On Pre- ρ -Continuity Where $\rho \in \{L, M, R, S\}$

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Abstract: The authors Selvi.R, Thangavelu.P and Anitha.M introduced the concept of ρ -continuity between a topological space and a non empty set where $\rho \in \{L, M, R, S\}$ [4]. Navpreet singh Noorie and Rajni Bala[3] introduced the concept of $f^{\#}$ function to characterize the closed, open and continuous functions. In this paper, the concept of pre- ρ -continuity is introduced and its properties are investigated and pre- ρ -continuity is further characterized by using $f^{\#}$ functions.

Keywords: Multifunction, Saturated set, ρ -continuity, pre-open, pre-closed and continuity

1. Introduction

By a multifunction F: $X \rightarrow Y$, We mean a point to set correspondence from X into Y with $F(x) \neq \phi$ for all $x \in X$. Any function f: $X \rightarrow Y$ induces a multifunction f⁻¹ O f: X $\rightarrow \mathcal{O}(X)$. It also induces another multifunction f O f⁻¹: Y \rightarrow (Y) provided f is surjective. The purpose of this paper is to introduce notions of pre-L-Continuity, pre- M-Continuity, pre-R-Continuity and pre-S-Continuity of a function f: $X \rightarrow$ Y between a topological space and a non empty set. Here we discuss their links with pre-open and pre-closed sets. Also we establish pasting lemmas for pre-R-continuous and pre-S-continuous functions and obtain some characterizations for pre- ρ -continuity. Navpreet singh Noorie and Rajni Bala [3] introduced the concept of f[#] function to characterize the closed, open and continuous functions. The authors [6] characterized ρ -continuity by using f[#] functions. In an analog way pre- ρ -continuity is characterized in this paper.

2. Preliminaries

The following definitions and results that are due to the authors [4] and Navpreet singh Noorie and Rajni Bala [3] will be useful in sequel.

Definition: 2.1

Let f: $(x, \tau) \rightarrow Y$ be a function. Then f is (i) L-Continuous if f⁻¹(f (A)) is open in X for every open set A in X. [4]

(ii) M-Continuous if $f^{-1}(f(A))$ is closed in X for every closed set A in X. [4]

Definition: 2.2

Let f: $X \rightarrow (Y, \sigma)$ be a function. Then f is

(i) R-Continuous if f (f⁻¹(B)) is open in Y for every open set B in Y. [4]

(ii) S-Continuous if f (f $^{-1}(B)$) is closed in Y for every closed set B in Y. [4]

Definition 2.3:

Let f: $X \rightarrow Y$ be any map and E be any subset of X. then the following hold. (i) $f^{\#}(E) = \{y \in Y: f^{-1}(y) \subseteq E\}$; (ii) $E^{\#} = f^{-1}(f^{\#}(E))$. [3]

Lemma 2.4:

Let E be a subset of X and let f: $X \rightarrow Y$ be a function. Then the following hold. (i) $f^{\#}(E) = Y \setminus f(X \setminus E)$; (ii) $f(E) = Y \setminus f^{\#}(X \setminus E)$. [3]

Lemma 2.5:

Let E be a subset of X and let f: $X \rightarrow Y$ be a function. Then the following hold. (i) $f^{-1}(f^{\#}(E)) = X \setminus f^{-1}(f(X \setminus E))$; (ii) $f^{-1}(f(E)) = X \setminus f^{-1}(f^{\#}(X \setminus E))$. [6]

Lemma 2.6:

Let E be a subset of X and let f: $X \rightarrow Y$ be a function. Then the following hold. (i) $f^{*}(f^{-1}(E)) = Y \setminus f(f^{-1}(Y \setminus E))$; (ii) $f(f^{-1}(E)) = Y \setminus f^{*}(f^{-1}(Y \setminus E))$. [6]

Definition 2.7

Let $f : X \rightarrow Y$, $A \subseteq X$ and $B \subseteq Y$. we say that A is f-saturated if $f^{-1}(f(A)) \subseteq A$ and B is f^{-1} -saturated if $f(f^{-1}(B)) \supseteq$ B. Equivalently A is f-saturated if and only if $f^{-1}(f(A))=A$, and B is f^{-1} -saturated if and only if $f(f^{-1}(B)) = B$.

Definition 2.8

Let A be a subset of a topological space (X, \mathcal{T}) . Then A is called (i) semi-open if $A \subseteq cl(int(A))$ and semi-closed if $int(cl(A)) \subseteq A$; [1]. (ii) pre-open if $A \subseteq int(cl(A))$ and pre-closed if $cl(int(A)) \subseteq A$; [2].

Definition 2.9:

Let f: $(X, l) \rightarrow (Y, \sigma)$ be a function. Then f is precontinuous if f⁻¹(B) is open in X for every pre-open set B in Y. [2]

Definition: 2.10:

Let f: $(X, l) \rightarrow (Y, \sigma)$ be a function. Then f is pre-open (resp. pre-closed) if f(A) is pre-open(resp. pre-closed) in Y for every pre-open(resp. pre-closed) set A in X.

3. Pre- ρ -Continuity Where $\rho \in \{L, M, R, S\}$

Definition: 3.1

Let f: (X, \mathcal{I}) \rightarrow Y be a function. Then f is

(I) pre-L-Continuous if $f^{1}(f(A))$ is open in X for every preopen set A in X.

(ii) pre-M-Continuous if $f^{1}(f(A))$ is closed in X for every pre-closed set A in X.

