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The Maximum and Minimum Conditions in Fuzzy Soft Ideals of a Fuzzy Soft Lattice

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Abstract: ***In this paper, we define the concept of fuzzy soft ideals over a collection of soft sets, study their related properties . We also define the maximum and minimum conditions in fuzzy soft lattice. In addition, we characterize fuzzy soft modularity and fuzzy soft distributivity of fuzzy soft lattices of fuzzy soft ideals.***

Keywords: Soft sets, Soft lattices, Soft sublattices, Soft lattice of Soft ideals. Fuzzy soft lattices, Fuzzy soft sublattices, Complete fuzzy soft lattices, Modular fuzzy soft lattices, Distributive fuzzy soft lattices.

1. Introduction

To solve complex problems in economy, engineering, environmental science and social science, the methods in classical mathematics may not be successfully modeled because of various types of uncertainties. There are some mathematical theories for dealing with uncertainties such as; fuzzy set theory, soft set theory , fuzzy soft set theory and so on.

Soft set theory [12] was firstly introduced by Molodtsov in 1999 as a general mathematical tool for dealing with uncertainty. At present, research works on the soft set theory and its applications are making progress rapidly. Faruk Karaaslan et al.[7] defined the concept of soft lattices, modular soft lattices and distributive soft lattices over a collection of soft sets. The operations of soft sets are defined in [1,4]. The algebraic structures of soft sets have been studied by some authors [2]. By embedding the ideas of fuzzy sets, many interesting applications of soft set theory have been expanded [6]. And also algebraic structures of fuzzy soft sets have been studied [3].

Faruk Karaaslan et al.[7] defined the concept of soft lattices over a collection of soft sets by using the operations of soft sets defined by Cagman et al.[1]. Serife Yilmaz et al. [15] defined and discussed soft lattices (ideals,) using soft set theory.

In this paper, we define the concept of fuzzy soft ideals, prime fuzzy soft ideals, and principal fuzzy soft ideals over a collection of soft sets and study their related properties with some examples. also define the notion of maximum and minimum conditions in fuzzy soft lattice and prove that some of the results related their.

Sarala and Suganya, [2014] presented some properties of fuzzy soft groups . Further Sarala and Suganya, [2014] introduced on normal fuzzy soft groups. Sarala and Suganya, [2015] presented Q- Fuzzy Soft Ring, In this paper, we study fuzzy soft lattices theory by using fuzzy soft sets and studied some of algebraic properties.

Throughout this work, U refers to the initial universe, $P(U)$ is the power set of U, E is a set of parameters and $A \subseteq E$. $S(U)$ denotes the set of all soft sets over U.

2. Preliminaries

In this section, we have presented the basic definition and results of fuzzy sets, soft sets the notion of fuzzy soft lattices is defined and fuzzy soft sets theory which are useful for subsequent discussions.

Definition 2.1 [6]
Let E be a crisp set. Then a fuzzy set μ over E is a function from E into [0, 1].

Definition 2 2:[5]
Let U be an initial universe, P(U) be the power set of U, E be a set of all parameters and. Then, a soft set f_A over U is a function from E into P(U) such that $f_A(x) = \phi$, $x \notin A$. Where f_A is called approximate function of the soft set f_A and the value $f_A(x)$is a set called x-element of the soft set for all $x \in E$.

Definition 2.3: [9]
Let U be an initial universe, F(U) be the set of all fuzzy sets over U, E be a set of parameters and $A \subset E$. Then, a fuzzy soft set (f,A) over U is a function from E into F(U).

Definition 2.4: [9]

Let f_A and f_B be two fuzzy soft sets. Then, f_A is a fuzzy soft subset of f_B, denoted by $f_A \subseteq f_B$, if $\mu_A \subseteq \mu_B$ and $f_A(x) \subseteq f_B(x)$ for all $x \in E$.

Definition 2.5:[9]

Let f_A and f_B be two fuzzy soft sets. Then, union of f_A and f_B, denoted by $f_A \cup f_B$, is defined by its fuzzy approximate function

$f_{A\cup B}(x) = f_A(x) \cup f_B(x)$, for all $x \in E$

Definition 2.6:[9]

Let f_A and f_B be two fuzzy soft sets. Then, intersection of f_A and f_B, denoted by $f_A \cap f_B$, is defined by its approximate function

$f_{A\cap B}(x) = f_A(x) \cap f_B(x)$, for all $x \in E$.

Definition 2.7:[9]

Let f_A be a fuzzy soft set over U. Then, the complement f_A^c of f_A is an fuzzy soft set such that $f_A c= f_A^c(x)$, for all $x \in E$, where $f_A^c(x)$ is complement of the set $f_A(x)$.

