waves, we have the line element as

defined in (3.1) and (3.2).And,  $T_i^j$  the energy momentum tensor for Bulk viscous fluid is given by

$$T_{i}^{j} = T_{i \ bulk}^{j} = \left(\rho + \overline{p}\right) v_{i} v^{j} + \overline{p} g_{i}^{j} \qquad (4.10)$$

Where  $p = p - \xi v^{j}$ ; *i* together with  $g_{ij}v^{i}v^{j} = -1$  ie  $v_{n}v^{n} = -1$  where  $v_{i}$  are the n-vector representing the velocity of the bulk viscous fluid and

 $\rho$ , p, p, p and  $\xi$  are the energy density, isotropic pressure, effective pressure and bulk viscous coefficient. In co-moving coordinate system we have

$$T_1^1 = T_2^2 = T_3^3 = \dots = T_{n-1}^{n-1} = p,$$
  
$$T_n^n = -\rho \text{ and } T_i^j = 0 \text{ for } i \neq j$$

Using equations (2.1) to (2.4) and (4.10) with (3.1) and (3.2), We get the field equations as

$$\left(\frac{n-1}{2}\right)D\left\{\begin{array}{l} \left(\overline{\frac{A}{A^{2}}}-\overline{\frac{A}{A}}\right)\\ -\frac{(n-4)}{2}\left(\overline{\frac{C}{C^{2}}}-\overline{\frac{C}{C}}\right)\end{array}\right\}$$
(4.11)
$$=-8\pi\kappa\overline{p}$$

$$\left(\frac{n-1}{2}\right)D\left\{\begin{array}{l}2\left(\frac{\overline{A}^{2}}{A^{2}}-\frac{\overline{A}}{A}\right)+\\\frac{(n-6)}{2}\left(\frac{\overline{C}^{2}}{C^{2}}-\frac{\overline{C}}{C}\right)\end{array}\right\}^{(4.12)}\\=-8\pi\kappa\overline{p}\\\left(\frac{n-1}{2}\right)D\left\{\begin{array}{l}\left(\frac{\overline{A}^{2}}{A^{2}}-\frac{\overline{A}}{A}\right)\\-\frac{(n-4)}{2}\left(\frac{\overline{C}^{2}}{C^{2}}-\frac{\overline{C}}{C}\right)\end{array}\right\}^{(4.13)}\\=8\pi\kappa\rho$$

Using equation (4.11) to (4.13),

We get  $p + \rho = 0$  (4.14)

In view of the reality conditions i.e. p > 0,  $\rho > 0$  must hold. Using above conditions ,equation(4.14) is satisfied only when

$$p = \rho = 0 \tag{4.15}$$

Equation (4.15) immediately implies that Bulk viscous fluid does not exist in

$$Z = \left(\frac{\sqrt{n-1} t}{x^{1} + x^{2} + x^{3} + \dots + x^{n-1}}\right)$$

plane gravitational waves in Rosen's Bimetric theory of relativity. Using (4.15), the field equations (4.11) to (4.13) reduces to the vacuum solutions

$$\left(\frac{\overline{A}^2}{A^2} - \frac{\overline{A}}{A}\right) = 0 \tag{4.16}$$

and

$$\left(\frac{\overline{C}^2}{C^2} - \frac{\overline{C}}{C}\right) = 0 \tag{4.17}$$

Solving equations (4.16) and (4.17), we get the same solution defined in (4.9).

Thus, it is found that in plane gravitational wave Z = 
$$\left(\frac{\sqrt{n-1} t}{x^1 + x^2 + x^3 + \dots + x^{n-1}}\right)$$
, Bulk viscous

fluid does not survive in Bimetric theory of relativity and only vacuum model can be constructed.

This study can further be extended with the introduction of cosmological constant  $\lambda$  in the field equation which is defined

$$_{\rm as} N_i^j = \lambda g_i^j$$

Thus we get

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plane

$$\left(\frac{\overline{A}^{2}}{A^{2}} - \frac{\overline{A}}{A}\right) D = \lambda$$

And 
$$\left(\frac{\overline{C}^2}{C^2} - \frac{\overline{C}}{C}\right)D = \lambda$$
 (4.19)

On solving equation (4.17) we have

$$A = \exp\left[D'\lambda Z^2 + EZ + F\right] \quad (4.20)$$

Where 
$$D' = \frac{1}{D} = \left[ \frac{\left(x^1 + x^2 + \dots + x^{n-1}\right)^4}{(n-1)t^2 - \left(x^1 + x^2 + \dots + x^{n-1}\right)^2} \right]$$

On solving (4.18) we obtain

$$C = \exp\left[D'\lambda Z^2 + GZ + H\right]_{(4.21)}$$

Where E, F, G and H are constants of integration.

Thus substituting the value of A and C [using (4.20)-(4.21)], the line element (3.1) reduces to

$$ds^{2} = -\exp\left[D'\lambda Z^{2} + EZ + F\right] \left(dx^{1^{2}} + dx^{2^{2}} + dx^{3^{2}}\right)$$
$$-\exp\left[D'\lambda Z^{2} + GZ + H\right] \sum_{i=4}^{n-4} dx^{i^{2}}$$
(4.22)

$$+\exp\left[D^{2}\lambda Z^{2} + EZ + F\right]dt^{2}$$
Thus  $Z = \left(\frac{\sqrt{n-1} t}{x^{1} + x^{2} + x^{3} + \dots + x^{n-1}}\right)$ 

gravitational wave exists in Bimetric relativity with or without cosmological constant  $\lambda$  respectively.

## 4. Conclusion

In the study of

$$Z = \left(\frac{x^1 + x^2 + x^3 + \dots + x^{n-1}}{\sqrt{n-1}} - t\right)$$
  
N dimensional plane gravitational wave there

is nil N-dimensional plane gravitational wave, there contribution from cosmological constant  $\lambda$  and in

$$\mathbf{Z} = \left(\frac{\sqrt{n-1} \ t}{x^1 + x^2 + x^3 + \dots + x^{n-1}}\right) \text{ type N-}$$

dimensional plane gravitational wave, there is nil contribution from the matter Bulk viscous fluid in Bimetric theory of relativity respectively. It is observed that the matter Bulk viscous fluid cannot be a source of gravitational field in the Rosen's bimetric theory but only vacuum model exists. Also we observed that the wave exists in this theory with or without cosmological constant  $\lambda$  respectively.

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