Definition: 3.2

Let f: $X \rightarrow (Y, \sigma)$ be a function. Then f is

(i) pre-R-Continuous if $f(f^{-1}(B))$ is open in Y for every preopen set B in Y.

(ii) pre-S-Continuous if f (f $^{-1}(B)$) is closed in Y for every pre-closed set B in Y.

Example: 3.3

Let X = {a, b, c, d} and Y = {1, 2, 3, 4}. Let $\tau = \{ \Phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\} \}$. Let f: $(X, \tau) \rightarrow Y$ defined by f(a)=1, f(b)=2, f(c)=3, f(d)=4. Then f is pre-L-Continuous and pre-M-Continuous.

Example: 3.4

Let X = {a, b, c, d} and Y = {1, 2, 3, 4}. Let $\sigma = {\Phi, Y, {1}, {2}, {1,2}, {1,2,3}}$. Let g : X \rightarrow (Y, σ) defined by g(a)=1, g(b)=2, g(c)=3, g(d)=4. Then g is pre-R-Continuous and pre-S-Continuous.

Definition: 3.5

Let f: (X, l) \rightarrow (Y, σ) be a function, Then f is

(i) pre-LR-Continuous, if it is both pre-L-Continuous and pre-R-Continuous. (ii) pre-LS -Continuous, if it is both pre-L-Continuous and pre-S-Continuous. (iii) pre-MR-Continuous, if it is both pre-M-Continuous and pre-R-Continuous. (iv) pre-MS-Continuous, if it is both pre-M-Continuous and pre-S-Continuous.

Theorem: 3.6 (i) Every injective function f: $(X, t) \rightarrow (Y, \sigma)$ is pre-L -Continuous and pre-M-Continuous. (ii) Every surjective function f: $(X, t) \rightarrow (Y, \sigma)$ is pre-R-Continuous and pre-S-Continuous. (iii) Any constant function f: $(X, t) \rightarrow (Y, \sigma)$ is pre-R-Continuous and pre-S-Continuous. Proof:

(i)Let f: $(X, \ell) \rightarrow (Y, \sigma)$ be injective function. Then pre-L-Continuity and pre-M Continuity follow from the fact that f⁻¹(f (A)) =A. This proves (i). (ii)Let f: $(X, \ell) \rightarrow (Y, \sigma)$ be surjective function. Since f is surjective, f (f⁻¹(B)) =B for every subset B of Y. Then f is both pre R-Continuous and pre-S-Continuous. This proves (ii). (iii)Suppose f(x) =y₀ for every x in X. Then f (f⁻¹(B)) =Y if y₀ \in B and f (f⁻¹(B)) = Φ , if y₀ \in Y\B. This proves (iii).

Corollary: 3.7

If f: $(X, t) \rightarrow (Y, \sigma)$ be bijective function then f is pre-L-Continuous, pre-M-Continuous, pre-R-Continuous and pre-S-Continuous.

Theorem: 3.8

Let f: $(X, t) \rightarrow (Y, \sigma)$. (i) If f is L-Continuous (resp. M-Continuous) then it is pre-L-Continuous (resp. pre-M-Continuous). (ii) If f is R-Continuous (resp. S-Continuous) then it is pre-R-Continuous (resp. pre-S-Continuous). Proof:

(i) Let $A \subseteq X$ be pre-open (resp. pre-closed) in X. since every pre-open (resp. pre-closed) set is open (resp. closed) and since f is L-continuous (resp. M-continuous), f⁻¹ (f(A)) is open (resp. closed) in X. Therefore f is pre-L-Continuous. (resp. pre-M-Continuous). (ii) Let $B \subseteq Y$ be pre-open (resp. pre-closed) in Y. since every pre-open (resp. pre-closed) set is open (resp. closed) and since f is R-continuous (resp. Scontinuous), f (f⁻¹(B)) is open (resp. closed) in Y. Therefore f is pre-R-Continuous (resp. pre-S-Continuous).

Theorem: 3.9

Let f: $(X, \mathcal{T}) \rightarrow Y$ be pre-L Continuous. Then int(cl(A)) is fsaturated whenever A is f-saturated and semi-closed. Proof:

Let $A \subseteq X$ be f-saturated. Since f is pre-L-Continuous $\Rightarrow A$ is pre-open set in $X \Rightarrow A \subseteq int(cl(A))$ and since A is semiclosed $\Rightarrow int(cl(A)) \subseteq A$. Therefore int(cl(A)) = A. since A is f-saturated $\Rightarrow f^{-1}(f(A)) = A$. That implies $int(cl(A))=f^{-1}(f(int(cl(A))))$. Therefore Hence int(cl(A)) is f-saturated whenever A is f-saturated and semi-closed.

Theorem: 3.10

Let f: $(X, l) \rightarrow Y$ be pre-M-Continuous. Then cl(int(A) is fsaturated whenever A is f-saturated and semi-open. Proof:

Let $A \subseteq X$ be f-saturated. Since f is pre-M-Continuous $\Rightarrow A$ is pre-closed set in $X \Rightarrow cl(int(A)) \subseteq A$ and since A is semiopen $\Rightarrow A \subseteq cl(int(A))$. Therefore cl(int(A))=A. since A is f-saturated $\Rightarrow f^{-1}(f(A))=A$. That implies $cl(int(A))=f^{-1}(f(cl(int(A))))$. Hence cl(int(A)) is f-saturated whenever A is f-saturated and semi-open.