Definition 2.8: [9]

Let f_L be a fuzzy soft set over U, and $\vee$and $\wedge$ be two binary operation on f_L. If, elements of f_L are equipped with two commutative and associative binary operations $\vee$ and $\wedge$ which are connected by the absorbtion law, then algebraic structure $(f_L, \vee, \wedge)$ is called a **fuzzy soft lattice**.

3. Fuzzy Soft Ideals and Fuzzy Soft Lattices

In this section we introduce the concept of every fuzzy soft ideal of a fuzzy soft lattice f_L is a convex fuzzy soft sublattice of f_L and conversely. Throughout this work, the fuzzy soft lattice f_L means the fuzzy soft lattice $(f_L, \vee, \wedge)$.

Definition 3.1

A non - empty soft subset I of a fuzzy soft lattice f_L is said to be a **fuzzy soft ideal** if

(I_1) $f_L(x), f_L(y) \in I$ implies $f_L(x) \vee f_L(y) \in I$

(I_2) $f_L(x) \in I$ implies $f_L(x) \wedge f_L(A) \in I$

for every element $f_L(A)$ of f_L or equivalently $f_L(x) \in I$ and $f_L(A) \leq f_L(x)$ implies $f_L(A) \in I$

Note:

Every fuzzy soft ideal of a fuzzy soft lattice f_L is a fuzzy soft sublattice of f_L.

Definition 3.2

A fuzzy soft ideal I of the fuzzy soft lattice f_L is said to be a **prime fuzzy soft ideal** if and only if atleast one of an arbitrary pair of elements whose meet is in f_L is contained in I. That is, $f_L(A) \wedge f_L(B) \in I$ implies $f_L(A) \in I$ **or** $f_L(B) \in I$

Definition 3.3

Let f_L be a fuzzy soft lattice. Let $f_L(x) \in f_L$ Then $\{ f_L(A) \in f_L / f_L(A) \leq f_L(x) \}$ is a fuzzy soft ideal and is called the **principal fuzzy soft ideal** generated by A f and is denoted by ( ]. A f

Definition 3.4

An element $f_L(x)$ of a fuzzy soft lattice f_L is called a **greatest element of the fuzzy soft lattice** f_L if $f_L(A) \leq f_L(x)$ for all $f_L(A) \in f_L$.

Similarly an element $f_L(x)$ of a fuzzy soft lattice f_L is called a **least element of fuzzy soft lattice** f_L if $f_L(x) \leq f_L(A)$ for all $f_L(A) \in f_L(A)$

Theorem 3.1

Every soft lattice has almost one minimal and one maximal element. These elements are at the same time the least and greatest element of that fuzzy soft lattice.

Proof

Let there be two minimal elements $f_L(M), f_L(N) \in f_L$, then $f_L(M) \wedge f_L(N) \leq f_L(M)$.

Since $f_L(M)$ is a minimal element, $f_L(M) \wedge f_L(N) < f_L(M)$

Therefore $f_L(M) \wedge f_L(N) = f_M$ and hence. $f_L(M) \leq f_L(N)$

Similarly we take , $f_L(M) \wedge f_L(N) \leq f_L(M)$, then $f_L(N) \leq f_L(M)$.

Therefore $f_L(M) = f_L(N)$.

Hence the fuzzy soft lattice f_L has atmost one minimal element and it is the least element of the lattice. By the principle of duality, every fuzzy soft lattice has atmost one maximal element and it is the greatest element of that fuzzy soft lattice.

4. The Maximum and Minimum Conditions In Fuzzy Soft Ideals of a Fuzzy Soft Lattice

In this section, we define the maximum and minimum conditions in fuzzy soft lattice. We also obtain a necessary and sufficient condition for a fuzzy soft lattice to satisfy the maximum condition. We also define $\vee$ and $\wedge$ of two fuzzy soft ideals and we prove that the set of all fuzzy soft ideals is a fuzzy soft lattice.

Definition 4.1

Let C_0 be any element of a poset P in the fuzzy soft lattice. Let us form the subchain of P in the following way: Let the greatest element of the subchain be C_0 Let C_k $(k \geq 1)$ be an element of P such that $C_k < C_{k-1}$. If each of the chains so formed, commencing at any C_0 is finite, then P is said to satisfy the maximum condition.

Definition 4.2

Let the least element of the subchain be C_0. Let C_k $(k \geq 1)$ be an element of P such that $C_{k-1} < C_k$. If each of the chains so formed, commencing at any C_0 is finite, then P is said to satisfy the minimum condition.

