An Advanced Method for Solving Fuzzy Transportation Problem with Minimum Cost Using Robust Ranking Method (RRT)

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Abstract: ***The aim of Fuzzy Transportation is to find the least transportation cost of some commodities through a capacitated network when the supply and demand of nodes and the capacity and cost of edges are represented as Fuzzy numbers. This paper deals about the Fuzzy transportation problem, where the transportation cost, supply and demand quantities are Fuzzy quantities. The objective of Fuzzy transportation problem is to determine the shipping schedule that minimizes the total transportation cost while satisfying Fuzzy supply and Fuzzy demand limits. Using Robust Ranking Method Fuzzy quantities are transformed into Crisp quantities. To obtain the Optimal Solution a new algorithm LOCFTP is proposed. Finally a Numerical illustration is given to check the validity of the proposal.***

Keywords: Fuzzy Transportation, Robust Ranking, Fuzzy numbers

1. Introduction

In today's highly competitive market, the pressure on organizations to find better ways to create and deliver value to customers becomes stronger. How and when to send the products to the customers in the quantities, they want in a cost-effective manner, become more challenging. Transportation models provide a powerful framework to meet this challenge. Transportation problem is a particular class of Linear Programming Problem, which is associated with routine activities in our real life and mainly deals with logistics. It helps in solving problems on distribution and transportation of resources from one place to another. The goods are transported from a set of sources to a set of destinations to meet the specific requirements. In other words, transportation problems deal with the transportation of a single product manufactured at different plants (origins) to a number of different warehouses (destinations). The objective is to satisfy the demand at destinations from the supply constraints at the minimum possible transportation cost. To reach this objective, we must know the quantity of available supplies and demand.

In this paper we shall study fuzzy transportation problem, and we introduce an approach for solving a wide range of such problem by using a method which apply it for ranking of the fuzzy numbers. Some of the quantities in a fuzzy transportation problem may be fuzzy or crisp quantities. In many fuzzy decision problems, the quantities are represented in terms of fuzzy numbers. Fuzzy numbers may be normal or abnormal, triangular or trapezoidal or any LR fuzzy number. Thus, some fuzzy numbers are not directly comparable. First, we transform the fuzzy quantities as the cost, coefficients, supply and demands, into crisp quantities by Robust ranking method which satisfies the properties of compensation, linearity and additivity, and then by using the classical algorithms we solve and obtain the solution of the problem. The new method (LOCFTP) is a systematic procedure, easy to apply and can be utilized for all types of transportation problem whether maximize or minimize objective function.

2. Preliminaries

2.1 Definition

A fuzzy set is characterized by a membership function mapping elements of a domain, space, or the universe of discourse X to the unit interval [0, 1] (i.e.) $A = \{x, \mu_A(x), x \in X\}$. Here $\mu_A: X \to [0,1]$ is a mapping called the degree of membership function of the Fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A. These membership grades are often represented by real numbers ranking from [0, 1].

2.2 Definition

A fuzzy set A is convex if and only if, for any $x_1, x_2 \in X$, the membership function of A satisfies the inequality $\mu_A\{\lambda x_1 + (1-\lambda)x_2\} \geq \min(\mu_A(x_1), \mu_A(x_2)), 0 \leq \lambda \leq 1$

2.3 Definition

For a triangular fuzzy number A(x), it can be represented by A (m,n,a;1) with membership function μ(x) is given by

$$\mu(x) = \begin{cases} \dfrac{x-m}{n-m}, & m \leq x \leq n \\ 1, & x = n \\ \dfrac{a-x}{a-n}, & n \leq x \leq a \\ 0, & Otherwise \end{cases}$$

2.4 Definition

The α-cut of a fuzzy number is defined as , $A(\alpha) = \{x: \mu(x) \geq \alpha, \alpha \in [0,1]\}$

2.5 Definition

Addition and Subtraction of two triangular fuzzy numbers can be performed as
$(a_1,b_1, c_1) + (a_2,b_2, c_2) = (a_1 +a_2, b_1 + b_2, c_1 + c_2)$
$(a_1,b_1, c_1) - (a_2,b_2, c_2) = (a_1 - a_2, b_1 - b_2, c_1- c_2)$

