Nuclear Shell Model Application to Calculate Energy Levels for nucleus ${}^{18}_{9}F_9$ by using Wildenthal Interaction

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Abstract: Energy Levels were calculated by applying nuclear shell model and using Wildenthal Interaction(USD) of Nucleus ${}^{18}_{9}F_9$ which contain two nucleons(one proton and the other one neutron) both of them outside close core ${}^{16}_{8}O_8$ and occupy the Space Model $(0d_{5/2,}1S_{1/2},0d_{3/2})$,the predicted energy levels of nucleus above are Compared with available experimental data, A reasonable agreement were obtained from these comparisons.

1. Introduction

The nuclear shell model is the basic theoretical tool for the microscopic description of nuclear structure. As well known ,this model is based on the assumption that ,as a first approximation ,to each nucleon inside the nucleus moves independently from the others in a spherically symmetric potential including a strong spin-orbit term .With this approximation the nucleus is considered as an inert core ,made up by shells filled up with neutrons and protons paired to angular momentum J=0 ,plus a certain number of external nucleons is called the "valence nucleons". As well known ,this extreme single –particle shell model, supplemented by empirical couple rules ,proved very soon to be to account for various nuclear properties [1,2]

2. Theory

The basic requirements for shell model calculation are set of single-particle energies and two-body interaction matrix elements and recently these set call effectives interaction [3],the nuclear shell model is based on the following assumptions[4]:-

1) The inner nucleons fill three closed shells($0S_{1/2,}0P_{1/2},0P_{3/2}$)to form an inert core with the spin J = 0, isospin T = 0,The extra-core nucleons move independently in the available orbitals $(0d_{5/2,}1S_{1/2},0d_{3/2})$ of the central spherical potential field supplied by the core.
2) The strong spin-orbit interaction splits each (j) level into $(j=l+1/2)$ and $(j=l-1/2)$ single-particle level,where $(\vec{j}=\vec{l}+\vec{s})$ The single-particle level (j_1) Is Lower in energy than (j_2)
3) The mutual interaction can be expressed as the sum of two-body interactions.

$$H = H_0 + H_n \ldots\ldots\ldots(1)$$

Where H_0 denotes the core part and Hn the contribution of the nucleons outside the core .Only the latter part will affect the relative energy eigen values of the nucleus. This is due to the interaction of the extra-core nucleons, Hobson Wildenthal obtained a universal model independent two –body interaction call USD interaction or W- interaction for the 0s0d shell by fitting single energies and two –body matrix element to 447 energies in the sd –shell.

The USD interaction Hamiltonian is defined by a set of 63 numbers given for the sd –shell two body matrix elements and three numbers for $0d_{5/2}$ $0d_{3/2}$ $1s_{1/2}$ single particle energies, the USD interaction has been the standard interaction for sd-shell and has been in interpretation of spectroscopic properties of the nuclei from A=18 to A=38. We employ a mass dependence of the two- body matrix elements of the from [5,6,7,8]:-

$$V_{JT}\langle\ ab,cd\ \rangle^{A} = V_{JT}\langle\ ab,cd\ \rangle^{A=18} \times (A/18)^{P} \ldots.(2)$$

Where A is mass number andP is equal 0.3

An appropriate expression of the shell-model Hamiltonian is given as the sum of one- and two-body operators[6]

$$H = \sum_{a} \varepsilon_a \hat{n}_a + \sum_{a\le b}\sum_{c\le d} V_{JT}\langle\ ab;cd\ \rangle \hat{T}_{JT}(ab;cd)\ldots\ldots(3)$$

Where ε_a are the single-particle energies, $\hat{n}_a$ is the number operator for the spherical orbit (a)with quantum number (n_a, l_a, j_a), $V_{JT}(ab;cd)$ is a two-body matrix element, and $\hat{T}_{JT}(ab;cd) = \sum_{MT_Z} A^{+}_{JMTT_Z}(ab) \times A_{JMTT_Z}(cd)$ is the scalar two-body transition density for nucleon pairs (*a, b*) and (*c, d*), each paircoupled to spin quantum numbers *JM.* In this work, the USD interaction matrix elements take the following form[9]