Theorem: 3.11

Let f: $X \rightarrow (Y, \sigma)$ be pre-R-Continuous. Then int(cl(B)) is f⁻¹ – saturated whenever B is f⁻¹ –saturated and semi-closed.

Proof:

Let $B \subseteq Y$ be f⁻¹-saturated. Since f is pre-R-Continuous \Rightarrow B is pre-open set in $Y \Rightarrow int(cl(B)) \supseteq B$, and since B is semi-closed $\Rightarrow int(cl(B)) \subseteq B$, Therefore int(cl(B))=B, since B is f⁻¹-saturated $\Rightarrow f(f^{-1}(B)) = B$ which implies that $f(f^{-1}(int(cl(B)))) = int(cl(B))$, Therefore hence int(cl(B)) is f⁻¹saturated.

Theorem: 3.12

Let f: $X \rightarrow (Y, \sigma)$ be pre-S-Continuous Then cl(int(B)) is f⁻¹ – saturated whenever B is f⁻¹–saturated and semi-open.

Proof:

Let $B \subseteq Y$ be f⁻¹ –saturated. Since f is pre-S-Continuous \Rightarrow_B is pre-closed set in $Y \Rightarrow$ cl(int(B)) \subseteq B, and since B is semi-open $\Rightarrow B \subseteq$ cl(int(B)), Therefore cl(int(B))=B, since B is f⁻¹ –saturated \Rightarrow f(f¹(B)) = B which implies that f(f¹(cl(int(B))))= cl(int(B)), Therefore hence cl(int(B)) is f⁻¹ –saturated

4. Properties

In this section we prove certain theorems related with preopen and pre-closed functions.

Theorem: 4.1

(i) Let f: $(X, \mathcal{I}) \rightarrow (Y, \sigma)$ be pre-open and pre-Continuous, Then f is pre-L-Continuous.

(ii) Let f: $(X, \mathcal{I}) \rightarrow (Y, \sigma)$ be open and pre-Continuous, Then f is pre-R-Continuous.

Proof:

(i) Let $A \subseteq X$ be pre-open in X. Let $f: (X, t) \rightarrow (Y, \sigma)$ be pre-open and pre-Continuous. Since f is pre-open $\Rightarrow f(A)$ is pre-open in Y and since f is pre-continuous $\Rightarrow f^{-1}(f(A))$ is open in X. Therefore f is pre-L Continuous.

This proves (i).

(ii) Let $B \subseteq Y$ be pre-open in Y. Let f: $(X, t) \rightarrow (Y, \sigma)$ be open and pre-Continuous. Since f is pre-continuous $\Rightarrow f^{-1}(B)$ is open in X and since f is open $\Rightarrow f(f^{-1}(B))$ is open in Y. Therefore f is pre-R Continuous, This proves (ii).

Theorem: 4.2

(i) Let f: $(X, \mathcal{I}) \rightarrow (Y, \sigma)$ be pre-closed and pre-Continuous, Then f is pre-M Continuous.

(ii)Let f: $(X, \mathcal{I}) \rightarrow (Y, \sigma)$ be closed and pre-Continuous, Then f is pre-S Continuous.

Proof :

(i) Let $A \subseteq X$ be pre-closed in X. Let f: $(X, \ell) \rightarrow (Y, \sigma)$ be pre-closed and pre-Continuous. Since f is pre-closed $\Rightarrow f(A)$ is pre-closed in Y and since f is pre-continuous $\Rightarrow f^{-1}(f(A))$ is closed in X. Therefore f is pre-M Continuous. This proves (i).

(ii) Let $B \subseteq Y$ be pre-closed in Y. Let f: $(X, l) \rightarrow (Y, \sigma)$ be closed and pre-Continuous. since f is pre-continuous $\Rightarrow f^{-1}(B)$ is closed in X and since f is closed $\Rightarrow f(f^{-1}(B))$ is closed in Y. Therefore f is pre-S Continuous, This proves (ii).

Theorem: 4.3

Let X be a topological space.

(i) If A is a pre-open subspace of X, the inclusion function j:
A→X is pre-L-continuous and pre-R-continuous.
(ii) If A is a pre-closed subspace of X, the inclusion function

(ii) If A is a pre-closed subspace of X, the inclusion function j: $A \rightarrow X$ is pre-M-continuous and pre-S-continuous.

Proof:

(i) Suppose A is a pre-open subspace of X. Let $j: A \rightarrow X$ be an inclusion function. Let $U \subseteq X$ be pre-open in X then j ($j^{-1}(U)$) = j (U n A) = U n A Which is open in X. Hence j is pre-R-continuous. Now, let $U \subseteq A$ be pre-open in A. Then j^{-1} ¹ (j (U)) = j⁻¹(U) =U which is open in A. Hence j is pre-Lcontinuous. This proves (i). (ii) Suppose A is a pre-closed subspace of X. Let j: $A \rightarrow X$ be an inclusion function. Let U $\bigcirc X$ be pre-closed in X then j (j⁻¹(U)) = j (U n A) =U n A, Which is closed in X. Hence j is pre-S-continuous. Now, let $U \bigcirc A$ be pre-closed in A Then j⁻¹(j(U)) = j⁻¹(U) =U which is closed in A. Hence j is pre-M-continuous. This proves (ii).

