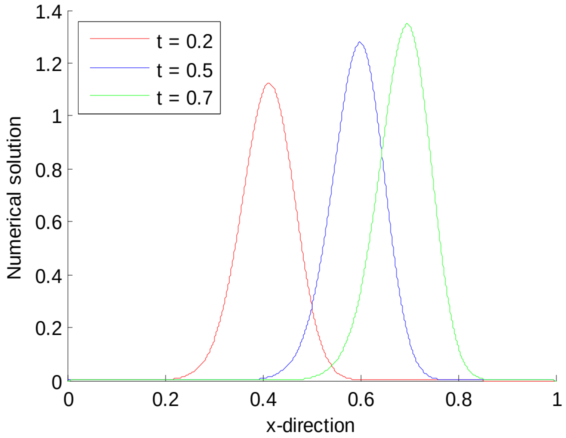

Figure 4: The numerical solution of Example 2 at different time levels for $\tau = .05$.

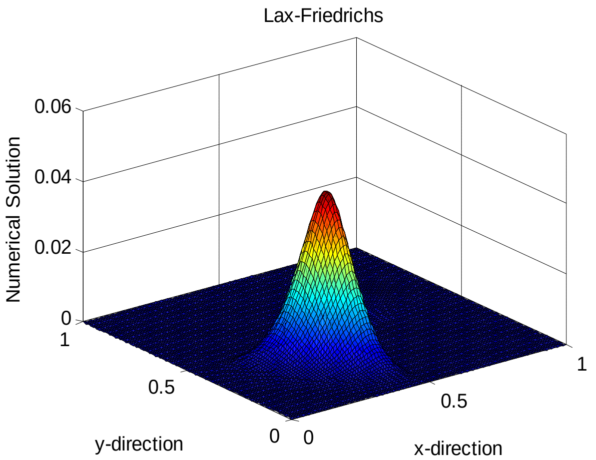

Figure 5: The numerical solution of Example 3 for $\delta = 0.5$ and $\tau = 0.5$ at $t = 0.5$.

5. Conclusion

In this paper we propose a numerical scheme based on Lax-Friedrichs finite difference approximations of order greater than one in space to solve hyperbolic partial differential equation with point-wise advance. The consistency, stability and convergence analysis prove that the proposed numerical schemes are consistent, stable with CFL condition and convergent in both space and time. This second order numerical scheme in space maintains the height and width better than a first-order scheme as author discussed in paper [9]. The effect of point-wise advance on the solution behavior is shown by the some test examples. Error tables illustrate the fact that the methods are convergent in space and time. The solutions are plotted in graphs which shown in figures 1-5. Also we extend our ideas in higher space dimensions and include numerical experiment to show the behavior of solution in two space dimension.

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