$$V_{IT}\langle ab;cd\rangle^{A=18} = \sum_{L,S,L',S'} [(1+\delta_{ab})\times(1+\delta_{cd})]^{-1/2} \times [(1+\delta_{AB})\times(1+\delta_{CD})]^{1/2} \times \begin{pmatrix} l_a & \frac{1}{2} & j_a \\ l_b & \frac{1}{2} & j_b \\ LSI' \end{pmatrix} \times \begin{pmatrix} l_c & \frac{1}{2} & j_c \\ l_d & \frac{1}{2} & j_d \\ L'S'I' \end{pmatrix} \times \sum_p (-1)^{J-J'} \begin{Bmatrix} LSI \\ S'L'p \end{Bmatrix} \times \begin{Bmatrix} LSI' \\ S'L'p \end{Bmatrix}^{-1}$$

$$\times \langle AB; LSI'IT | V_p) | CD; L'S'I'IT \rangle \ldots\ldots (4)$$

where the(a, b, c, d) are the orbital's of the particle under investigation in the model space ,I is the total two particle angular momentum and T is the isospin if isospin was conserved the matrix elements are characterized by their isospin T=0,T=1 If isospin is not conserved then there are three sets of two body matrix elements for the combination pppp ,nnnn ,pnpn where p =proton ,n=neutron.

$\langle AB; LSJ'T | V_p) | CD; L'S'J'JT \rangle$ is LS-coupled matrix elements are linear combinations of the jj- coupled matrix elements, the large square bracket is the jj-LS transformation coefficient, the component V_p is the two body interaction can be written in form[9]

$$V = \sum_p V_p = \sum_p U^p \cdot X^p \ldots (5)$$

Where the operators U,X are irreducible tensor of rank p in the space and spin coordinate respectively ,Interaction components are specified by p=0 for central ,p=1 for spin –orbit and antisymmetric spin –orbit ,and p=2 for tensor. The equation (3) show that the 63 two body matrix elements in the sd-shell can be written as linear combinations of 20 central, 9 spin –orbit ,16 tensor and 18 antisymmetric spin –orbit[10].

If the two particles occupy the same levels, the energy relative to the closed shell is [11,12]:

$$\langle jjIM | H | jjIM \rangle = 2\varepsilon_j + \langle jj | V | jj \rangle_I \ldots\ldots (6)$$

where ε_j is the single-particle energy and $\langle jj|V|jj\rangle_I$ is the matrix element of the residual two-body interaction in same orbit j

we assume there aretwo states denoted by | j1 j2 J M > and | j3 j4 J M > , then the energies with respect to the core are given by

$$\left.\begin{aligned} \langle H\rangle_{11} &= \varepsilon_{j1} + \varepsilon_{j2} + \langle j_1 j_2 | V_{(1,2)} | j_1 j_2 \rangle_I \\ \langle H\rangle_{22} &= \varepsilon_{j3} + \varepsilon_{j4} + \langle j_3 j_4 | V_{(3,4)} | j_3 j_4 \rangle_I \end{aligned}\right\} \ldots\ldots (7)$$

The above equation calculate the energies that consider only pure configuration states .However , the nucleons may be scattered from one state $|j_1 j_2 IM\rangle$ in to $|j_3 j_4 IM\rangle$.Thus a mixture of state must give the actual state. This mean that a term like $\langle H\rangle_{12}$ should be added to equ(7),which can be written as:

$$\langle H\rangle_{12} = \langle H\rangle_{21} = \langle j_1 j_2 | V_{(1,2)} | j_3 j_4 \rangle_I \ldots\ldots (8)$$

The allowable angular momentum states for two particles calculate from two theorems[11], First theorem: if two identical particles in the same single particle orbit j (j half integer) can only couple their spins to even values of I :

$$I = 0,2,4,\ldots\ldots(2j-1)\ldots\ldots (9)$$

Second theorem: for two particles in the states j1 and j2 (j1≠ j2)the allowable angular momentum values are

$$I = j_1 + j_2, j_1 + j_2 - 1, j_1 + j_2 - 2 \ldots\ldots |j_1 - j_2| \ldots\ldots (10)$$