Theorem: 4.4

Let g: $Y \rightarrow Z$ and f: $X \rightarrow Y$ be any two functions. Then the following hold. (i) If g: $Y \rightarrow Z$ is pre-L-continuous (resp. pre-M-continuous) and f: $X \rightarrow Y$ is pre-open (resp. pre-closed) and continuous, then gOf: $X \rightarrow Z$ is pre-L-continuous (resp. pre-M-continuous). (ii) If g: $Y \rightarrow Z$ is open (resp. closed) and pre-continuous and f: $X \rightarrow Y$ is R-continuous (resp. S-continuous), then gOf is pre-R-continuous (resp. pre-S-continuous).

Proof:

(i) Suppose g is pre-L-continuous (resp. pre-M continuous) and f is pre- open (resp. pre-closed) and continuous. Let A be pre-open (resp. pre-closed) in X. Then $(gOf)^{-1}$.(gO $f(A)=f^{-1}(g^{-1}(g(f(A))))$. Since f is pre-open (resp. pre-closed) \Rightarrow f (A) is pre-open (resp. pre-closed) in Y. since g is pre-L-continuous (resp. pre-M-continuous), $\Rightarrow g^{-1}(g(f(A)))$ is open (resp. closed) in Y, since f is continuous \Rightarrow f⁻¹(g⁻¹) $^{1}(g(f(A))))$ is open (resp. pre-closed) in X. Therefore, g O f is pre-L-continuous (resp. pre-M-continuous). This proves (i). (ii) Let f: $X \rightarrow Y$ be R-continuous (resp. S-continuous) and g: $Y \rightarrow Z$ be open (resp. closed) and pre-continuous. Let B be pre-open (resp. pre-closed) in Z. Then $(g O f) (g O f)^{-1}(B)$ = (g O f) (f⁻¹g⁻¹(B) is open (resp. closed) in Y. since f is Rcontinuous (resp. S-continuous) \Rightarrow f(f $^{-1}(g^{-1}(B))$) is open (resp. closed) in Y. since g is open (resp. closed) \Rightarrow g(f(f - $^{1}(g^{-1}(B)))$ is open (resp. closed) in Z. Therefore, g O f is pre-R-continuous (resp. pre-S-continuous). This proves (ii).

Theorem: 4.5

If f: $X \rightarrow Y$ is pre-L-continuous and if A is an open subspace of X, then the restriction of f to A is pre-L-continuous.

Proof:

Let $h = f/_A$. Then h = fOj, where j is the inclusion map j: A $\rightarrow X$. Since j is open and continuous and since f: $X \rightarrow Y$ is pre-L-continuous, using theorem (4.4(i)), h is pre-L-continuous.

Theorem: 4.6

If f: $X \rightarrow Y$ is pre-M-continuous and if A is a closed subspace of X, then the restriction of f to A is pre-M-continuous.

Proof:

Let $h = f/_A$. Then h = fOj, where j is the inclusion map j: A $\rightarrow X$. Since j is closed and continuous and since f: $X \rightarrow Y$ is pre-M-continuous, using theorem (4.4 (i)), h is pre-M-continuous.

Theorem: 4.7

Let f: $X \rightarrow Y$ be pre-R-continuous. Let $f(x) \subseteq Z \subseteq Y$ and f(X) be open in Z. Let h: $X \rightarrow Z$ be obtained by from f by restricting the co-domain of f to Z. Then h is pre-R-continuous.

Proof:

Clearly h = j O f where j: $f(x) \rightarrow Z$ is an inclusion map. Since f(X) is open in Z, the inclusion map j is both open and pre-continuous. Then by applying theorem 4.4(ii), h is pre-R-continuous.

Theorem: 4.8

Let f: $X \rightarrow Y$ be pre-S-continuous. Let $f(x) \subseteq Z \subseteq Y$ and f(X) be closed in Z. Let h: $X \rightarrow Z$ be obtained by from f by restricting the co-domain of f to Z. Then h is pre-S-continuous.

Proof:

Clearly h = j O f where j: $f(x) \rightarrow Z$ is an inclusion map. Since f(X) is closed in Z, the inclusion map j is both closed and pre-continuous. Then by applying theorem 4.4(ii), h is pre-S-continuous.

Now we establish the pasting lemmas for pre-R-continuous and pre-S-continuous functions.

Theorem: 4.9

Let $X=A \cup B$. Let f: $A \rightarrow (Y, \sigma)$ and g: $B \rightarrow (Y, \sigma)$ be pre-R-continuous (res. pre-S-continuous) f(x)=g(x) for every $x \in A \cap B$, then f and g combined to give a pre-Rcontinuous (res. pre-S-continuous) function h: $X \rightarrow Y$ defined by h(x)=f(x) if $x \in A$, and h(x)=g(x) if $x \in B$.

Proof:

Let C be a pre-open (res. Pre-closed) set in Y. Now $hh^{-1}(C) = h (f^{-1}(C) \bigcup g^{-1}(C)) = h (f^{-1}(C)) \bigcup h (g^{-1}(C)) = f (f^{-1}(C)) \bigcup g (g^{-1}(C))$. Since f is pre-R-continuous (res. pre-S-continuous), f (f^{-1}(C)) is open (resp. closed) in Y and Since g is pre-R-continuous (res. pre-S-continuous), g (g^{-1}(C)) is open (resp. closed) in Y. Therefore, $hh^{-1}(C)$ is open (resp. closed) in Y. Hence h is pre-R-continuous (resp. pre-S-continuous).