Addition and Subtraction of two trapezoidal fuzzy numbers can be performed as
$(a_1, b_1, c_1,d_1) + (a_2,b_2, c_2, d_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2,d_1 + d_2)$
$(a_1, b_1, c_1,d_1) - (a_2,b_2, c_2, d_2) = (a_1 - a_2, b_1 - b_2, c_1 - c_2,d_1 - d_2)$

3. Robust Ranking Technique – Algorithm

Using robust ranking technique fuzzy numbers can be converted into crisp ones. Robust ranking technique which satisfies compensation, linearity, and additive properties and provides results which are consistent with human intuition. Give a convex fuzzy number ã , the Robust Ranking Index is defined by

$$R(\tilde{a}) = \int_0^1 (0.5)(a_\alpha^L, a_\alpha^U)\,d\alpha$$

Where (a_α^L, a_α^U) is the α- level cut of the fuzzy number ã and (a_α^L, a_α^U) = {(n-m) α + m), (a- (a - n) α)}.In this paper the above mentioned method is used for ranking the fuzzy numbers. The Robust ranking index R(ã) gives the representative value of the fuzzy number ã . It satisfies the linearity and additive property.

4. Fuzzy Transportation Problem

The fuzzy transportation problems, in which a decision maker is uncertain about the precise values of transportation cost, availability and demand, can be formulated as follows

Minimize $z=\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$,

Subject to, $\sum_{j=1}^{n} x_{ij} = a_i$, i=1,2,3,…..m

$\sum_{i=1}^{m} x_{ij} = b_j$, j=1,2,3,………….n

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j, i = 1,2,3,.....m \text{ and } j = 1,2,3,......n \text{ and } x_{ij} \geq 0$$

Where m= total number of sources
n=total number of destinations
a_i=the fuzzy availability of the product at i^th source
b_j= the fuzzy demand of the product at j^th destination
c_{ij}= the fuzzy transportation cost for unit quantity of the product from i^th source to j^th destination
x_{ij}=the fuzzy quantity of the product that should be transported from ith source to j^th destination to minimize the total fuzzy transportation cost

$\sum_{i=1}^{m} a_i$ = total fuzzy availability of the product

$\sum_{j=1}^{n} b_j$ = total fuzzy demand of the product

$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$ = total fuzzy transportation cost

If $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ then the fuzzy transportation problem is said to be balanced fuzzy transportation problem, otherwise it is called unbalanced fuzzy transportation problem.This problem can be represented as follows:

Table 1

Origins/Destinations	D_1	D_2	……	D_n	Supply
O_1	C_{11}	C_{12}	……	C_{1n}	a_1
O_2	C_{21}	C_{22}	……	C_{2n}	a_2
: :	: :	: :	: :	: :	: :
O_m	C_{m1}	C_{m2}	……	C_{mn}	a_m
Demand	b_1	b_2	……	b_n	

5. Methodology of LOCFTP Method

Step 1: Construct the transportation problem from fuzzy transportation problem. Suppose the values of transportation problem are not integers, round off into integers.
Step 2: Select the minimum odd cost from all cost in the matrix. Suppose all the costs are even,
Multiply by 1/2 each column.
Step 3: Subtract selected least odd cost only from odd cost in the matrix. Now there will be at least one zero and remaining all cost become even.
Step 4: Allocate this minimum of supply / demand at the place of zero.
Step 5: After the allotment, multiply by 1/2 each row.
Step 6: Again select minimum odd cost in the remaining column except zeros in that column.
Step 7: Go to step III and repeat step IV and V till optimal solution are obtained.
Step8: Finally total minimum cost is calculated as sum of the product of the cost and corresponding allocated value of supply/demand

Total cost= $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$

6. Numerical Example

A factory has three origins O_1,O_2,O_3 four destinations D_1 ,D_2, D_3. The fuzzy transportation cost for unit quantity of the product from i^th Origin to j^th destination is C_{ij} where

Table 2

Origins/Destinations	D1	D2	D3
O1	(7,12,9,14)	(5,12,15,22)	(9,18,20,29)
O2	(12,14,17,19)	(9,15,24,30)	(2,5,21,24)
O3	(7,8,9,10)	(4,9,14,19)	(3,5,7,9)

Fuzzy availability of the product at origins are (20,50,80,110), (50,70,90,110), (30,50,70,90) and the fuzzy demand of the product at destinations are (10,30,90,110), (30,60,70,100), (160,70,90,100) respectively. The fuzzy transportation problem is given by

