

Fuzzy Multi-Attribute Approach in Project Assessment

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Abstract: *The point of departure in assessment of projects is the selection of planning and design advantages. So, the prime issues for the projects threefold. First, the evaluation criteria are generally multiple and often structured in multilevel hierarchies. Secondly, the evaluation process usually involves subjective assessments, resulting in the use of qualitative and imprecise data. Thirdly, other related interest groups input for planning and design alternative selection process should be considered. This paper presents a fuzzy multi-attribute approach in project assessment. The fuzzy analytic hierarchy process method is used to determine the weights for evaluation criteria, sub-criteria and alternatives. The vagueness in the alternatives selection process is dealt with using fuzzy numbers for linguistic terms. A crisp overall value is obtained for each alternative based on the concept of fuzzy multi-criteria decision making. A case study of project consisting of six criteria, twenty sub-criteria and five alternatives, illustrates the effectiveness of the proposed approach for these problems.*

Keywords: Project management, FAHP, fuzzy set theory, fuzzy numbers, project assessment

1. Introduction

Multiple criteria decision making is an analytic method to evaluate the advantages and disadvantages of alternatives based on multiple criteria. MCDM problems can be broadly classified into two categories: multiple objective programming and multiple criteria evaluation [1]. Since this study focuses mainly on the evaluation problem, the second category is emphasized. In any project, the planning and design phase is most critical to project success. Yet, when selecting an appropriate planning and design alternative, most public works owners lack the ability of effectively evaluate the candidates. Substandard planning and design work is often a direct result of inadequate tender selection. Thus, when initiating a construction project, most analysts must outsource engineering services in order to develop the preliminary plans and the associated design details [2,3,4].

The typical multiple criteria evaluation problem examines a set of feasible alternatives and considers more than one criterion to determine a priority ranking for alternative implementation. The five principles be considered when criteria are being formulated: completeness (the criteria must embrace all of the important characteristics of the decision-making problems), operational ability (the criteria will have to be meaningful for decision-makers and available for open study), decomposability (the criteria can be decomposed from higher hierarchy to lower hierarchy to simplify evaluation processes), non redundancy (the criteria must avoid duplicate measurement of the same performance) and minimum size (the number of criteria should be as small as possible so as to reduce the needed manpower, time and cost) [2]. Since the criteria of building Planning and design evaluation have diverse significance and meanings, we cannot assume that each evaluation criteria is of equal importance. There are many methods that can be employed to determine weights [1] such as the eigenvector method, weighted least square method, entropy method, Analytic Hierarchy Process (AHP), and Linear programming techniques for Multidimensional of Analysis Preference (LINMAP).

The selection of method depends on the nature of the problem. Building planning and design is a complex and wide-ranging problem, so this problem requires the most inclusive and flexible method. Since the AHP developed in [4-10] is a very useful decision analysis tool in dealing with multiple criteria decision problem, and has successfully been applied to many construction industry decision areas [11-20].

However, in operation process of applying AHP method, it is more easy and humanistic for evaluators to assess "criterion A is much more important than criterion B" than to consider "the importance of principle A and principle B is seven to one". Hence, [21] extended Saaty's AHP to the case where the evaluators are allowed to employ fuzzy ratios in place of exact ratios to handle the difficulty for people to assign exact ratios when comparing two criteria and derive the fuzzy weights of criteria by geometric mean method.

Many decision-making and problem-solving tasks are too complex to be understood quantitatively; however, people succeed by using knowledge that is imprecise rather than precise. Fuzzy set theory resembles human reasoning in its use of approximate information and uncertainty to generate decisions. It was specifically designed to mathematically represent uncertainty, vagueness and provide formalized tools for dealing with the imprecision intrinsic to many problems. By contrast, traditional computing demands precision down to each bit. Since knowledge can be expressed in a more natural by using fuzzy sets, many engineering and decision problems can be greatly simplified. Fuzzy Analytic hierarchy Process methods have been proposed by various authors. These methods are systematic approaches to the alternative selection and justification problem by using the concepts of fuzzy set theory and hierarchical structure analysis. Decision makers usually find that it is more confident to give interval judgments than fixed value judgments. This is because usually he/she is unable to explicit about his/her preferences due to the fuzzy nature of the comparison process.

Thus, this study applied fuzzy set theory [22,23] to managerial DM problem of alternative selection, with the intention of establishing a framework of incorporating FAHP and FMCDM, in order to help the decision maker to select the most appropriate planning and design candidate for building investment. This paper uses the FAHP to determine the criteria weights from subjective judgments of each DM group. Since the evaluation criteria of building planning and design have diverse connotations and meanings, there is no logical reason to treat them as if they are each of equal importance. Furthermore, the FMCDM was used to evaluate the synthetic performance of building planning and design alternatives, in order to handle qualitative criteria that are difficult to describe in crisp values, thus strengthening the comprehensiveness and reasonableness of the DM process.

This paper is organized as follows: Section 2 provides discussion on the fuzzy set theory with the fuzzy numbers and linguistic variables able to uses in the planning and design of projects. In Section 3, brief introduction previous research of FAHP and FMCDM methods, establishment of a hierarchical structure for planning and design specifications in projects, proposed technique for planning and design of project, illustration to demonstrate the synthesis decision using integration of FAHP and FMCDM approach are introduced. Section 4 then provides the concluding remarks of this study.

