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R-Restricted Steiner Problem is NP-Complete

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Abstract: ***This work aims to arrive a Steiner minimum tree with*** R ***-terminals. Every full component of a Steiner tree contains almost 4 terminals. In this work, we propose to flash R-restricted Steiner problem which is NP-complete.***

Keywords: Steiner minimum tree, NP-complete, R-restricted Steiner tree, satisfiability.

1. Introduction

A number of optimization problems in different application areas can be modeled by the Steiner tree problem in graphs. Given a connected, undirected distance graph with two types of vertices the required vertices and the Steiner vertices find a shortest connected sub graph that contains all vertices. This sub graph is called a Steiner minimum tree; it need not contain Steiner vertices, but may contain some of them to shorten the total length of the tree. In the Steiner minimum tree problem, the vertices are divided into two parts, terminals and non terminal vertices. The terminals are the given vertices which must be included in the solution. In this paper, planned to discuss the R-restricted 3SAT is NP-complete. First to prove that Steiner problem stays NP-complete. Using reducibility we will prove that R-restricted Steiner problem is NP-complete

2. Preliminaries

Definition 2.1

Let a connected graph $G=(V.E)$ and a set $K \subseteq V$ of terminals. Then the Steiner minimum tree for K in G that is Steiner tree τ for K such that

$$|E(T)| = \min \left\{ E(T^1) / T^1 is\, a\, steiner tree\ for\ K\ in G \right\}$$

In the Steiner minimum tree problem, the vertices are divided into two parts, terminals and non terminal vertices. The terminals are the given vertices which must be included in the solution.

Example 2.1

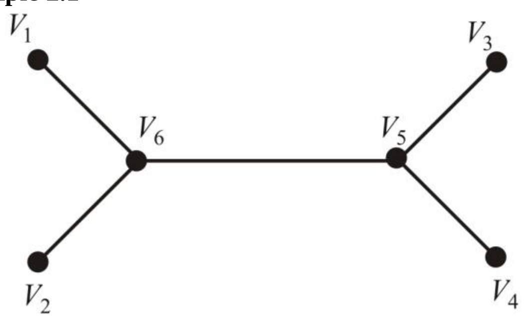


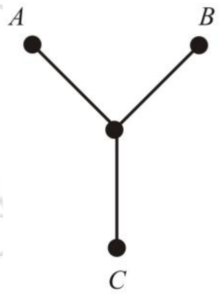
Figure 1: Steiner minimal tree

V_1, V_2, V_3, V_4 are terminals V_5 and V_6 are non-terminals.

Definition 2.2

A Steiner tree is a tree in a distance graph which spans a given subset of vertices. (Steiner point with the minimum total distance on its edges)

Example 2.2



Steiner tree for three points [Note there is no direct connection between A, B and C].

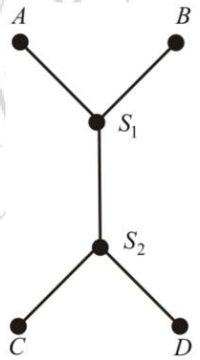


Fig.2 Steiner Tree

Solution for four points [Note that there are two Steiner points S_1 and S_2]

Definition 2.3
A formula is satisfiable if there is an assignment of truth values to the variables that makes every clause tree

Definition 2.4
A class of problems solvable by non deterministic polynomial time algorithm is called NP.

Definition 2.5
A problem is NP-complete if
1. It is an element of the class NP.

2. Another NP-complete problem is polynomial time reducible to it.

Definition 2.6

A 3SAT formula F such that every variable occurs in at most 3-clauses. Moreover, it is required that each variable appears both negated and un negated such that there exists a satisfying assignment for F.

A Steiner minimum tree for K is given such that some of the terminals are interior points. Then we can decompose this tree (by splitting terminals) into components so that terminals only occur as leaves of these components. Such a component is called a full component. Figure 3 illustrates the decomposition of a Steiner tree into full components.

