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Smart Antenna System for DOA Estimation using Nyström Based MUSIC Algorithm

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Abstract: ***This paper presents the high efficiency and low complexity MUltiple Signal Classification (MUSIC) algorithms for accurate direction of arrival (DOA) estimation. In this work, we proposed modified MUSIC algorithm for high resolution DOA estimation of coherent source signals under low signal to noise ratio (SNR) scenario using less array elements and snapshots. The subspace based method requires intensive calculations especially for large arrays to compute singular vector decomposition (SVD) of sample covariance matrix (SCM). The proposed Nyström based MUSIC method computes SVD of SCM without computing SCM. This reduces the computational complexity and makes it more robust. The simulated results are compared with existing algorithms which shows that the proposed methods are computationally efficient and simple.***

Keywords: Direction of Arrival (DOA) estimation, root MUSIC, MUSIC, signal subspace, smart antenna

1. Introduction

The Music [1] algorithm for DOA estimation in array signal processing is popular, efficient and relatively simple method. It has many variations and is perhaps the most studied method in its class [2]. But this algorithm deviates from its performance under low SNR conditions and for small snapshots. For large arrays and snapshots, subspace based algorithm like MUSIC require intensive computations for calculations of sample covariance matrix (SCM) and Eigen Vector Decomposition (EVD) to evaluate signal subspace and noise subspace [3]. The complexity of MUSIC needs to be reduced in order to make it more suitable for practical applications like mobile communication, RADAR, biomedical, satellite etc. Many algorithms and modifications have been proposed in the literature to reduce the computational cost and to enhance the DOA resolution. Cheng Qian et al [3] have proposed improved DOA estimation using pseudo-noise resampling (PR) for high resolution estimations for small snapshots. DOA estimation in an impulsive noise is always a challenging task. Zeng et al [4] have proposed *l*p-MUSIC which replaces the Frobenius norm of conventional MUSIC by the *l*p –norm of the residual error matrix for DOA estimation. Frequency selective MUSIC (F-MUSIC) [5] shows increased robustness under low SNR and colored noise. It uses frequency selective data model for subspace decomposition.

Application of Nyström approximations to subspace methods increases the speed of algorithms by generating low rank approximations [7]-[10]. In this work we proposed the two algorithms namely modified MUSIC and Nyström based MUSIC methods for increasing the resolution of DOA estimation and to reduce the computational complexity for large arrays.

2. Problem Formulation

2.1 System Model

Let us consider system model with uniform linear array (ULA) consisting of 'M' isotropic sensors. Let 'm' (m<M) be the unconstrained signal with frequency f_o impinging on a ULA. Consider 'd' as element spacing between array elements and its value in this work is $\lambda/2$. Here $\lambda = c/f_o$, where 'c' is the speed of light and f_o is the frequency of received signals respectively. Consider Cx1 dimension steering vector for DOA estimation for Azimuth directions $\theta(\theta_1, \theta_2, ..., \theta_m)$ in far fields and Nx1 dimension array observation vector which can be modeled for K snapshots as:

$$x(l) = \mathrm{B}s(l) + n(l) \quad l = 1,2,...K \tag{1}$$

Where $s(l) = [s_2(l),...s_m(l)]^{\mathrm{T}}$ is source vector, here $(\cdot)^{\mathrm{T}}$ is the transpose; $n(l) \in C^{\mathrm{Mx1}}$ is the complex noise vector and it is given by $n(l) = [n_1(l), n_2(l),..., n_m(l)]^{\mathrm{T}}$ is the noise vector; $\mathrm{B} = [b(\theta_1), b(\theta_2),..., b(\theta_m)]$ is the steering matrix with steering vector

$$b(\theta) = \left[1, e^{j2\pi \sin(\theta) d/\lambda}, ..., e^{j2\pi(M-1)\sin(\theta)d/\lambda}\right]$$

Let us assume that the noise is white Gaussian with zero mean and $\sigma_s^{\ 2}$ variance.

2.2 Conventional MUSIC algorithm

The SCM of received signal is given by

$$\Phi = \mathrm{E}\{x_j x_j^{\ H}\} \tag{2}$$

Here $(\cdot)$ is the expectation which can be obtained from K snapshots as:

$$\Phi_X = \frac{1}{K}\sum_{j=1}^{K} x_j x_j^H = \frac{1}{K} XX^H \quad (3)$$

Here $(\cdot)^H$ denotes the Hermetain transpose. Since noise and signal has no correlation, SCM can be written as:

$$\Phi = \mathrm{E}\{x_j x_j^H\} = \mathrm{B}\Phi_v \mathrm{B}^H + \sigma_s^2 \mathrm{I}_M \quad (4)$$

Where $\Phi_v = \mathrm{E}\{s_j s_j^H\}$ is the source matrix.