2. Fuzzy Set Theory

“Not very clear”, “probably so”, “very likely”, these terms of expression can be heard very often in daily life, and their commonality is that they are more or less tainted with uncertainty. With different daily decision making problems of diverse intensity, the results can be misleading if the fuzziness of human decision-making is not taken into account. However, since [22] was first proposed fuzzy set theory, and [23] described the decision-making method in fuzzy environments, an increasing number of studies have dealt with uncertain fuzzy problems by applying fuzzy set theory. With such an idea in mind, this study includes fuzzy decision-making theory, considering the possible fuzzy subjective judgment during planning and design constraints evaluation. Fuzzy set theory implements classes or groupings of data with boundaries that are not sharply defined (i.e., fuzzy). Any methodology or theory implementing “crisp” definitions such as classical set theory, arithmetic and programming may be “fuzzified” by generalizing the concept of a crisp set to a fuzzy set with blurred boundaries. The benefit of extending crisp theory and analysis methods to fuzzy techniques is the strength in solving real-world problems, which inevitably entail some degree of imprecision and noise in the variables and parameters measured and processed for the application.

Accordingly, linguistic variables are a critical aspect of some fuzzy logic applications, where general terms such as “large,” “medium,” and “small” are each used to capture a range of numerical values. Fuzzy set theory encompasses fuzzy logic, fuzzy arithmetic, fuzzy mathematical programming, fuzzy topology, fuzzy graph theory, and fuzzy data analysis, though the term fuzzy logic is often used to

describe all of these. The applications of fuzzy theory in this study are elaborated as follows:

2.1 Fuzzy Number

Fuzzy numbers are a fuzzy subset of real numbers and they represent the expansion of the idea of confidence interval. According to the definition, the following is the explanation for the features and calculation of the Triangular Fuzzy Number (TFN). According to the nature of TFN and the extension principle put forward by [22], the triangular fuzzy number is represented as shown in Fig.1.



Figure 1: The membership function of the triangular fuzzy number

A fuzzy number \tilde{A} on R be a TFN if its membership function $\mu_{\tilde{A}}(x): R \rightarrow [0,1]$ validates the conditions in equation (1).

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < L, \\ (x-L)/(M-L), & L \leq x \leq M, \\ (U-x)/(U-M), & M \leq x \leq U, \\ 0, & x > U, \end{cases} \quad (1)$$

where L and U stand for the lower and upper bounds of the fuzzy number \tilde{A} , respectively, and M for the modal value (see Fig.1). The TFN can be denoted by $\tilde{A} = (L, M, U)$ and the following is the operational laws of two TFNs, $\tilde{A}_1 = (L_1, M_1, U_1)$ and $\tilde{A}_2 = (L_2, M_2, U_2)$ as follows:

Addition of a fuzzy numbers \oplus

$$\begin{aligned} \tilde{A}_1 \oplus \tilde{A}_2 &= (L_1, M_1, U_1) \oplus (L_2, M_2, U_2) \\ &= (L_1 + L_2, M_1 + M_2, U_1 + U_2) \end{aligned} \quad (2)$$

Multiplication of a fuzzy numbers \otimes

$$\begin{aligned} \tilde{A}_1 \otimes \tilde{A}_2 &= (L_1, M_1, U_1) \otimes (L_2, M_2, U_2) \\ &= (L_1 L_2, M_1 M_2, U_1 U_2) \\ &\text{for } L_i > 0, M_i > 0, U_i > 0. \end{aligned} \quad (3)$$

Subtraction of a fuzzy numbers $(-)$

$$\begin{aligned} \tilde{A}_1 - \tilde{A}_2 &= (L_1, M_1, U_1) - (L_2, M_2, U_2) \\ &= (L_1 - L_2, M_1 - M_2, U_1 - U_2) \end{aligned} \quad (4)$$

Division of a fuzzy numbers $(/)$

$$\begin{aligned} \tilde{A}_1 / \tilde{A}_2 &= (L_1, M_1, U_1) / (L_2, M_2, U_2) \\ &= (L_1 / U_2, M_1 / M_2, U_1 / L_2) \\ &\text{for } L_i > 0, M_i > 0, U_i > 0. \end{aligned} \quad (5)$$

Reciprocal of a fuzzy numbers

$$\begin{aligned} \tilde{A}_1^{-1} &= (L_1, M_1, U_1)^{-1} = (1/U_1, 1/M_1, 1/L_1) \\ &\text{for } L_i > 0, M_i > 0, U_i > 0. \end{aligned} \quad (6)$$

2.2. Linguistic Variable

Table 1: Membership function of linguistic variables

TFN	Linguistic variables	Fuzzy Scale	Reciprocal scale
$\tilde{1}$	Equally important	(1,1,3)	(1/3,1,1)
$\tilde{3}$	Weakly important	(1,3,5)	(1/5,1/3,1)
$\tilde{5}$	Essentially important	(3,5,7)	(1/7,1/5,1/3)
$\tilde{7}$	Very strongly important	(5,7,9)	(1/9,1/7,1/5)
$\tilde{9}$	Absolutely important	(7,9,9)	(1/9,1/9,1/7)

According to [23], it is very difficult for conventional quantification to express reasonably those situations that are overtly complex or hard to define; thus, notion of a linguistic variable is necessary in such situations.