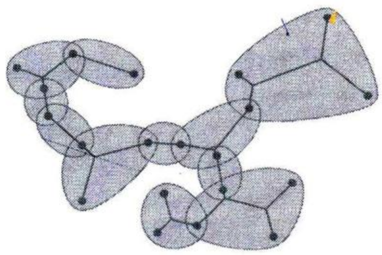


Figure 3: Full components of a Steiner Tree

Definition 2.7

Clause gadget consisting of a vertex C_i that is connected to the literals contained in the clause C_i by paths of length $t=2n+1.$ as terminal set we choose $k=\{y,y_6\}\cup\{C_1...C_4\}$ and set $B=2n+t.m.$

In Fig 4 the clause gadget for the clause $C_1=x_1\vee\overline{x_2}\vee x_3$. The dashed lines indicated paths of length $t=2n+1$ from C_i to the appropriate vertices on the variable path.

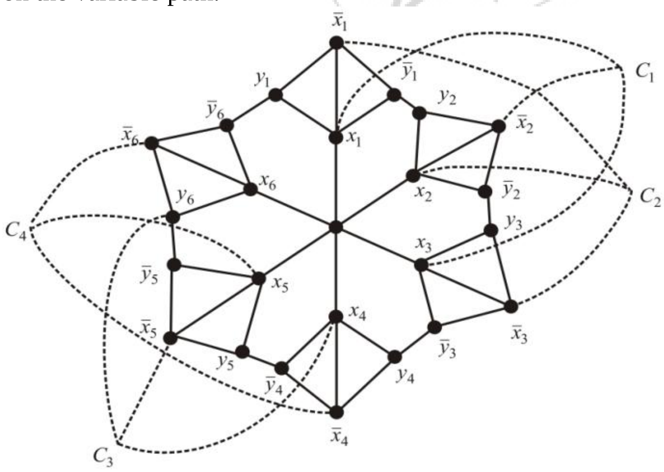


Figure 4

3. R-Restricted Steiner Problem is NP-Complete

Result 1

For every $r\geq 4,$ R-Restricted Steiner problem is NP-complete.

Proof

First we have to prove that Steiner problem in Graphs is NP-Complete.

Applying 3 SAT Steiner problems in Graphs.

Let $x_1,x_2,...x_n$ be the variables and $c_1,...c_m$ the clause in an arbitrary instance of 3 SAT.

Our aim is to construct a graph $G=(V,E),$ a terminal set K, and a bound B such that G contains a Steiner Tree T for K of size at most B if and only if the given 3 SAT instance is satisfiable.

Construct the graph as follows in fig 3.1.

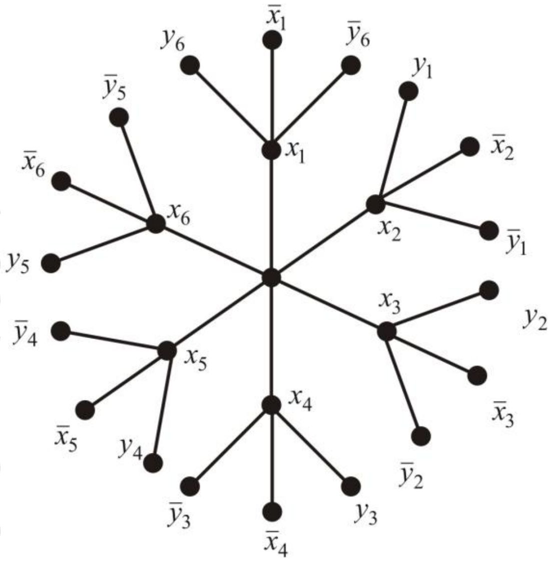


Figure 3.1

$x_1,x_2,...x_6$ and $y_1,y_2,...y_6$ are variables.

First we connect two vertices y_6 and y_1 by variable path as in the figure.3.2.

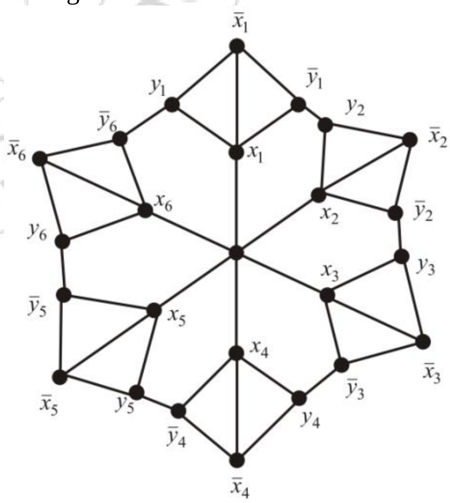


Figure 3.2

Figure 3.2. Transforming 3 SAT to Steiner Problem in graph with the variable path.