2.3 Modified MUSIC Algorithm

Conventional MUSIC algorithm deviates from its performance under low SNR condition especially for large arrays. Either it makes bad estimation or fails completely to estimate DOA of required signals. To overcome this problem we proposed Modified MUSIC method which incorporates Jordon canonical matrix as follows.

$$\tilde{W} = [\mathrm{P}^{-1}\Lambda^{-1}\mathrm{P}^{-1}Z_{12}]$$

Consider the transition matrix T, T of the M^th order as:

$$T = \begin{bmatrix} 0 & 0 & - & 1 \\ 0 & 0 & - & 0 \\ - & - & - & - \\ 1 & 0 & - & 0 \end{bmatrix} \quad (5)$$

Let, $\mathrm{Y} = \mathrm{X}^*$, Where X^* is the complex conjugate of X.

Then we define $\Phi_y = \mathrm{E}[\mathrm{YY}^H] = T\Phi_X X^* T$

Then the SCM using above relations can be written as:

$$\Psi = \Phi_X \Phi_y = [\Phi_v B^H \mathrm{T}] * \mathrm{T} + 2\sigma_s^2 \mathrm{I}_M \quad (6)$$

The matrices Φ_X, Φ_y and Ψ provides new subspace for the construction of spatial spectrum which gives accurate DOA estimation even under low SNR condition.

2.4 Nyström Method

Consider $Z \in C^{M\times M}$ as acquire matrix. Let us decompose Z as:

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \quad (7)$$

Here: $Z_{11} \in C^{K\times K}$, $Z_{12} \in C^{K\times(M-K)}$, $Z_{21} \in C^{(M-K)\times K}$ and $Z_{22} \in C^{(M-K)(M-K)}$,Consider $\mathrm{P}\Lambda\mathrm{P}^{-1}$ as EVD of Z_{11}, where $Z \in C^{K\times K}$ is the matrix of eigenvectors and $\Lambda \in C^{K\times K}$ is the matrix of eigen values. The main aim is to obtain the eigenvectors of column of Z with respect to P.

Now let us define

$$\hat{\mathrm{P}} = Z_{21}\Lambda^{-1} \quad (8)$$

and

$$\hat{W} = \Lambda^{-1}\mathrm{P}^{-1}Z_{12} \quad (9)$$

Let us extend equation (8) and (9) into matrix $\tilde{\mathrm{P}}$ and $\tilde{W}$ as below:

$$\tilde{\mathrm{P}} = \begin{bmatrix} \mathrm{P} \\ Z_{21}\mathrm{P}\Lambda \end{bmatrix}$$

Now we can represent Nyström form as follows:

$$\hat{V} = \hat{\mathrm{P}}\Lambda\hat{W} = \begin{bmatrix} \mathrm{P} \\ Z_{21}\mathrm{P}\Lambda^{-1} \end{bmatrix} \wedge [\mathrm{P}^{-1}\Lambda^{-1}\mathrm{P}^{-1}Z_{12}]$$

$$= \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{21}Z_{11}^{+}Z_{12} \end{bmatrix} \quad (10)$$

Here $(\cdot)^+$ represent pseudo inverse. We should note that the values Z_{11}, Z_{12} and Z_{21} are not affected by the Nyström method, but at the same time Z_{22} is replaced by $Z_{21}Z_{11}^{+}Z_{12}$.

2.5 Proposed Nyström based MUSIC Algorithm

The SCM $\Phi = \mathrm{E}\{x_j x_j^H\}$ can be written as:

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{12}^H & \Phi_{22} \end{bmatrix} \quad (11)$$

The received matrix 'S' can be portioned as [3]:

$$S = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} \quad (12)$$

Where $S_1 \in C^{u\times n}$ and $S_2 \in C^{(u-n)\times n}$ are sub matrices of data obtained from the first u array elements and (M-u) array elements respectively. Here we should note that u is the user defined parameter [3] that satisfies $u \in (1, 2, \ldots M)$.

Let us define

$\Phi_{11} = \mathrm{E}[S_1 S_1^H]$, $\Phi_{12} = \mathrm{E}[S_1 S_2^H]$, $\Phi_{22} = \mathrm{E}[S_2 S_2^H]$.

