(v) holds. Let B be a pre-closed subset of $Y \Rightarrow f^{\#}(f^{-1}(cl(int(B)))) \subseteq f^{\#}(f^{-1}(B))$, since B is semi-open in Y, by (v), we see that $cl(f^{\#}(f^{-1}(B))) \subseteq f^{\#}(f^{-1}(cl(int(B))))$, $\Rightarrow cl(f^{\#}(f^{-1}(B))) \subseteq f^{\#}(f^{-1}(cl(int(B)))) \subseteq f^{\#}(f^{-1}(B)) \Rightarrow cl(f^{\#}(f^{-1}(B))) \subseteq f^{\#}(f^{-1}(B))$, Therefore $f^{\#}(f^{-1}(B))$ is closed in Y. This proves (ii).

Theorem: 5.8

Let f: $X \rightarrow (Y, \sigma)$ be a function. Then the following are equivalent. (i) f is pre-S-continuous, (ii) for every pre-open subset V of Y, $f^{\#}(f^{-1}(V))$ is open in Y, (iii) $cl(f(f^{-1}(B))) \subseteq f(f^{-1}(cl(int(B))))$ for every semi-open subset B of Y. (iv) $f^{\#}(f^{-1}(int(cl(B)))) \subseteq int(f^{\#}(f^{-1}(B)))$ for every semi-closed subset B of Y.

Proof: (i)$\Leftrightarrow$(ii): follows from theorem 5.4. (i)$\Leftrightarrow$(iii) :Suppose f is pre-S-continuous. Let B be a semi-open set in Y. Since f is pre-S-continuous, $f(f^{-1}(cl(int(B))))$ is closed in Y, $\Rightarrow cl(f(f^{-1}(cl(int(B))))) \subseteq f(f^{-1}(cl(int(B))))$. Since B is semi-open in Y, we see that $f(f^{-1}(B)) \subseteq f(f^{-1}(cl(int(B))))$, $\Rightarrow cl(f(f^{-1}(B))) \subseteq cl(f(f^{-1}(cl(int(B)))))$, It follows that, $cl(f(f^{-1}(B))) \subseteq cl(f(f^{-1}(cl(int(B))))) \subseteq f(f^{-1}(cl(int(B))))$. This proves (iii).

conversely, suppose (iii) holds. Let B be pre-closed subset in $Y \Rightarrow f(f^{-1}(cl(int(B)))) \subseteq f(f^{-1}(B))$ Since B is semi-open by applying (iii) $\Rightarrow cl(f(f^{-1}(B))) \subseteq f(f^{-1}(cl(int(B)))) \Rightarrow cl(f(f^{-1}(B))) \subseteq f(f^{-1}(cl(int(B)))) \subseteq f(f^{-1}(B)) \Rightarrow cl(f(f^{-1}(B))) \subseteq f(f^{-1}(B))$ That implies $f(f^{-1}(B))$ is closed set in Y. This completes the proof for (i)$\Leftrightarrow$(iii).
(ii)$\Leftrightarrow$(iv): Suppose (ii) holds. Let B be a semi-closed subset of Y. Then int(cl(B)) is pre-open in Y. By (ii), $f^{\#}(f^{-1}(int(cl(B))))$ is open in $Y \Rightarrow f^{\#}(f^{-1}(int(cl(B)))) \subseteq int(f^{\#}(f^{-1}(int(cl(B)))))$. Since B is a semi-closed, it follows that $f^{\#}(f^{-1}(int(cl(B)))) \subseteq f^{\#}(f^{-1}(B)) \Rightarrow int(f^{\#}(f^{-1}(int(cl(B))))) \subseteq int(f^{\#}(f^{-1}(B)))$, $\Rightarrow f^{\#}(f^{-1}(int(cl(B)))) \subseteq int(f^{\#}(f^{-1}(int(cl(B))))) \subseteq int(f^{\#}(f^{-1}(B)))$, we see that $f^{\#}(f^{-1}(int(cl(B)))) \subseteq int(f^{\#}(f^{-1}(B)))$. This proves (iv).

Conversely, suppose (iv) holds. Let V be pre-open in $Y \Rightarrow f^{\#}(f^{-1}(V)) \subseteq f^{\#}(f^{-1}(int(cl(V))))$. Since V is semi-closed in Y, by using (iv), we see that $f^{\#}(f^{-1}(int(cl(V)))) \subseteq int(f^{\#}(f^{-1}(V)))$, $\Rightarrow f^{\#}(f^{-1}(V)) \subseteq f^{\#}(f^{-1}(int(cl(V)))) \subseteq int(f^{\#}(f^{-1}(V)))$, $\Rightarrow f^{\#}(f^{-1}(V)) \subseteq int(f^{\#}(f^{-1}(V)))$ Then it follows that $f^{\#}(f^{-1}(V))$ is open in Y. This proves (ii).

6. Conclusion

In this paper the notions of Pre-L-Continuity, Pre-M-Continuity, Pre-R-Continuity and Pre-S-Continuity of a function f: $X \rightarrow Y$ between a topological space and a non empty set are introduced. The purpose of this paper is to introduce, Pre-ρ-continuity. Here we discuss their links with Pre-open, Pre-closed sets. Also we establish pasting lemmas for Pre-R-continuous and Pre-s-continuous functions and obtain some characterizations for, Pre-ρ-continuity. We have put forward some examples to illustrate our notions

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