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Algorithmic Research and Application Using the Rayleigh Method

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Abstract: ***The aim of this article is to present the general notions and algorithm about power (Rayleigh) method. The solutions for a numerical example are given and the C++ program illustrated the facility of this method. We can concluded, that the small number of iterations resulted to determined the equation solutions, indicated us, that the chosen of power method is a good decision.***

Keywords: Matrixes, Rayleigh numerical method, algorithm, power method, research, DevC++ program.

1. Introduction

This document presents an algorithmic research of the direct power numerical method [1-2]. Direct power method allows the determination of the proper values ($\lambda \in C$) of maximum module, assuming that the matrix proper vectors create a basis, so there are linearly independents [3]. This always happens when the proper values are distinct. If there are multiple values, then the proper vectors of the considered matrix, may or may not be linearly independent. For instance, the unit matrix, I, has all proper values equal ($\lambda_i = 1, i = \overline{1,n}$), but the proper vectors are linearly independent [1-3].

The dominant proper value (main proper value) is that who has maximum value in module. To calculate the dominant proper value, the proper vector associated of the matrix, and its spectral range, is indicated to use the direct power method (or Rayleigh's method) or direct iterative method [1-3].

2. The Direct Power Method (Rayleigh's)

We consider a matrix "A" with n dimensions, which has $\lambda \in C$, proper values and $(\exists)\underline{x} \in R^n, \underline{x} \neq 0$ proper vectors, so that is satisfy the conditions [4-5]:

$$A \bullet x = \lambda \bullet x \tag{1}$$

where the algorithm will produce a number λ (the eigen value) and a nonzero vector "x" (the eigenvector) [6], and further it takes the following form:

$$(\underline{A} - \lambda \cdot \underline{I})\underline{x} = 0, \tag{2}$$

The relation (2) was consisted a linear and homogeny system, where "I" is the unit matrix of n dimension.
We note $\underline{u}^0$ the proper vector. The first iteration of the solution consists the dominant vector of the $\underline{u}^0$ proper value, and its meaning is a linear unknown combination (α_i coefficients) of $\underline{x}_i$ proper vectors, supposedly independent linear quadratic matrix $\underline{A}$ with n dimension and real elements.

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dots \\ 0 \end{bmatrix} \tag{3}$$

And

$$\underline{u}^0 = \sum_{i=1}^{n} \alpha_i \underline{x}_i . \tag{4}$$

The next iterations are calculated with the relations:

$$\underline{u}^1 = \underline{A}\underline{u}^0, \underline{u}^2 = \underline{A}\underline{u}^1, \dots, \underline{u}^k = \underline{A}\underline{u}^{k-1}, \dots \tag{5}$$

with repeated substitutions relations (4) in relation (5) and with relation (6):

$$\underline{A}\underline{x}_i = \lambda \underline{x}_i, i = 1 \dots n, \tag{6}$$

we obtain the expresion of proper vector for dominant proper value at k iteration:

$$\underline{u}^k = \lambda_1^k \left[\alpha_1 \underline{x}_1 + \sum_{i=2}^{n} \alpha_i (\lambda_i / \lambda_1)^k \underline{x}_i \right]. \tag{7}$$

Suppose that the proper values of the $\underline{A}$ matrix, are arranged so to check ((possibly a permutation):

$$|\lambda_1| > |\lambda_2| > \dots > |\lambda_n|, \tag{8}$$

Ascertain that for a sufficient number of iterations (for sufficiently large k) the report $(\lambda_i / \lambda_1)^k$ will converge to zero, therefore the sum from (7) relation converges to zero, therefore,

$$\underline{u}^k \to \lambda_1^k \alpha_1 \underline{x}_1 = p^k \underline{x}_1 \ \ with \ \ p^k = \lambda_1^k \alpha_1 . \tag{9}$$

so, $\underline{u}^k$ is proportional to $\underline{x}_1$ and

$$(p^k / p^{k-1}) \to \lambda_1 . \tag{10}$$

and, the spectral range will be:

$$\rho(\underline{A}) = |\lambda_1| . \tag{11}$$

The implementation pass of as follows: after each iteration runs normalization vector $\underline{u}^k$ by dividing with the maximum module element, such in relation (12):

$$\underline{v}^k = A\underline{u}^k, \underline{u}^{k+1} = \left[1/\max\left(\underline{v}^k\right)\right]\cdot \underline{v}^k, k = 0,1,2,..., \qquad (12)$$

where $\max\left(\underline{v}^k\right)$ represent the element of maximum modulo $\underline{v}^k$ vector.