5. Characterizations

Theorem: 5.1

A function f: $X \rightarrow Y$ is pre-L-continuous if and only if f ${}^{1}(f^{\#}(A))$ is closed in X for every pre-closed subset A of X. Proof:

Suppose f is pre-L-continuous. Let A be pre-closed in X. Then G = X\A is pre-open in X. Since f is pre-L-continuous and since G is pre-open in $X \Longrightarrow f^{-1}(f(G))$ is open in X. By applying lemma ((2.5)-(i)), $f^{-1}(f^{*}(A)) = X \setminus f^{-1}(f(X \setminus A)) = X$ $\setminus f^{-1}(f(G))$. That implies $f^{-1}(f^{*}(A))$ is closed in X. Conversely, we assume that $f^{-1}(f^{*}(A))$ is closed in X for every pre-closed subset A of X. Let G be a pre-open in X. By our assumption, $f^{-1}(f^{*}(A))$ is closed in X, where A = X\G. By using lemma ((2.5)-(ii)) $\Longrightarrow f^{-1}(f(G)) = X \setminus f^{-1}(f^{*}(X \cap G)) = X \setminus f^{-1}(f^{*}(A))$. That implies $f^{-1}(f(G)) = X \setminus f^{-1}(f^{*}(A))$. That implies $f^{-1}(f(G)) = X \setminus f^{-1}(f^{*}(A))$.

Theorem: 5.2

A function f: $X \rightarrow Y$ is pre-M-continuous if and only if $f^{-1}(f^{\#}(G))$ is open in X for every pre-open subset G of X.

Proof:

Suppose f is pre-M-continuous. Let G be pre- open in X. Then A = X \G is pre-closed in X. Since f is pre-Mcontinuous and since A is pre-closed in X \Longrightarrow f⁻¹(f (A)) is closed in X. By lemma ((2.5)-(i)) \Longrightarrow f⁻¹(f [#](G))=X\f⁻¹(f(X\G))=X\f⁻¹(f(A)). That implies f⁻¹(f [#](G)) is open in X. Conversely, we assume that f⁻¹(f [#](G)) is open in X for every pre-open subset G of X. Let A be a pre-closed in X. By our assumption, f⁻¹(f [#](G)) is open in X, where G = X\A. By using lemma ((2.5) - (ii)) \Longrightarrow f⁻¹(f(A)) = X \f⁻¹(f [#](X\A)) = X \f⁻¹(f [#](G)). That implies f⁻¹(f(A)) is closed in X. Therefore, hence f is pre-M-continuous.

Theorem: 5.3

The function f: $X \rightarrow Y$ is pre-R-continuous if and only if f[#] (f⁻¹(B)) is closed in Y for every pre-closed subset B of Y.

Proof:

Suppose f is pre-R-continuous. Let B be pre-closed in Y. Then G=Y\B is pre-open in Y. since f is pre-R-continuous and since G is pre-open in Y \Longrightarrow f (f ⁻¹(G)) is open in Y. Now by using lemma ((2.6)(i)) \Longrightarrow f [#](f ⁻¹(B)) = Y \ f(f ⁻¹(Y\B)) = Y \ f(f ⁻¹(G)). That implies f [#](f ⁻¹(B)) is closed in Y. Conversely, we assume that f [#](f ⁻¹(B)) is closed in Y for every pre-closed subset B of Y. Let G be pre-open in Y. Let B = Y\G. By our assumption, f [#](f ⁻¹(B)) is closed in Y. By lemma ((2.6) (ii)) \Longrightarrow f (f¹(G)) = Y \ (f [#](f ⁻¹(Y\G))) = Y \ f [#](f ⁻¹(B)), This proves that f(f ⁻¹(G)) is open in Y. Therefore, hence f is pre -R-continuous.

Theorem: 5.4

The function f: $X \rightarrow Y$ is pre-S-continuous if and only if f[#] (f⁻¹(G)) is open in Y for every pre-open subset G of Y. Proof:

Suppose f is pre-S-continuous. Let G be pre-open in Y. Then B=Y\G is pre-closed in Y. Since f is pre-S-continuous and since B is pre-closed in $Y \Longrightarrow f(f^{-1}(B))$ is open in Y. Now by using lemma ((2.6)(i)) $\Longrightarrow f^{\#}(f^{-1}(G)) = Y \setminus f(f^{-1}(Y)) = Y \setminus f(f^{-1}(B))$. That implies $f^{\#}(f^{-1}(G))$ is open in Y. Conversely, we assume that $f^{\#}(f^{-1}(G))$ is open in Y for every pre -open subset G of Y. Let B be pre-closed in Y. Let G = Y\B. By our assumption, $f^{\#}(f^{-1}(G))$ is open in Y. By lemma ((2.6) (ii)) $\Longrightarrow f(f^{-1}(B)) = Y \setminus (f^{\#}(f^{-1}(Y\setminus B))) = Y \setminus f^{\#}(f^{-1}(G))$, This proves that $f(f^{-1}(B))$ is closed in Y. Therefore, hence f is pre -S-continuous.