Table 3

Origins/Destinations	D1	D2	D3	Fuzzy Supply
O1	(7,12,9,14)	(5,12,15,22)	(9,18,20,29)	(20,50,80,110)
O2	(12,14,17,19)	(9,15,24,30)	(2,5,21,24)	(50,70,90,110)
O3	(7,8,9,10)	(4,9,14,19)	(3,5,7,9)	(30,50,70,90)
Fuzzy Demand	(10,30,90,110)	(30,60,70,100)	(60,70,90,100)	

In conformation to model the fuzzy transportation problem can be formulated as:

Min
Z=R(7,12,9,14)x_{11}+R(5,12,15,22)x_{12}+R(9,18,20,29)x_{13}+R(12,14,17,19)x_{21}+R(9,15,24,30)x_{22}+
R(2,5,21,24)x_{23}
+R(7,8,9,10)x_{31}+R(4,9,14,19)x_{32}+R(3,5,7,9)x_{33}

Step1: Applying Robust ranking method, we get

R (ã) = $\int_0^1 (0.5)(a_\alpha^L, a_\alpha^U)d\alpha$ where $\left(a_\alpha^L, a_\alpha^U\right)$= {(n-m) α + m), (b- (b -a) α)}

R(7,12,9,14) =

$$\int_0^1 (0.5)(5\alpha+7\ ,14-5a)d\alpha \quad = \int_0^1 (0.5)(21)d\alpha \quad = 10.5 = 11$$

Similarly we get, R(5,12,15,22)=13.5=14 , R(9,18,20,29)=19 , R(12,14,17,19)=15.5=16 , R(9,15,24,30)=19.5=20 , R(2,5,21,24)=13 , R(7,8,9,10)=8.5=9 , R(4,9,14,19)=11.5=12 , R(3,5,7,9)=6 , Rank of all Supplies R(20,50,80,110)=65 , R(50,70,90,110)=80 , R(30,50,70,90)=60,Rank of all Demands R(10,30,90,110)=60 , R(30,60,70,100)=65 , R(60,70,90,100)=80
Substitute these values in fuzzy transportation problem we get the crisp transportation problem as follows

Table 4

Origins/Destinations	D_1	D_2	D_3	*Fuzzy Supply*
O_1	11	14	19	65
O_2	16	20	13	80
O_3	9	12	6	60
Fuzzy Demand	60	65	80	205

Step 2: Here the minimum odd cost is 9.
Step 3: Subtract 9 from all odd cost, which is shown below.

Table 5

Origins/Destinations	D_1	D_2	D_3	*Fuzzy Supply*
O_1	2	14	10	65
O_2	16	20	4	80
O_3	0	12	6	60
Fuzzy Demand	60	65	80	205

Allocate the cell(O_3, D_1), Min(60,60)=60, we get x_{31}=60 and delete column D_1 and O_3 which is shown below

Table 6

Origins/Destinations	D_2	D_3	*Fuzzy Supply*
O_1	14	10	65
O_2	20	4	80
Fuzzy Demand	65	80	

Step IV: Multiply by 1/2 each column which is shown below

Table 7

Origins/Destinations	D_2	D_3	*Fuzzy Supply*
O_1	7	5	65
O_2	10	2	80
Fuzzy Demand	65	80	

Now the minimum odd cost is 5.Go to step III and repeat step IV and V Until and unless all the demands are satisfied and all the supplies are exhausted.

Table 8

Origins/Destinations	D_1	D_2	D_3	*Fuzzy Supply*
O_1	11	14	19 [65]	65
O_2	16	20 [65]	15 [15]	80
O_3	9 [60]	12	6	60
Fuzzy Demand	60	65	85	205

The minimum transportation cost associated with this solution is
Z = (65x19)+(65x20)+(15x13)+(60x9)
= Rs.3270

7. Conclusion

In this paper, the fuzzy numbers are used in transportation problem as a cost. Moreover, the fuzzy transportation problem of trapezoidal numbers has been transformed into crisp transportation problem using Robust's ranking indices. A simple LOCFTP algorithm has been developed to find the optimal solution in transportation problem. A numerical example shows that the comparison of VAM and LOCFTP, by using LOCFTP method we have determined that the total minimum cost obtained is optimal. So it will be very helpful for decision makers. Moreover, this technique can also be used in solving other types of problems like, project schedules, assignment problems and so on.

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