The main objective of this research is to approximate the eigenvalues and eigenvectors using low complexity method. Let us assume that Φ_{11} is the nonsingular matrix and its rank is 'm'. Consider $G = \begin{bmatrix} S_{11} \\ S^H{}_{12} \end{bmatrix} S_{11}^{-1/2}$ be the EVD of matrix $G^H G$ which is $\mathrm{P}_G\Lambda_G\mathrm{P}_G{}^H$ Let

$Q = \Lambda_G^{1/2}\mathrm{P}_G^H\mathrm{P}_G\Lambda_G^{1/2}$ and EVD of Q is $\mathrm{P}_Q\Lambda_Q\mathrm{P}_Q{}^H$, now the signal subspace $\mathrm{P}_S \in C^{m\times n}$ is $\mathrm{P}_S = G\mathrm{P}_G\Lambda_G^{-1/2}\mathrm{P}_Q$. Hence from the above equations we can obtain the covariance estimator of Nyström based approximation is:

$$\chi_{NCE} = \mathrm{P}_S\Lambda_G\mathrm{P}_S^H$$

$$= \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^{H} & S_{12}^{H} S_{11}^{-1} S_{12} \end{bmatrix} \tag{13}$$

(15)

3. Simulation Results

We developed Modified MUSIC and Nyström based MUSIC methods using MATLAB software. Let us assume that the noise is white Gaussian with zero mean and σ_s^2 variance. Root Mean Square Error (RMSE) of all methods is computed using Monte Carlo simulation. The simulation results obtained are compared with conventional and other MUSIC algorithms.

3.1 Performance Analysis of Modified MUSIC Algorithm

Modified MUSIC algorithm can be used for DOA estimation of coherent source signals under severe environmental scenario. Let us consider four coherent source signals with azimuth angles -20^o, 0^o, 20^o and 40^o impinges a ULA of array elements M=10, array element spacing is d=0.5λ, snapshots K=100 and SNR=5dB. The simulated result obtained for conventional and Modified MUSIC algorithms for above data is shown in figure1 and 2 respectively.

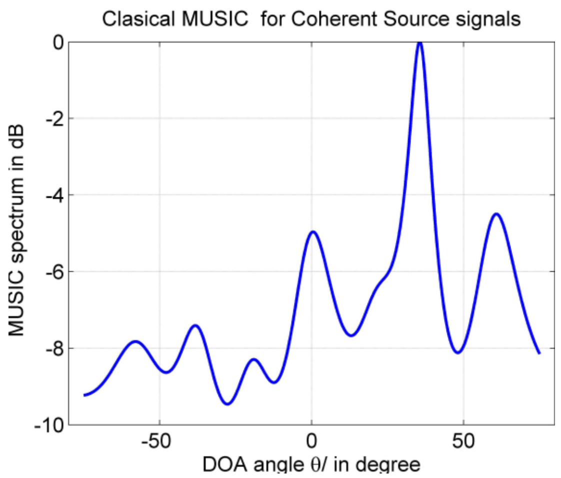

Figure 1: Spectrum of Classical MUSIC for coherent source signals

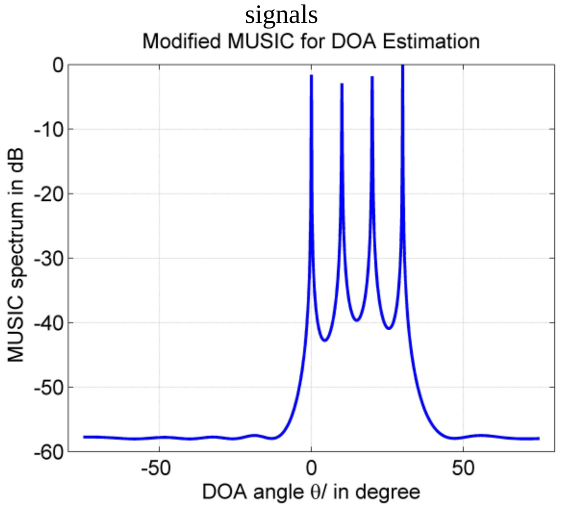

Figure 2: Spectrum of Modified MUSIC for coherent source signals

From figure 1 and 2, it is clear that the conventional MUSIC can make the good estimation when the signals are uncorrelated. For coherent sources it loses its effectiveness and deteriorates from its performance.

3.2 Performance Analysis of Nyström method based MUSIC Algorithm

Let us consider two narrowband source signals of true DOA 10^o and 20^o impinges a ULA of array elements M=20, array element spacing d=0.5λ, snapshots K=100 and SNR is varied from -40dB to 20dB. RMSE is evaluated using Monte Carlo simulation using trails L=500. The RMSE can be calculated as:

$$RMSE = \sqrt{\frac{1}{L}\sum_{m=1}^{L}\left|\hat{\theta}_m - \theta_m\right|^2} \tag{14}$$

Figure 3 and 4 shows RMSE versus SNR for various algorithms for small and large array cases respectively.