Result 4.1

If a poset P in a fuzzy soft lattice satisfies the minimum condition then for any $f_L(A) \in P$,there exists atleast one minimal element $f_L(M)$ of P such that $f_L(M) \leq f_L(A)$

Result 4.2
If a poset P in a fuzzy soft lattice satisfies the maximum condition then for any $f_L(A) \in P$, there exists atleast one maximal element $f_L(M)$ of P such that $f_L(A) \le f_L(M)$

Corollary 4.1
Every fuzzy soft lattice satisfying minimum (maximum) condition has a least (greatest) element.

Note
By a fuzzy soft ideal chain of a fuzzy soft lattice f_L, we shall mean a set of fuzzy soft ideals in L in which one of every pair of fuzzy soft ideals includes the other.

Theorem 4.1

The fuzzy soft union of any fuzzy soft ideal chain of a fuzzy soft lattice f_L is itself a fuzzy soft ideal in L.

Proof:
Let C be a chain of fuzzy soft ideals of L. Let I denote the fuzzy soft union of all fuzzy soft ideals of L in C.
Let $f_L(x)$, $f_L(y) \in$ I. Then there exists fuzzy soft ideals $I_1$and I_2 in C such that $f_L(x) \in I_1$ and
$f_L(y) \in I_2$ since $I_1, I_2 \in C$, either $I_1 \subseteq I_2$ or $I_2 \subseteq I_1$
Let $I_1 \subseteq I_2$ .then $f_L(x) \in I_1 \subseteq I_2$. Therefore $f_L(x) \in I_2$ since $f_L(x), f_L(y) \in I_2$, $f_L(x) \vee f_L(y) \in I_2$.
Hence $f_L(x) \vee f_L(y) \in I$.

Let $f_L(A) \in f_L$. Then $f_L(x) \wedge f_L(A) \in I_1 \subseteq I$. Therefore $f_L(x) \wedge f_L(A) \in I$.
Hence *I* is a fuzzy soft ideal.

Theorem 4.2
A necessary and sufficient condition for a fuzzy soft ideal I in a fuzzy soft lattice f_L to be a principal fuzzy soft ideal is that the fuzzy soft lattice f_L satisfies the maximum condition.

Proof
Suppose the fuzzy soft lattice f_L satisfies the maximum condition. Then it is also satisfied in every fuzzy soft ideal *I* of f_L.

By corollary, the fuzzy soft ideal *I* includes a greatest element $f_L(x)$ .Then $I = (f_L(x)]$. Hence every fuzzy soft ideal of *L* is a principal fuzzy soft ideal.

Conversely, suppose that every fuzzy soft ideal is a principal fuzzy soft ideal. We have to prove fuzzy soft lattice satisfies the maximum condition.

Suppose not, then we can find an infinite subchain of the form $C = C_0 < C_1 < \ldots$. The set $I = \cup^{\infty}_{n=0} (C_n\ ]$ being the fuzzy soft union of the elements of the fuzzy soft ideal chain is itself a fuzzy soft ideal Hence *I* cannot be a principal fuzzy soft ideal since every one of its elements is less than the other of its elements. Therefore *I* has no greatest element which is a contradiction.

Definition 4.3
Let $(f_L, \vee, \wedge, \le)$ be a fuzzy soft lattice. Then f_L is said to be a ***modular fuzzy soft lattice*** if $f_L(x) \le f_L(z) \Rightarrow f_L(x) \vee [f_L(y) \wedge f_L(z)] = [f_L(x) \vee f_L(y)] \wedge f_L(z)$, for all $f_L(x), f_L(y), f_L(z) \in f_L$ or equivalently,
$f_L(x) \ge f_L(z) \Rightarrow f_L(x) \wedge [f_L(y) \vee f_L(z)] = [f_L(x) \wedge f_L(y)] \vee f_L(z)$, for all $f_L(x), f_L(y), f_L(z) \in f_L$

Definition 4.4
Let $(f_L, \vee, \wedge, \le)$ be a fuzzy soft lattice. Then f_L is said to be a ***distributive fuzzy soft lattice*** if

$$f_L(x) \vee [f_L(y) \wedge f_L(z)] = [f_L(x) \vee f_L(y)] \wedge [f_L(x) \vee f_L(z)]$$

or equivalently,

$$f_L(x) \wedge [f_L(y) \vee f_L(z)] = [f_L(x) \wedge f_L(y)] \vee [f_L(x) \wedge f_L(z)]$$