A linguistic variable is a variable whose values are words or sentences in a natural or artificial language. Here, we use this kind of expression to compare the four levels of hierarchy (Fig.3) by five basic linguistic terms, (Fig.2) as “absolutely important,” “very strongly important,” “essentially important,” “weakly important” and “equally important” with respect to a fuzzy five level scale (Fig.2) [21] with its reciprocal FN as shown in Table 1. In this paper, the computational technique is based on the following fuzzy numbers defined in Table 1 [24]. Here each membership function (scale of fuzzy number) is defined by three parameters of the symmetric triangular fuzzy number, the left point, middle point and right point of the range over which the function is defined.

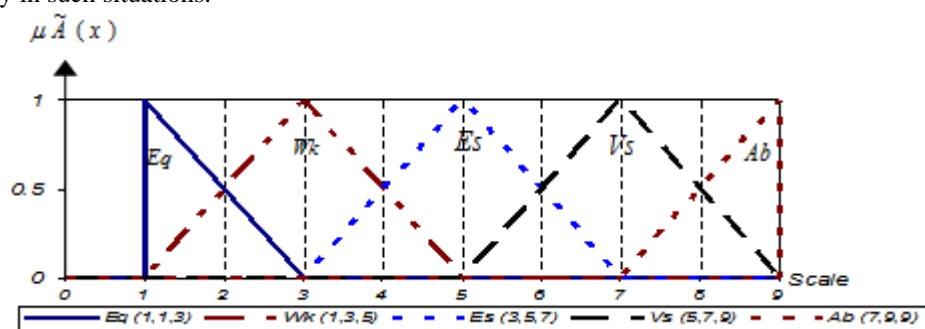


Figure 2: Membership functions of linguistic variables.

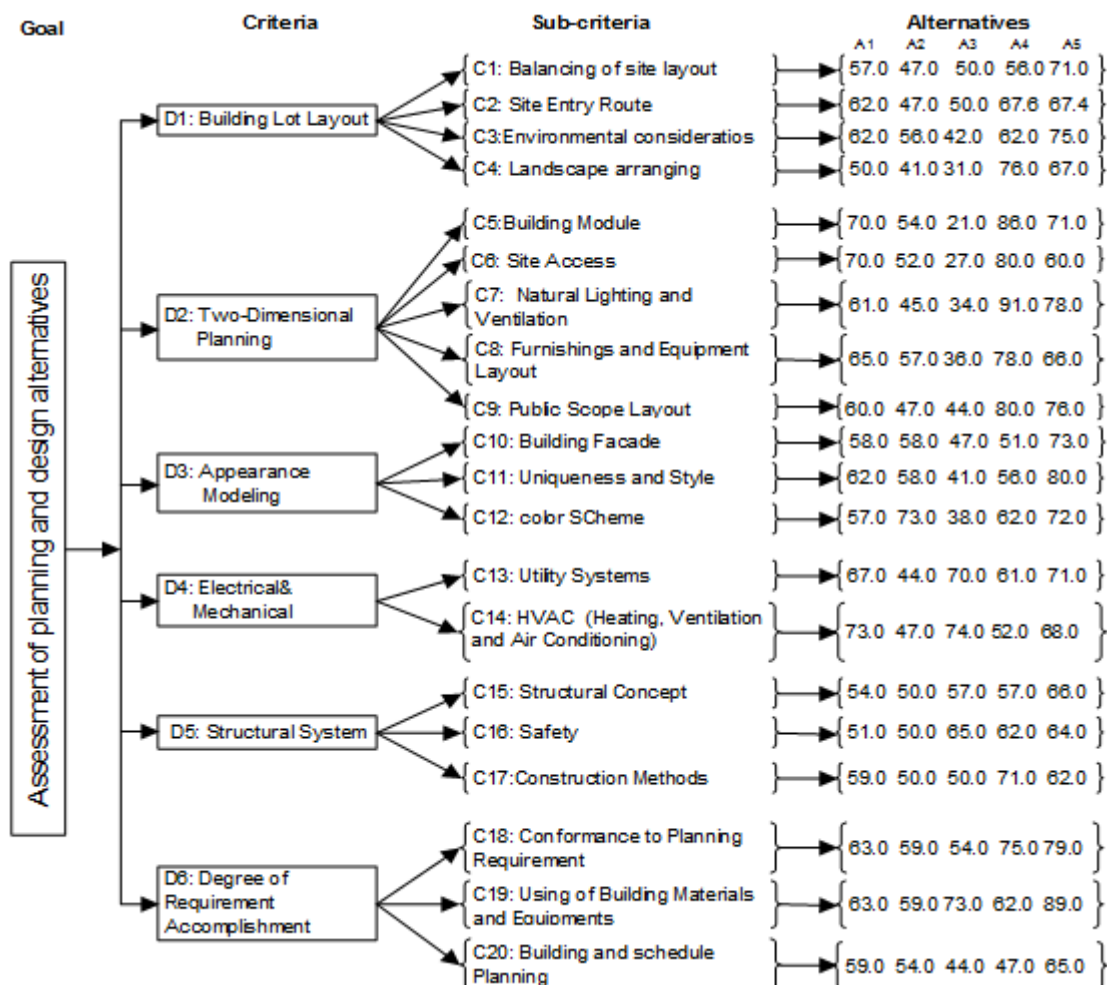


Figure 3: Hierarchical structural for building and design alternatives assessment