3. Calculations and Results:

Shell-model calculations for the sd-shell nuclei to be presented in this study for nucleus $^{18}_{9}F_9$ in this case there are one proton and one neutron outside the inert core $^{16}_{8}O_8$,which occupy the model space $(0d_{5/2}, 1S_{1/2}, 0d_{3/2})$,From equations 9 and 10, we can determined the possible total angular momentum values which are:

$$J^+ = 1,2,3,4,5$$

In this work, we calculated the spectrum of this nucleus by mixing and pure configuration of energy levels**.** Matrix elements of the USD interaction are calculated by applying (eq.2) and from these elements plus the single - particle energies of this nucleus. We obtain the energy matrix elements by applying the equations 7&8 for the mixing configuration for calculate energies levels w.r.t.(g.s.) shown in table (1) in comparison with the experimental values. as well as that energies levels w.r.t.(g.s.) in the pure configuration is calculated by applying the equations (7) shown in table (2) in comparison with the experimental data .In figures (1) and (2) show the Comparison between calculated excitation energy levels for mixing and pure configuration and with experimental data respectively .

4. Conclusions

*From table (1) and fig(1) , we can show the following conclusions for mixing configuration:

a. We predict certainty parity for experimental energy level(3) which uncertainty experimentally with the value 5.502MeV as well as certainty total angular momentum and parity experimental energy level(2)with the value 20.10 MeV.
b. We predict determined parity for experimental energies levels 1,2 with the energies values 6.633MeV, 9.5MeV as well as ,We predict determined total angular momentum and parity for experimental energies levels{4_2,1_6,1_5,2_4,3_4} with energies Values {7.447MeV,7.729MeV,8.115MeV,10.58MeV }respectively.
c. We predict good agreement for the experimental energy level 4^+ for value 8.238MeV with calculate energy level 8.410 MeV.

*From table (2) and fig(2) for pure configuration:

1) We predict good agreement for the experimental energies levels{1^+,3^+,2^+} for energies values{6.163MeV,2.523MeV}with the calculated data.
2) We predict determined total angular momentum and parity for experimental energy level for value 8.114MeV which is undetermined experimentally.

*The shell model calculation using the USD interaction is quite successful in introducing the energy spectrum of this nuclei

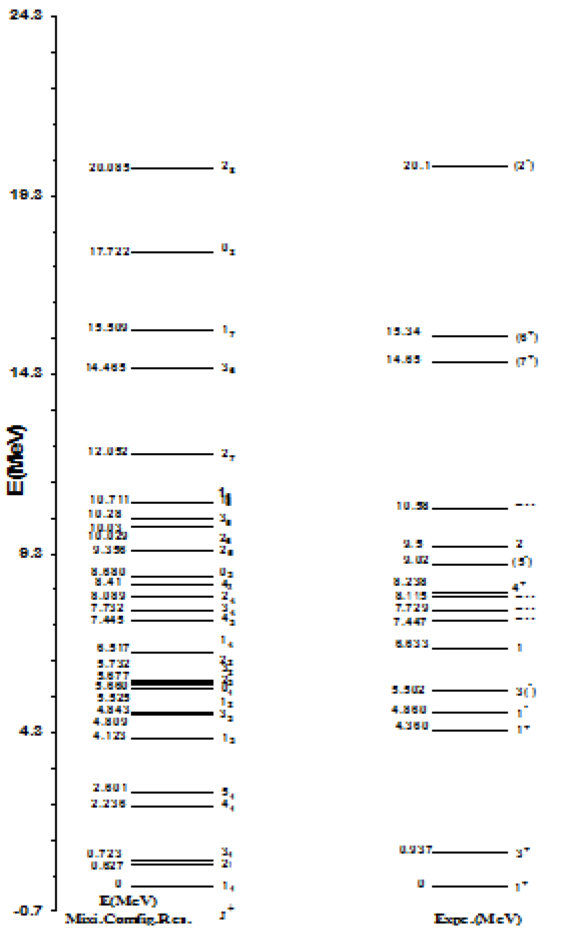
Table 1: Comparison between calculated excitation energy levels for mixing configuration with experimental data for ${}^{18}_{9}F_{9}$ nucleus