Theorem 4.3
A fuzzy Soft lattice f_L is modular if and only if for all $f_L(x)$, $f_L(y)$,$f_L(z) \in f_L$ With $f_L(x) \le f_L(z)$,
$[f_L(x) \wedge f_L(y)] \vee [f_L(y) \wedge f_L(z)] \vee [f_L(z) \wedge f_L(x)] = [f_L(x) \vee f_L(y)] \wedge [f_L(y) \vee f_L(z)] \wedge [f_L(z) \vee f_L(x)]$

Proof
Suppose that f_L is a fuzzy soft lattice in which every triplet of elements, $f_L(x)$, $f_L(y)$,$f_L(z)$ With
$f_L(x) \le f_L(z)$, has a median

To prove that f_L is a modular fuzzy soft lattice. If $f_L(x) \le f_L(z)$, then
$[f_L(x) \wedge f_L(y)] \vee [f_L(y) \wedge f_L(z)] \vee [f_L(z) \wedge f_L(x)] = [f_L(x) \wedge f_L(y)] \vee [f_L(y) \wedge f_L(z)] \vee f_L(x)$
$= f_L(x) \vee [f_L(y) \wedge f_L(z)]$ ----------------------- (1)

$[f_L(x) \vee f_L(y)] \wedge [f_L(y) \vee f_L(z)] \wedge [f_L(z) \vee f_L(x)] = [f_L(x) \vee f_L(y)] \wedge [f_L(y) \vee f_L(z)] \wedge f_L(z)$
$= [f_L(x) \vee f_L(y)] \wedge f_L(z)$ ----------------------- (2)

From (1) & (2)
$f_L(x) \vee [f_L(y) \wedge f_L(z)] = [f_L(x) \vee f_L(y)] \wedge f_L(z)$

Hence f_L is a modular fuzzy soft lattice.

Conversely, suppose that f_L is a modular fuzzy soft lattice, Then $f_L(x) \le f_L(z) \Rightarrow f_L(x) \vee [f_L(y) \wedge f_L(z)] = [f_L(x) \vee f_L(y)] \wedge f_L(z)$ to prove that every triplet of elements, $f_L(x)$, $f_L(y)$,$f_L(z)$ With $f_L(x) \le f_L(z)$, has a median
Now,
$f_L(x) \vee [f_L(y) \wedge f_L(z)] = f_L(x) \vee [f_L(x) \wedge f_L(y)] \vee [f_L(y) \wedge f_L(z)]$
$= [f_L(x) \wedge f_L(z)] \vee [f_L(x) \wedge f_L(y)] \vee [f_L(y) \wedge f_L(z)]$
$= [f_L(x) \wedge f_L(y)] \vee [f_L(y) \wedge f_L(z)] \vee [f_L(z) \wedge f_L(x)]$ ----(3)

$[f_L(x) \vee f_L(y)] \wedge f_L(z) = [f_L(x) \vee f_L(y)] \wedge [f_L(y) \vee f_L(z)] \wedge f_L(z)$
$= [f_L(x) \vee f_L(y)] \wedge [f_L(y) \vee f_L(z)] \wedge [f_L(x) \vee f_L(z)]$
$= [f_L(x) \vee f_L(y)] \wedge [f_L(y) \vee f_L(z)] \wedge [f_L(z) \vee f_L(x)]$ -----(4)

From (3) & (4)
$[f_L(x) \wedge f_L(y)] \vee [f_L(y) \wedge f_L(z)] \vee [f_L(z) \wedge f_L(x)] = [f_L(x) \vee f_L(y)] \wedge [f_L(y) \vee f_L(z)] \wedge [f_L(z) \vee f_L(x)]$

Theorem 4.4

A fuzzy soft lattice f_L is distributive if and only if every one of its triplets has a median. (or) A fuzzy soft lattice f_L is distributive $\Leftrightarrow$ $(f_L(x) \vee f_L(y)) \wedge (f_L(y) \vee f_L(z)) \wedge (f_L(z) \vee f_L(x)) = (f_L(x) \wedge f_L(y)) \vee (f_L(y) \wedge f_L(z)) \vee (f_L(z) \wedge f_L(a))$ for all $f_L(x), f_L(y), f_L(z) \in f_L$

5. Conclusion

In this paper, we defined fuzzy soft ideals and fuzzy soft lattices are discussed their properties. We have shown that the set of fuzzy soft ideals is a fuzzy soft lattice. We have given characterization theorems for modular fuzzy soft lattices and distributive fuzzy soft lattices. We are studying about these fuzzy soft lattices and are expected to give some more results.

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