3. Fuzzy Analytic hierarchy Process

3.1. Fuzzy AHP

Review the basic ideas behind the AHP by [24] Based on these ideas, they introduce the concept of comparison interval and propose a methodology based on stochastic optimization to achieve global consistency and to accommodate the fuzzy nature of the comparison process. a new method is proposed for evaluating weapon systems by analytical hierarchy process based on linguistic variable weight [25] and make a discussion on extent analysis method and applications of fuzzy AHP. A technology selection algorithm is presented to quantify both tangible and intangible benefits in fuzzy environment [26]. They describe an application of the theory of fuzzy sets to hierarchical structural analysis and economic evaluations. By aggregating the hierarchy, the preferential weight of each alternative technology is found, which is called fuzzy appropriate index. The fuzzy appropriate indices of different technologies are then ranked and preferential ranking orders of technologies are found. From the economic evaluation perspective, a fuzzy cash flow analysis is employed. [26] reported an integrated approach for the automatic design of FMS, which uses simulation and multi-criteria decision-making techniques. The design process consists of the construction and testing of alternative designs using simulation methods. The selection of the most suitable design (based on AHP) is employed to analyze the output from the FMS simulation models. Intelligent tools (such as expert systems, fuzzy systems and neural networks) are developed for supporting the FMS design process. a fuzzy consistency definition with consideration of a tolerance deviation is described, essentially, the fuzzy ratios of relative importance, allowing certain tolerance deviation, are formulated as constraints on the membership values of the local priorities [26]. The fuzzy local and global weights are determined via the extension principle. The alternatives are ranked on the basis of the global weights by application of maximum–minimum set ranking method. The AHP is one of the extensively used multi-criteria decision-making methods. One of the main advantages of this method is the relative ease with which it handles multiple criteria. In addition to this, AHP is easier to understand and it can effectively handle both qualitative and quantitative data. The use of AHP does not involve cumbersome mathematics. AHP involves the principles of decomposition, pairwise comparisons, and priority vector generation and synthesis. Though the purpose of AHP is to capture the expert's knowledge, the conventional AHP still cannot reflect the human thinking style. Therefore, fuzzy AHP, a fuzzy extension of AHP, was developed to solve the hierarchical fuzzy problems. In the fuzzy-AHP procedure, the pairwise comparisons in the judgment matrix are fuzzy numbers that are modified by the designer's emphasis. The procedure calculates a corresponding set of scores and determines one composite score that is the average of these fuzzy scores. In the following, first the outlines of the extent analysis method on fuzzy AHP are with the application to assess planning and design of project that introduced in [2] as introduced in Fig.3. Planning and design alternatives evaluation, the weights of the dimension hierarchy and criteria hierarchy can be analyzed. The decision maker can

define their own range for linguistic variables employed in this study according to their subjective judgments within a fuzzy Saaty's scale as introduced in Table 1.

3.2 Multi Attribute Comparisons of Planning and Design

The definitions inside the methodology are introduced as follows: Let $X = \{x_1, x_2, \dots, x_n\}$ be an object set, and $U = \{u_1, u_2, \dots, u_n\}$ be a goal set. According to the method of [3-8, 27]), and extent analysis, each object is taken and extent analysis for each goal g_i is performed, respectively. Therefore, m extent analysis values for each object can be obtained, with the following signs:

$$M_{g_i}^1, M_{g_i}^2, \dots, M_{g_i}^m, \quad i = 1, 2, \dots, n \quad (7)$$

where all $M_{g_i}^j$, ($j = 1, 2, \dots, m$) are TFNs which constructs a reciprocal matrix. From the introduced example the first progress of application starts with the pairwise comparisons on the descending hierarchy constructed for the criteria (dimension) with respect to goal. So the pairwise comparison matrix of six criteria are introduced with the appropriate linguistic variables and also with its fuzzy numbers as shown in Table 2 and also the summation of components of fuzzy numbers (SL , SM and SU) are introduced. This technique of linguistic pairwise comparisons is also applied for comparisons of sub-attributes with its related criteria and alternatives with its related sub-criteria as shown in Table 3.

Table 3: Fuzzy pairwise comparisons of sub-criteria

D1					D2					
a	C1	C2	C3	C4	b	C5	C6	C7	C8	A9
C1	1	3	7	9	C5	1	3	5	9	7
C2	3 ⁻¹	1	3	7	C6	3 ⁻¹	1	3	7	3
C3	7 ⁻¹	3 ⁻¹	1	3	C7	5 ⁻¹	3 ⁻¹	1	3	3
C4	9 ⁻¹	7 ⁻¹	3 ⁻¹	1	C8	9 ⁻¹	7 ⁻¹	3 ⁻¹	1	3 ⁻¹
					A9	7 ⁻¹	3 ⁻¹	3 ⁻¹	3	1

D3				D4		
c	C10	C11	C12	d	C13	C14
C10	1	3	3	C13	1	7
C11	3 ⁻¹	1	3	C14	7 ⁻¹	1
C12	3 ⁻¹	3 ⁻¹	1			

D5				D6			
e	C15	C16	C17	f	C18	C19	C20
C15	1	3 ⁻¹	3 ⁻¹	C18	1	7	9
C16	3	1	7	C19	7 ⁻¹	1	3
C17	3	7 ⁻¹	1	C20	9 ⁻¹	3 ⁻¹	1

3.3 Fuzzy synthetic

The value of fuzzy synthetic with respect to the i^{th} object is defined as

Table 2: Fuzzy pairwise comparisons of criteria

	D_1	D_2	D_3	D_4	D_5	D_6
	(L, M, U)	(L, M, U)	(L, M, U)	(L, M, U)	(L, M, U)	(L, M, U)
D_1	1, 1, 1	1/5, 1/3, 1	3, 5, 7	1, 3, 5	1/9, 1/7, 1/5	1/5, 1/3, 1
D_2		1, 1, 1	5, 7, 9	3, 5, 7	1/7, 1/5, 1/3	1/5, 1/3, 1
D_3			1, 1, 1	1/5, 1/3, 1	1/9, 1/7, 1/5	1/9, 1/7, 1/5
D_4				1, 1, 1	1/7, 1/5, 1/3	1/5, 1/3, 1
D_5					1, 1, 1	1, 3, 5
D_6						1, 1, 1