$J^{+}_{theor.}$	$E(MeV).Pre.$ $Mixi.Conf$	$E(MeV).Exp.$	$J^{\pi}_{\exp}$
1_1	0 g.s=-9.2254MeV	0	1^+
2_1	0.627	——	——
3_1	0.723	0.937	3^+
4_1	2.236	——	——
5_1	2.601	——	——
1_2	4.123	4.360	1^+
3_2	4.809	——	——
1_3	4.843	4.860	1^-
0_1	5.525	——	——
2_2	5.660	——	——
3_3	5.677	5.502	3(-)
2_3	5.732	——	——
1_4	6.517	6.633	1
4_2	7.445	7.447	——
3_4	7.732	7.729	——
2_4	8.089	8.115	——
4_3	8.410	8.238	4^+
0_2	8.680	9.020	(5^-)
2_5	9.356	9.5	2
2_6	10.029	——	——
3_5	10.030	——	——
1_5	10.28|	10.58	——
1_6	10.711	10.58	——
2_7	12.052	——	——
3_6	14.465	14.650	( ) 7^+
1_7	15.509	15.340	(6^+)
0_3	17.722	——	——
2_8	20.028	20.10	( ) 2^-

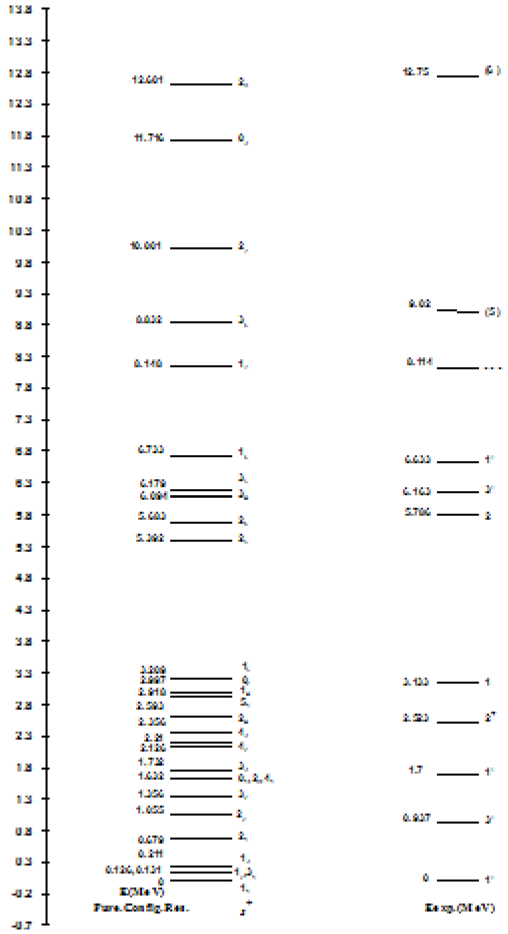
Table 2: Comparison between calculated excitation energy levels for pure configuration with experimental data for ${}^{18}_{9}F_{9}$ nucleus

$J^{+}_{theor.}$	$E(MeV).Pre.$ $Pur..Conf$	$E(MeV).Exp.$	$J^{\pi}_{\exp}$
1_1	0 g.s=-9.2254MeV	0	1^+
1_2	0.1264	——	——
3_1	0.1310	——	——
1_3	0.211	——	——
2_1	0.679	——	——
2_2	1.055	——	——

3_2	1.356	0.937	3^+
$(0_1,2_3,4_1)$	1.632	1.700	1^+
3_3	1.732	——	——
4_2	2.126	——	——
4_3	2.210	——	——
2_4	2.356	2.523	2^+
5_1	2.593	——	——
1_4	2.918	3.133	1^-
0_2	2.997	——	——
1_5	3.209	——	——
2_5	5.392	——	——
2_6	5.683	5.786	2^-
3_4	6.094	6.163	3^+
3_5	6.179	6.163	3^+
1_6	6.733	6.633	1^+
1_7	8.148|	8.114	——
3_6	8.832	9.02	(5^-)
2_7	10.001	——	——
0_3	11.716	——	——
2_8	12.601	12.75	(6^-)



fig(1):A comparison between theoretical energy levels and the experimental values taken from[18] for ^{18}F nucleus



Fig(2):A comparison between theoretical energy levels and the experimental values taken from [13] for ^{18}F nucleus.

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