Theorem: 5.5

Let f: $(X, \ell) \rightarrow Y$ be a function. Then the following are equivalent. (i) f is pre-L-continuous, (ii)for every pre-closed subset A of X, f⁻¹(f[#](A) is closed in X, (iii)for every $x \in X$ and for every pre-open set U in X with $f(x) \in f(U)$ there is an open set G in X with $x \in G$ and $f(G) \subseteq f(U)$, (iv) f⁻¹(f(int(cl(A)))) \subseteq int(f⁻¹(f(A))) for every semi-closed subset A of X. (v) cl(f⁻¹(f[#](A))) \subseteq f⁻¹(f[#](cl(int(A)))) for every semi-open subset A of X. Proof: (i) \Leftrightarrow (ii) : follows from theorem 5.1. (i) \Leftrightarrow (iii): Suppose f is pre-L-continuous. Let U be pre-open set in X such that $f(x) \in f(U)$. Since f is pre-L-continuous, f⁻¹(f(U)) is open in X. Since $x \in f^{-1}(f(U))$ there is an open set G in X, such that $x \in G \subseteq f^{-1}(f(U)) \Rightarrow$ $f(G)-f(f^{-1}(f(U))) \subseteq f(U)$. This proves (iii) conversely, suppose (iii) holds. Let U be pre-open set in X and $x \in f^{-1}(f(U))$. Then $f(x) \in f(U)$. By using (iii), there is an open set G in X containing x such that $f(G) \subseteq f(U)$. Therefore $x \in G$ $\subseteq f^{-1}(f(G) \subseteq f^{-1}(f(U))$. That implies $f^{-1}(f(U))$ is open set in X, This completes the proof for (i) \Leftrightarrow (iii).

(i) \Leftrightarrow (iv):Suppose f is pre-L-continuous. Let A be a semiclosed subset of X. Then int(cl(A)) is pre-open set in X. By the pre-L-continuity of f, we see that f⁻¹ (f (int(cl(A)))) is open in X, \Rightarrow f⁻¹ (f (int(cl(A)))) \subseteq int(f⁻¹ (f (int(cl(A))))). since A is semi-closed in X, We have f⁻¹(f(int(cl(A)))) \subseteq f⁻¹ (f(A)), \Rightarrow int(f⁻¹(f(int(cl(A)))) \subseteq int(f⁻¹(f(A))), It follows that f⁻¹ (f (int(cl(A)) \subseteq int(f⁻¹(f(A))). This proves (iv).

Conversely, we assume that (iv) holds. Let U be pre-open set in $X \Longrightarrow f^{-1}(f(U)) \subseteq f^{-1}(f(int(cl(U))))$, since U is semi-closed by applying (iv) we get $f^{-1}(f(int(cl(U)))) \subseteq int(f^{-1}(f(U)))$, Therefore $f^{-1}(f(U)) \subseteq int(f^{-1}(f(U)))$ and hence $f^{-1}(f(U))$ is open in X. This proves that f is pre-L-continuous. (ii) \Leftrightarrow (v): Suppose (ii) holds. Let A be a semi-open subset of X. By using (ii) $f^{-1}(f^{*}(cl(int(A))))$ is closed in X. since A is semi-open $f^{-1}(f^{*}(A)) \subseteq f^{-1}(f^{*}(cl(int(A))))$, it follows that $cl(f^{-1}(f^{*}(A))) \subseteq f^{-1}(f^{*}(cl(int(A))))$. This proves (v), Conversely, let us assume that (v) holds. Let A be a preclosed subset of X, $\Rightarrow f^{-1}(f^{*}(cl(int(A)))) \subseteq f^{-1}(f^{*}(A))$, since A is semi-open, by (v), we see that $cl(f^{-1}(f^{*}(A))) \subseteq f^{-1}$ $f^{*}(cl(int(A))))$, $\Rightarrow cl(f^{-1}(f^{*}(A))) \subseteq f^{-1}(f^{*}(A))$, Therefore $f^{-1}(f^{*}(A))$ is closed in X. This proves (ii)

Theorem: 5.6

Let f: $(X, \mathcal{T}) \rightarrow Y$ be a function. Then the following are equivalent. (i) f is pre-M-continuous, (ii) for every pre-open subset G of X, f⁻¹ (f[#](G) is open in X, (iii) cl(f⁻¹(f(A))) \subseteq f⁻¹(f(cl(int(A)))) for every semi-open subset A of X. (iv) f⁻¹(f[#](int(cl(A)))) \subseteq int(f⁻¹(f[#](A))) for every semi-closed subset A of X.

Proof:

(i) \Leftrightarrow (ii): follows from theorem 5.2. (i) \Leftrightarrow (iii) :Suppose f is pre-M-continuous . Let A be a semi-open set in X. cl(int(A)) is pre-closed in X, Since f is pre-M-continuous \Rightarrow f⁻¹(f(cl(int(A)))) is closed in X, \Rightarrow cl(f⁻¹(f(cl(int(A))))) = f⁻¹(f(cl(int(A)))), Since A is semi-open in X, we see that f⁻¹(f(A)) \subseteq f⁻¹(f(cl(int(A)))) \Rightarrow cl(f⁻¹(f(A))) \subseteq cl(f⁻¹(f(A))) \subseteq cl(f⁻¹(f(cl(int(A))))), it follows that, cl(f⁻¹(f(A))) \subseteq cl(f⁻¹(f(cl(int(A))))) = f⁻¹(f(cl(int(A)))). This proves (iii).

conversely, suppose (iii) holds. Let A be pre-closed subset in $X \Longrightarrow f^{-1}(f(cl(int(A)))) \subseteq f^{-1}(f(A))$, Since A is semi-open by applying (iii), $cl(f^{-1}(f(A))) \subseteq f^{-1}(f(cl(int(A))))$, $\Longrightarrow cl(f^{-1}(f(A))) \subseteq f^{-1}(f(a))$,

 $^{1}(f(A))) \subseteq f^{-1}(f((A)))$, That implies $f^{-1}(f(A))$ is closed set in X. This completes the proof for (i) \Leftrightarrow (iii).