Table 4: Alternatives fuzzy pairwise comparisons with respect to sub-criteria from C1 up to C20

$$\begin{array}{c}
 \text{goal} \\
 \left. \begin{array}{c}
 x_1 \quad M_{g_1}^1 \quad M_{g_1}^2 \quad M_{g_1}^3 \quad \dots M_{g_1}^m = \sum_{j=1}^{j=m} M_{g_1}^j \\
 x_2 \quad M_{g_2}^1 \quad M_{g_2}^2 \quad M_{g_2}^3 \quad \dots M_{g_2}^m = \sum_{j=1}^{j=m} M_{g_2}^j \\
 \vdots \\
 x_n \quad M_{g_n}^1 \quad M_{g_n}^2 \quad M_{g_n}^3 \quad \dots M_{g_n}^m = \sum_{j=1}^{j=m} M_{g_n}^j
 \end{array} \right\} \text{object} \quad (9)
 \end{array}$$

where $\sum_{j=1}^{j=m} M_{g_i}^j = \left(\sum_{j=1}^m l_j, \sum_{j=1}^m M_j, \sum_{j=1}^m U_j \right) = (L_i, M_i, U_i)$ as a fuzzy

number, and also to obtain $\left(\sum_{i=1}^{i=n} \left[\sum_{j=1}^{j=m} M_{g_i}^j \right] \right)^{-1}$, perform addition

operation of $M_{g_i}^j$, ($j=1,2,\dots,m$) values as indicated in equation (7) such that

$$\sum_{i=1}^{i=n} \left[\sum_{j=1}^{j=m} M_{g_i}^j \right] = \sum_{i=1}^{i=n} \left(\sum_{j=1}^m L_j, \sum_{j=1}^m M_j, \sum_{j=1}^m U_j \right) = (L_f, M_f, U_f) \quad (10)$$

Triangular fuzzy number that extracted from double summation is the final summation in the matrix, where

$$\begin{array}{c}
 \left. \begin{array}{c}
 x_1 \quad M_{g_1}^1 \quad M_{g_1}^2 \quad M_{g_1}^3 \quad \dots M_{g_1}^m = \sum_{j=1}^{j=m} M_{g_1}^j \dots \dots \dots S_1 = \left(\sum_{j=1}^{j=m} M_{g_1}^j \right) / \left(\sum_{i=1}^{i=n} \left[\sum_{j=1}^{j=m} M_{g_i}^j \right] \right) \\
 x_2 \quad M_{g_2}^1 \quad M_{g_2}^2 \quad M_{g_2}^3 \quad \dots M_{g_2}^m = \sum_{j=1}^{j=m} M_{g_2}^j \dots \dots \dots S_2 = \left(\sum_{j=1}^{j=m} M_{g_2}^j \right) / \left(\sum_{i=1}^{i=n} \left[\sum_{j=1}^{j=m} M_{g_i}^j \right] \right) \\
 \vdots \\
 x_n \quad M_{g_n}^1 \quad M_{g_n}^2 \quad M_{g_n}^3 \quad \dots M_{g_n}^m = \sum_{j=1}^{j=m} M_{g_n}^j \dots \dots \dots S_n = \left(\sum_{j=1}^{j=m} M_{g_n}^j \right) / \left(\sum_{i=1}^{i=n} \left[\sum_{j=1}^{j=m} M_{g_i}^j \right] \right)
 \end{array} \right\} \text{object} \quad (12)
 \end{array}$$

3.4. Degree of Possibility

The degree of possibility of any two membership function $A_2 = (Ls_2, Ms_2, Us_2) \geq A_1 = (Ls_1, Ms_1, Us_1)$ is defined as

$$V(A_2 \geq A_1) = \sup_{y \geq x} [\min (\mu_{A_2}(x), (\mu_{A_1}(y)))] \quad (13)$$

	$C1$					$C2$					$C3$			
	$A1$	$A2$	$A3$	$A4$	$A5$	$A1$	$A2$	$A3$	$A4$	$A5$	$A1$	$A2$	$A3$	$A4$
$A1$	1	3	3	3	3 ⁻¹	1	7	3	3 ⁻¹	3 ⁻¹	1	3	3	3 ⁻¹
$A2$	3 ⁻¹	1	3 ⁻¹	3 ⁻¹	3 ⁻¹	3 ⁻¹	1	3 ⁻¹	3 ⁻¹	3 ⁻¹	3 ⁻¹	1	3	3 ⁻¹
$A3$	3 ⁻¹	3	1	3 ⁻¹	3 ⁻¹	3 ⁻¹	3	1	3 ⁻¹	3 ⁻¹	3 ⁻¹	3 ⁻¹	1	3 ⁻¹
$A4$	3 ⁻¹	3	3	1	3 ⁻¹	3	3	3	1	3	3 ⁻¹	3	3	1

$$S_i = \sum_{j=1}^{j=m} M_{g_i}^j \otimes \left(\sum_{i=1}^{i=n} \left[\sum_{j=1}^{j=m} M_{g_i}^j \right] \right)^{-1} \quad (8)$$