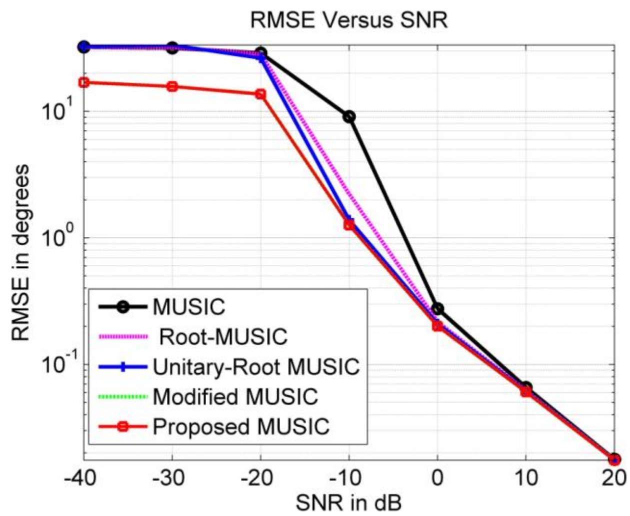

Figure 3: RMSE performance versus SNR for small array case (M =10, m = 2, K = 100, SNR = -40: 20)

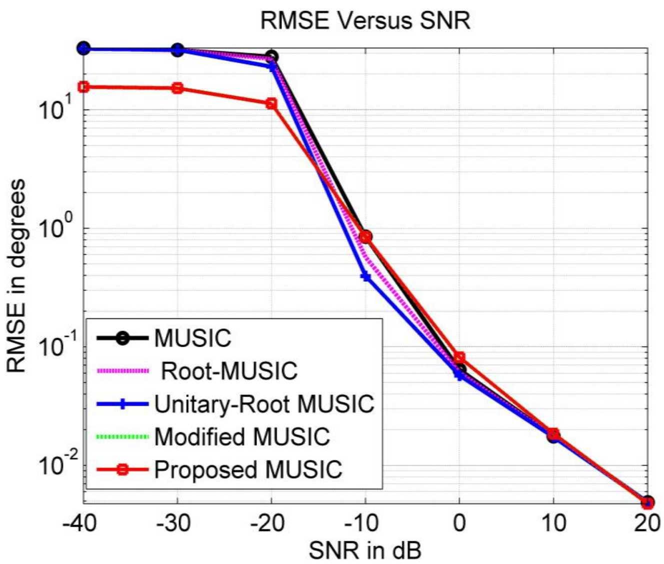

Figure 4: RMSE performance versus SNR for large array case (M =10, m = 2, K = 100, SNR = -40: 20)

Angle error performance of various MUSIC algorithms over SNR varying from -40dB to 20dB in figure 3 and 4 reflects that the proposed Modified MUSIC and Nyström method based MUSIC algorithms have almost similar performance as compared to existing methods.

3.3 Complexity of computation

The conventional MUSIC requires $O(M^3)+O(M^2K)$ flops to compute SVD of SCM. Whereas proposed Nyström based MUSIC method computes SVD of SCM without computing SCM. Hence it requires $O(Mm^2+Mm)$ flops, provided that $m<M$. The complexity of commutation for five mentioned MUSIC algorithms versus number of array elements is shown in figure 5. Time complexities of all methods are processed using intel i3-3110M CPU with 2.40GHz capacity.

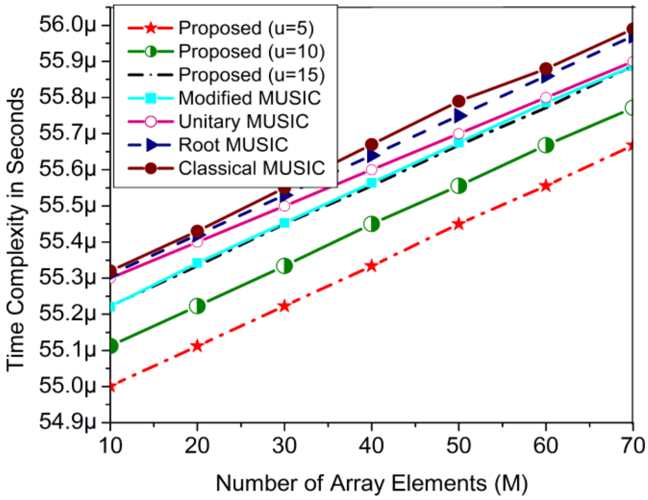

Figure 5: Complexity of computation versus number of array elements. (M =10, m = 2, K = 100, SNR = 20 dB, u=[5,10,15])

From figure 5 we observe that the proposed Nyström based MUSIC method is computationally efficient and simple.

4. Conclusion

A smart antenna for DOA estimation using low complexity method has been devised. The proposed modified MUSIC method provides the high resolution DOA estimation under low SNR condition for fewer snapshots. This makes communication system efficient and robust. The proposed Nyström based MUSIC method is computationally efficient and simple which requires only $O(Mm^2+Mm)$ flops to compute SVD of SCM which is very less as compared to existing methods. This makes it more suitable for practical array signal processing applications.

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