(ii) \Leftrightarrow (iv): Suppose (ii) holds. Let A be a semi-closed subset of X. Then int(cl(A)) is pre-open in X . By (ii), f⁻¹(f "(int(cl(A)))) is open in X, \Rightarrow f⁻¹(f"(int(cl(A)))) \subseteq int(f⁻¹(f "(int(cl(A)))) Since A is semi closed \Rightarrow f⁻¹(f"(int(cl(A)))) \subseteq f⁻¹(f"(A)) \Rightarrow int(f⁻¹(f"(int(cl(A)))) \subseteq int(f⁻¹(f"(A))), we see that f⁻¹(f"(int(cl(A)))) \subseteq int(f⁻¹(f"(A))). This proves (iv). conversely, suppose (iv) holds. Let G be preopen in X \Rightarrow f⁻¹(f"(G)) \subseteq f⁻¹(f"(int(cl(G)))). Since G is semi-closed in X, by using (iv) \Rightarrow f⁻¹(f"(int(cl(G)))) \subseteq int(f -¹(f"(G))). we see that f⁻¹(f"(G)) \subseteq f⁻¹(f"(int(cl(G)))) \subseteq int(f -¹(f"(G))), \Rightarrow f⁻¹(f"(G)) \subseteq int(f⁻¹(f"(G))), Then it follows that f⁻¹(f"(G)) is open in X. This proves (ii).

Theorem: 5.7

Let f: $X \rightarrow (Y, \sigma)$ be a function and σ be a space with a base consisting of f⁻¹saturated open sets. Then the following are equivalent. (i) f is pre-R-continuous, (ii) for every preclosed subset B of X, f[#](f⁻¹(B) is closed in Y, (iii)for every $x \in X$ and for every pre-open set V in Y with $x \in f^{-1}(V)$ there is an open set G in Y with $f(x) \in G$ and f⁻¹(G) \subseteq f⁻¹ (V), (iv) f(f⁻¹(int(cl(B)))) \subseteq int(f(f⁻¹(B))) for every semiclosed subset B of Y. (v) cl(f[#](f⁻¹(B))) \subseteq f[#](f⁻¹ (cl(int(B)))) for every semi-open subset B of Y. proof:

(i) \Leftrightarrow (ii): follows from theorem 5.3. (i) \Leftrightarrow (iii) :Suppose f is pre-R-continuous. Let V be a pre-open set in Y such that x \in f⁻¹(V). Since f is pre-R-continuous, f (f⁻¹(V)) is open in Y. f (x) \in f(f⁻¹(V)) there is an open set G in Y such that $f(x) \in G \subseteq f(f^{-1}(V))$. That implies $x \in f^{-1}(G) \subseteq f^{-1}(f(f^{-1}(V)))$ $^{1}(V)) \in f^{-1}(V)$. This proves (iii). Conversely, suppose (iii) holds. Let V be pre-open in Y and $y \in f(f^{-1}(G))$, Then y=f(x) for some $x \in f^{-1}(V)$. By using (iii) there is an open set G in Y containing f(x) such that $f^{-1}(G) \subseteq f^{-1}(V)$. We choose G to a f⁻¹-saturated in Y. Then G= f (f⁻¹(G)) \subseteq f(f⁻¹ $^{1}(V)$). This proves that f (f $^{-1}(V)$) is open in Y. This proves that f is pre-R-continuous. (i) \Leftrightarrow (iv): Suppose f is pre-Rcontinuous. Let B be semi-closed subset in Y. Then int(cl(B)) is pre-open set in Y. By the pre-R-continuity of f, we see that, f (f $^{-1}(int(cl(B))))$ is open in $Y \Longrightarrow f$ (f $^{1}(int(cl(B)))) \subseteq int(f(f^{-1}(int(cl(B)))))$ since B is semi-closed in $Y \Longrightarrow f$ (f $^{-1}(int(cl(B)))) \subseteq f(f {}^{-1}(B))$, \Longrightarrow int(f (f $^{1}(int(cl(B))))) \subseteq int(f(f^{-1}(B)))$. Then It follows that f (f - $^{1}(int(cl(B)))) \subset int(f(f^{-1}(B)))$. This proves (iv). Conversely, we assume that (iv) holds. Let B be pre-open set in $Y \Rightarrow f(f)$ $^{-1}(B)) \subseteq f$ (f $^{-1}(int(cl(B))))$. Since B is semi-closed by applying (iv) we get f (f $^{-1}(int(cl(B)))) \subseteq int(f(f {}^{-1}(B)))$ Therefore $f(f^{-1}(B)) \subseteq int(f(f^{-1}(B)))$ and hence $f(f^{-1}(B))$ is open in Y. This proves that f is pre-R-continuous. (ii) \Leftrightarrow (v): Suppose (ii) holds. Let B be a semi-open subset of Y. By using (ii) $f^{\#}(f^{-1}(cl(int(B))))$ is closed in Y. Since B is semi-open in Y, we see that, $f^{\#}(f^{-1}(B)) \subseteq f^{\#}(f^{-1}(B))$ $^{1}(cl(int(B))))$, it follows that $cl(f^{\#}(f^{-1}(B))) \subseteq f^{\#}(f^{-1}(B))$ ¹(cl(int(B)))). This proves (v). Conversely, let us assume that