To obtain $\sum_{j=1}^{j=m} M_{g_i}^j$, perform the fuzzy addition operation of

m extent analysis values for a particular matrix (9).

$$\left(\sum_{i=1}^{i=n} \left[\sum_{j=1}^{j=m} M_{g_i}^j \right] \right)^{-1} = \left(\frac{1}{U_f}, \frac{1}{M_f}, \frac{1}{L_f} \right), S_1 = \sum_{j=1}^{j=m} M_{g_1}^j / \left(\sum_{i=1}^{i=n} \left[\sum_{j=1}^{j=m} M_{g_i}^j \right] \right),$$

$$S_2 = \sum_{j=1}^{j=m} M_{g_2}^j / \left(\sum_{i=1}^{i=n} \left[\sum_{j=1}^{j=m} M_{g_i}^j \right] \right), \dots \dots S_i = \sum_{j=1}^{j=m} M_{g_i}^j / \left(\sum_{i=1}^{i=n} \left[\sum_{j=1}^{j=m} M_{g_i}^j \right] \right) \quad (11)$$

and also can be represented as follows;

$$S_i = (L_i, M_i, U_i) \otimes \left(\frac{1}{U_f}, \frac{1}{M_f}, \frac{1}{L_f} \right) = \left(\frac{L_i}{U_f}, \frac{M_i}{M_f}, \frac{U_i}{L_f} \right) = (Ls, Ms, Us)$$

for each $i=1,2,\dots,n$.

The description of the fuzzy synthetic values is introduced in equations 11 and 12. Therefore, the estimations of fuzzy synthetic values for six criteria are introduced in Table 5.

Table 5: Fuzzy synthetic values

S_i		
Ls	Ms	Us
0.05	0.12	0.31
0.09	0.20	0.47
0.01	0.02	0.06
0.02	0.06	0.18
0.18	0.37	0.77
0.08	0.21	0.53

When a pair (x, y) exists such that $y \geq x$ and $\mu_{A_2}(x) = \mu_{A_1}(y)$, then we have $V(A_2 \geq A_1) = 1$. Since A_2 and A_1 are convex fuzzy numbers we have that:

$$V(A_2 \geq A_1) = 1 \text{ if } M_2 \geq M_1 \quad (14)$$

$$V(A_2 \geq A_1) = hgt(A_2 \cap A_1) = \mu_{A_2}(d) \quad (15)$$

$$V(A_2 \geq A_1) = \text{hgt}(A_2 \cap A_1) = \mu_{A_2}(d) =$$

$$\begin{cases} 1 & \text{if } M_2 \geq Ms_1 \\ 0 & \text{if } Ls_1 \geq Us_2 \\ \frac{Ls_1 - Us_2}{(Ms_2 - Us_2) - (Ms_1 - Ls_1)}, \dots \text{otherwise} \end{cases} \quad (16)$$

where d is the ordinate of the higher intersection point D between μ_{A_2} and μ_{A_1} see Fig.4. So, the degree of possibility can be equivalently expressed as follows;

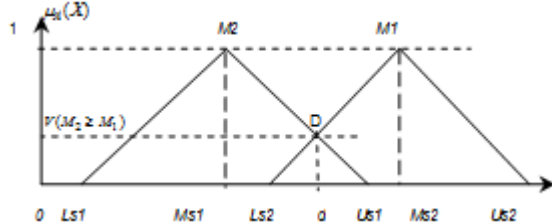


Figure 4: The intersection between M_1 and M_2

To compare the performance of A_2 and A_1 , we need both the values of $V(A_2 \geq A_1)$ and $V(A_1 \geq A_2)$. If we have a comparison of three object say criteria, values in Table 6 are required:

Table 6: Degree of Possibility Pairwise Matrix

$V(C_i \geq C_j)$	C_1	C_2	C_3
C_1	-	$V(M_1 \geq M_2)$	$V(M_1 \geq M_3)$
C_2	$V(M_2 \geq M_1)$	-	$V(M_2 \geq M_3)$
C_3	$V(M_3 \geq M_1)$	$V(M_3 \geq M_2)$	-

3.5. Weighted Vector

the degree possibility for a convex fuzzy number to be greater than k convex fuzzy numbers $A_i, (i=1,2,\dots,k)$ can be defined by

$$\begin{aligned} V(A \geq A_1, A_2, \dots, A_k) &= \\ V[(A \geq A_1) \cap (A \geq A_2) \cap \dots \cap (A \geq A_k)] &= \min(V(A \geq A_i)), \\ i=1,2,\dots,k. \end{aligned} \quad (17)$$

Assume that $d(Z_i) = \min(V(S_i \geq S_k)) = \text{minimum value (MV)}$ for $k=1,2,\dots,n, k \neq i$. Then the weight vector is given by

$$W' = (d'(Z_1), d'(Z_2), \dots, d'(Z_n))^T \quad (18)$$

where $Z_i (i=1,2,\dots,n)$ are elements. Via normalization, the normalized weight vector are

$$W = (d(Z_1), d(Z_2), \dots, d(Z_n))^T \quad (19)$$

where W is non fuzzy number.