Clause gadget consisting of a vertex C_i that is connected to the literals contained in the clause C_i by paths of length $t=2n+1.$ as terminal set we choose $k=\{y,y_6\}\cup\{C_1...C_4\}$ and set $B=2n+t.m.$

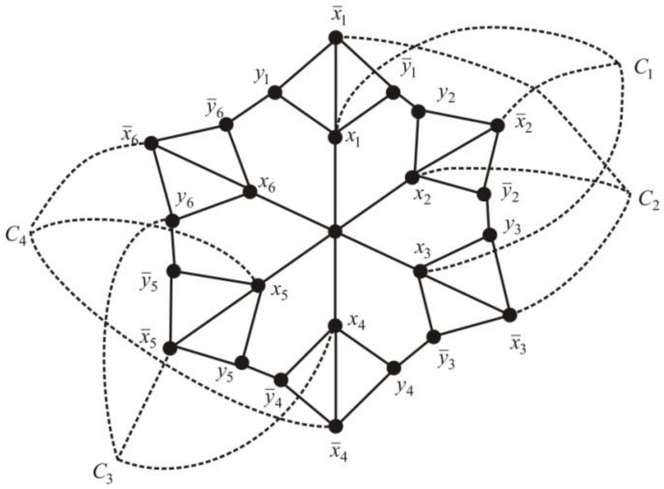


Figure 3.3

In Fig 3.3 the clause gadget for the clause $C_1 = x_1 \vee \overline{x_2} \vee x_3$. The dashed lines indicated paths of length $t = 2n+1$ from C_i to the appropriate vertices on the variable path.

First assume that the 3 SAT instance is satisfiable. To construct a Steiner tree for K we start with a $y_1 - y_6$ path P reflecting a satisfying assignment. That is let $x_i \in P$ if x_i is set to true in this assignment, and $\overline{x_i} \in P$ otherwise. Next observe that for every clause the vertex C_i can be connected to P by path of Length t. In this way we obtain a Steiner tree for K of Length $2n + t.m = B.$

$t = 2n+1$

$n = 6 \quad m = 4. \quad t = 2(6)+1 \quad = 13$

$2(6) + 13.4 \quad = B$

$B = 12 + 52$

$B = 64.$

On the other hand, assume now that T is a Steiner tree for K of Length at most $B,$ Trivially for each clause the vertex C_i has to be connected to the variable path.

Assume for the moment that there exists a clause C_{io} connected to the variable path by at least two of its paths.

Then $|E(T)| \geq (m+1).t > B,$

By figure 3.3 $\geq (4+1).13 > B$

$\geq 5.13 > B$

$\geq 65 > 64$

This show that $y_1 - y_6$ can only be connected along the variable path, we conclude that the $y_1 - y_6$ path contains exactly 2n edges and that each clause gadget is connected to this path using exactly t edges. Thus $y_1 - y_6$ path reflects a satisfying assignment.

In the figure 3.4, we can split the graph into components.

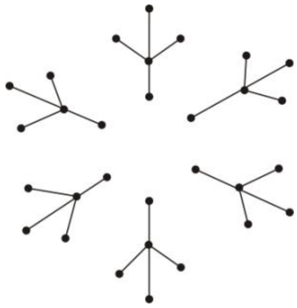


Figure 3.4

This implies that every full component of a graph contains at most four terminals. Hence

Result 1

R-Restricted Steiner Problem is NP-complete.

Result 2

A Steiner minimum tree for the new terminal set, in which every full component contains at most 4 terminals.

4. Conclusion

In this work we conclude that, in a Steiner minimum tree with R-terminals with every full component contains at most 4 terminals are R-Restricted Steiner problem which is NP Complete.

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