(v) holds. Let B be a pre-closed subset of $Y \Longrightarrow f^{\#}(f^{-1}(cl(int(B)))) \subseteq f^{\#}(f^{-1}(B))$, since B is semi-open in Y, by (v), we see that $cl(f^{\#}(f^{-1}(B))) \subseteq f^{\#}(f^{-1}(cl(int(B)))) \Longrightarrow cl(f^{\#}(f^{-1}(B))) \subseteq f^{\#}(f^{-1}(cl(int(B)))) \subseteq f^{\#}(f^{-1}(B)) \Longrightarrow cl(f^{\#}(f^{-1}(B))) \subseteq f^{\#}(f^{-1}(B))$, Therefore $f^{\#}(f^{-1}(B))$ is closed in Y. This proves (ii).

Theorem: 5.8

Let f: $X \rightarrow (Y, \sigma)$ be a function. Then the following are equivalent. (i) f is pre-S-continuous, (ii) for every pre-open subset V of Y, f [#](f ⁻¹(V)) is open in Y, (iii) cl(f(f ⁻¹(B))) \subseteq f(f ⁻¹(cl(int(B)))) for every semi-open subset B of Y. (iv) f [#] (f ⁻¹(int(cl(B)))) \subseteq int (f [#](f ⁻¹(B))) for every semi-closed subset B of Y.

Proof: (i) \Leftrightarrow (ii): follows from theorem 5.4. (i) \Leftrightarrow (iii) :Suppose f is pre-S-continuous. Let B be a semi-open set in Y. Since f is pre-S-continuous, f(f⁻¹(cl(int(B)))) is closed in Y, \Rightarrow cl(f(f⁻¹(cl(int(B)))) \subseteq f(f⁻¹(cl(int(B)))). Since B is semi-open in Y, we see that f(f⁻¹(B)) \subseteq f(f⁻¹(cl(int(B)))), \Rightarrow cl(f(f⁻¹(B))) \subseteq cl(f(f⁻¹(cl(int(B)))), It follows that, cl(f(f⁻¹(B))) \subseteq cl(f(f⁻¹(cl(int(B))))) \subseteq f(f⁻¹(cl(int(B)))). This proves (iii).

conversely, suppose (iii) holds. Let B be pre-closed subset in $Y \Longrightarrow f(f^{-1}(cl(int(B)))) \subseteq f(f^{-1}(B))$ Since B is semi-open by applying (iii) $\Longrightarrow cl(f(f^{-1}(B))) \subseteq f(f^{-1}(cl(int(B)))) \Longrightarrow cl(f(f^{-1}(B))) \subseteq f(f^{-1}(cl(int(B)))) \subseteq f(f^{-1}(B)) \Longrightarrow cl(f(f^{-1}(B))) \subseteq f(f^{-1}(B)) \Longrightarrow cl(f(f^{-1}(B))) \subseteq f(f^{-1}(B)) \Longrightarrow cl(f(f^{-1}(B))) \subseteq f(f^{-1}(B))$ That implies $f(f^{-1}(B))$ is closed set in Y. This completes the proof for (i) \Leftrightarrow (iii).

(ii) \Leftrightarrow (iv): Suppose (ii) holds. Let B be a semi-closed subset of Y. Then int(cl(B)) is pre-open in Y. By (ii), f [#](f ⁻¹(int(cl(B)))) is open in Y \Rightarrow f [#](f ⁻¹(int(cl(B)))) \subseteq int(f [#](f ⁻¹(int(cl(B))))). Since B is a semi-closed, it follows that f [#](f ⁻¹(int(cl(B))))) \subseteq f [#](f ⁻¹(B)) \Rightarrow int(f [#](f ⁻¹(int(cl(B))))) \subseteq int(f [#](f ⁻¹(B))), we see that f [#](f ⁻¹(int(cl(B))))) \subseteq int(f [#](f ⁻¹(B))). This proves (iv).

Conversely, suppose (iv) holds. Let V be pre-open in $Y \Longrightarrow f^{#}(f^{-1}(V)) \subseteq f^{#}(f^{-1}(int(cl(V))))$. Since V is semi-closed in Y, by using (iv), we see that $f^{#}(f^{-1}(int(cl(V)))) \subseteq int(f^{#}(f^{-1}(V)))$, $\Longrightarrow f^{#}(f^{-1}(V)) \subseteq f^{#}(f^{-1}(int(cl(V)))) \subseteq int(f^{#}(f^{-1}(V)))$, $\Longrightarrow f^{#}(f^{-1}(V)) \subseteq int(f^{#}(f^{-1}(V)))$ Then it follows that $f^{#}(f^{-1}(V))$ is open in Y. This proves (ii).

6. Conclusion

In this paper the notions of Pre-L-Continuity, Pre-M-Continuity, Pre-R-Continuity and Pre-S-Continuity of a function f: $X \rightarrow Y$ between a topological space and a non empty set are introduced. The purpose of this paper is to introduce, Pre- ρ -continuity. Here we discuss their links with Pre-open, Pre-closed sets. Also we establish pasting lemmas for Pre-R-continuous and Pre-s-continuous functions and obtain some characterizations for, Pre- ρ -

continuity. We have put forward some examples to illustrate our notions

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