Thus, as a solution of the proposed technique from equations 12-19, for planning and design alternatives for building, the Fuzzy synthetic ,degree of possibility and priority vector of attributes are introduced in table 7, of sub-attributes are introduced in Tables 8-13 and of alternatives are introduced in Tables 14- 33 respectively.

Table 7: Fuzzy synthetic ,degree of possibility and priority vector of criteria

S_i				$V(M_i \geq M_j)$							
Ls	Ms	Us	Obj	$D1$	$D2$	$D3$	$D4$	$D5$	$D6$	MV	WV
0.05	0.12	0.31	$D1$	-	0.72	1.00	1.00	0.35	0.71	0.35	0.17
0.09	0.20	0.47	$D2$	1.00	-	1.00	1.00	0.64	0.98	0.64	0.32
0.01	0.02	0.06	$D3$	0.1	0.0	-	0.47	0.00	0.00	0.00	0.00
0.02	0.06	0.18	$D4$	0.68	0.37	1.00	-	0.00	0.38	0.00	0.00
0.18	0.37	0.77	$D5$	1.00	1.00	1.00	1.00	-	1.00	1.00	0.50
0.08	0.21	0.53	$D6$	1.00	1.00	1.00	1.00	0.69	-	0.69	0.35

Table 8: Fuzzy synthetic ,degree of possibility and priority vector of sub-attributes under D1

S_i				$V(M_i \geq M_j)$				MV	WV
Ls	Ms	Us	Obj	$C1$	$C2$	$C3$	$C4$		
0.28	0.51	0.89	$C1$	-	1.00	1.00	1.00	1.00	0.50
0.18	0.34	0.67	$C2$	0.70	-	1.00	1.00	1.00	0.50
0.04	0.11	0.24	$C3$	0.0	0.21	-	1.00	0.00	0.00
0.03	0.04	0.09	$C4$	0.00	0.00	0.38	-	0.00	0.00

Table 9: Fuzzy synthetic ,degree of possibility and priority vector of sub-attributes under D2

S_i				$V(M_i \geq M_j)$					MV	WV
Ls	Ms	Us	Obj	$C5$	$C6$	$C7$	$C8$	$C9$		
0.22	0.44	0.85	$C5$	-	1.00	1.00	1.00	1.00	1.00	0.37
0.13	0.28	0.63	$C6$	0.73	-	1.00	1.00	1.00	0.73	0.27
0.07	0.17	0.39	$C7$	0.4	0.69	-	1.00	1.00	1.00	0.37
0.02	0.03	0.07	$C8$	0.00	0.00	0.04	-	0.45	0.00	0.00
0.03	0.08	0.21	$C9$	0.00	0.27	0.62	1.00	-	0.00	0.00

Table 10: Fuzzy synthetic ,degree of possibility and priority vector of sub-attributes under D3

S_i				$V(M_i \geq M_j)$			MV	WV
Ls	Ms	Us	Obj	$C10$	$C11$	$C12$		
0.20	0.48	1.04	$C10$	-	1.00	1.00	1.00	0.50
0.24	0.45	0.88	$C11$	0.95	-	1.00	1.00	0.50
0.05	0.07	0.12	$C12$	0.0	0.00	-	0.00	0.00

Table 11: Fuzzy synthetic ,degree of possibility and priority vector of sub-attributes under D4

S_i				$V(M_i \geq M_j)$		MV	WV
Ls	Ms	Us	Obj	$C13$	$C14$		
0.54	0.88	1.41	$C13$	-	1.00	1.00	1.00
0.10	0.13	0.17	$C14$	0.00	-	0.00	0.00

Table 12: Fuzzy synthetic ,degree of possibility and priority vector of sub-attributes under D5

S_i				$V(M_i \geq M_j)$			MV	WV
Ls	Ms	Us	Obj	$C15$	$C16$	$C17$		
0.05	0.06	0.13	$C15$	-	0.00	0.00	0.00	0.00
0.48	0.75	1.16	$C16$	1.00	-	1.00	1.00	1.00
0.08	0.18	0.38	$C17$	1.0	0.00	-	0.00	0.00

Table 13: Fuzzy synthetic ,degree of possibility and priority vector of sub-attributes under D6

S_i				$V(M_i \geq M_j)$			MV	WV
Ls	Ms	Us	Obj	$C15$	$C16$	$C17$		
0.45	0.70	1.03	$C15$	-	1.00	1.00	1.00	1.00
0.14	0.25	0.45	$C16$	0.00	-	1.00	0.00	0.00
0.04	0.05	0.08	$C17$	0.00	0.00	-	0.00	0.00

Table 14: Fuzzy synthetic ,degree of possibility and priority vector of alternatives under C1

S_i			$V(M_i \geq M_j)$							
Ls	Ms	Us	Obj	A1	A2	A3	A4	A5	MV	WV
0.08	0.23	0.56	A1	-	1.00	1.00	1.11	0.59	0.59	0.28
0.02	0.04	0.09	A2	0.04	-	0.50	0.11	0.00	0.00	0.00
0.03	0.09	0.25	A3	0.6	1.00	-	0.67	0.07	0.07	0.04
0.07	0.18	0.44	A4	0.88	1.00	1.00	-	0.43	0.43	0.21
0.22	0.47	1.01	A5	1.00	1.00	1.00	1.00	-	1.00	0.48

Table 20: Fuzzy synthetic ,degree of possibility and priority vector of alternatives under C7