The algorithm of the power method involves the repeated relations (12) to achieve convergence, until the termination condition is satisfied iterative calculation process:

$$\max\left\{\left|u_j^{k+1} - u_j^k\right|\right\} \le \varepsilon, i = 1...n, \qquad (13)$$

with an default error $\varepsilon > 0$.

Once satisfied the relation (13), the parameter, $\max(\underline{v}^k)$ will be dominant proper value and its module will be spectral range.

3. The Power Method Algorithm

The Power method (or Rayleigh) steps are such as followings:

3.1 Initial we notice $\underline{x}_1$ with $\underline{u}^0$ (arbitrarily chosen):

$$\underline{x}_1 = \underline{u}^k, \qquad (14)$$

where, the upper index means the number of the current iteration.

3.2 At a some step k, the iterative calculation process, k = 0,1,2,…, determine the current value of $\underline{v}^k$ vector and a new value for $\underline{x}_1$, $\underline{u}^{k+1}$, vector, from (12) relation.

3.3 The calculation is considered complete when it stabilizes $\underline{x}_1$ at proper vector, namely is verified condition (13) to termination the iterative calculation process.

3.4 The main proper value, is determined as equal to the last normalization factor:

$$\lambda_1 = \max(\underline{v}^k), \qquad (15)$$

And corresponding proper vector is the last vector $\underline{u}^{k+1}$:

$$\underline{x}_1 = \underline{u}^{k+1}, \qquad (16)$$

and spectral radius is obtained by applying the formula:

$$\rho(\underline{A}) = |\lambda_1|. \qquad (17)$$

Observations:

1. Speed of convergence of the algorithm is even greater as reports λ_i / λ_1, i = 2, …, n are less.
2. The algorithm is efficient for non-symmetrical matrix.

4. The Computational Implementation

We can write the program in DevC++ language, for implementation the Rayleigh method.

```
#include <stdio.h>
#include <iostream.h>
#include <conio.h>
#include <iomanip.h>
#include<math.h>
void main()
{
clrscr();
int A[100][100],X[10],Y[10],n,m,i,j,k,itermax;
float lambda,eps,difm;
printf("******power method************");
printf(" \n maximum of iterations k=");
scanf("%d", &itermax);
printf("\n maximum error admisible eps=0.001");
printf("\n Matrix is:");
for(i=1;i<=10;i++)
 { for (j=1;j<=10;j++) printf("\n matrix elements A raw [%d]
column [%d]=%d ",i,j, A[i] [j]);
 }
 /*initialization of X vector */ for(i=1;i<=10;i++) ≡ File Edit
Search Run
printf("\n vector elements X [%d] = %d ",i,X[i]); /*the start
of iterativ process for calculate the dominant proper value */
for(k=0; k<4; k++); printf("iteration k="); scanf("%d", &k);
for(i=1;i<=10;i++)
printf("\n %d ",i);
lambda=0;
for (i=1;i<10;i++)
Y[i]=0; for(j=1;j<10;j++){ Y[i]=Y[i]+A[i][j]*X[j];
printf("\n vector elements Y [%d]= %d " ,Y[i]); }
Y[i]=0;
 if abs(Y[i])>abs(A[i][j]) lambda = Y[i]; printf("\n lambda=
%f", lambda);
difm=0;
 for (i=1;i<10;i++){ Y[i]=Y[i]/lambda;
 if
abs(Y[i]-X[i])>difm difm=abs(Y[i]-X[i]) printf("%f",difm }
if difm<eps
{
 printf("%d",itermax) printf("\n Number of iterations is
over")
}
getch();
 }
```

5. Results and Discussions

In detailed program we use the algoritm steps of the power method (Rayleigh) determined the vectors and proper values of a matrix of size m × n, with real components. We choose the maximum number of iterations necessary to repeat instructions determine vectors and proper values lambda, X[i] and Y[j], where i =1,…, m , and j =1, …, n.

The maximum permissible error is chosen to be eps=0.001. The C programm is over when the number of iterations has been completed.

6. Conclusions and Final Remarks

The recent research and experiments were advocating to the algorithm method of various calculations, especially by using Rayleigh method to provide more precise and more valuable results / numerically solutions within a short time. One can evaluate easily any calculations by using above said method, and it is also a proper mathematical approach to determine the power vectors and their proper values of any metrics of m × n dimensions.

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