S_i			$V(M_i \geq M_j)$							
Ls	Ms	Us	Obj	A1	A2	A3	A4	A5	MV	WV
0.14	0.22	0.32	A1	-	1.00	1.00	0.31	0.71	0.31	0.16
0.07	0.10	0.18	A2	0.23	-	1.00	0.00	0.01	0.00	0.00
0.02	0.02	0.03	A3	0.0	0.00	-	0.00	0.00	0.00	0.00
0.25	0.39	0.58	A4	1.00	1.00	1.00	-	1.00	1.00	0.51
0.17	0.28	0.44	A5	1.00	1.00	1.00	0.64	-	0.64	0.33

Table 15: Fuzzy synthetic ,degree of possibility and priority vector of alternatives under C2

S_i			$V(M_i \geq M_j)$							
Ls	Ms	Us	Obj	A1	A2	A3	A4	A5	MV	WV
0.11	0.23	0.49	A1	-	1.00	1.00	0.73	0.81	0.81	0.29
0.02	0.03	0.07	A2	0.00	-	0.46	0.00	0.00	0.00	0.00
0.03	0.08	0.17	A3	0.3	1.00	-	0.00	0.00	0.00	0.00
0.16	0.35	0.75	A4	1.00	1.00	1.00	-	1.00	1.00	0.36
0.15	0.31	0.65	A5	1.00	1.00	1.00	1.00	-	1.00	0.36

Table 21: Fuzzy synthetic ,degree of possibility and priority vector of alternatives under C8

S_i			$V(M_i \geq M_j)$							
Ls	Ms	Us	Obj	A1	A2	A3	A4	A5	MV	WV
0.12	0.23	0.38	A1	-	1.00	1.00	0.57	1.00	0.57	0.24
0.11	0.18	0.28	A2	0.73	-	1.00	0.30	0.87	0.30	0.12
0.02	0.02	0.04	A3	0.0	0.00	-	0.00	0.00	0.00	0.00
0.19	0.37	0.65	A4	1.00	1.00	1.00	-	1.00	1.00	0.41
0.13	0.20	0.41	A5	0.89	1.00	1.00	0.56	-	0.56	0.23

Table 16: Fuzzy synthetic ,degree of possibility and priority vector of alternatives under C3

S_i			$V(M_i \geq M_j)$							
Ls	Ms	Us	Obj	A1	A2	A3	A4	A5	MV	WV
0.12	0.20	0.36	A1	-	1.00	1.00	0.89	0.26	0.26	0.17
0.05	0.10	0.19	A2	0.37	-	1.00	0.27	0.00	0.00	0.00
0.02	0.02	0.03	A3	0.0	0.00	-	0.00	0.00	0.00	0.00
0.14	0.23	0.36	A4	1.00	1.00	1.00	-	0.28	0.28	0.18
0.28	0.45	0.69	A5	1.00	1.00	1.00	1.00	-	1.00	0.65

Table 22: Fuzzy synthetic ,degree of possibility and priority vector of alternatives under C9

S_i			$V(M_i \geq M_j)$							
Ls	Ms	Us	Obj	A1	A2	A3	A4	A5	MV	WV
0.11	0.19	0.35	A1	-	1.00	1.00	0.40	0.47	0.40	0.17
0.03	0.06	0.13	A2	0.15	-	1.00	0.00	0.00	0.00	0.00
0.02	0.02	0.05	A3	0.0	0.36	-	0.00	0.00	0.00	0.00
0.22	0.38	0.62	A4	1.00	1.00	1.00	-	1.00	1.00	0.44
0.21	0.34	0.54	A5	1.00	1.00	1.00	0.89	-	0.89	0.39

Table 17: Fuzzy synthetic ,degree of possibility and priority vector of alternatives under C4

S_i			$V(M_i \geq M_j)$							
Ls	Ms	Us	Obj	A1	A2	A3	A4	A5	MV	WV
0.10	0.17	0.27	A1	-	1.00	1.00	0.15	0.21	0.15	0.07
0.05	0.09	0.16	A2	0.41	-	1.00	0.00	0.00	0.00	0.00
0.02	0.02	0.03	A3	0.0	0.00	-	0.00	0.00	0.00	0.00
0.24	0.38	0.59	A4	1.00	1.00	1.00	-	1.00	1.00	0.49
0.23	0.34	0.51	A5	1.00	1.00	1.00	0.89	-	0.89	0.44

Table 23: Fuzzy synthetic ,degree of possibility and priority vector of alternatives under C10

S_i			$V(M_i \geq M_j)$							
Ls	Ms	Us	Obj	A1	A2	A3	A4	A5	MV	WV
0.08	0.18	0.44	A1	-	1.00	1.00	1.00	0.33	0.33	0.21
0.07	0.18	0.39	A2	1.00	-	1.00	1.00	0.24	0.24	0.16
0.02	0.03	0.08	A3	0.0	0.02	-	0.45	0.00	0.00	0.00
0.03	0.09	0.22	A4	0.59	0.61	1.00	-	0.00	0.00	0.00
0.28	0.52	0.90	A5	1.00	1.00	1.00	1.00	-	1.00	0.64

Table 18: Fuzzy synthetic ,degree of possibility and priority vector of alternatives under C5

S_i			$V(M_i \geq M_j)$							
Ls	Ms	Us	Obj	A1	A2	A3	A4	A5	MV	WV
0.14	0.22	0.34	A1	-	1.00	1.00	0.33	1.00	0.33	0.19
0.10	0.14	0.20	A2	0.44	-	1.00	0.00	0.41	0.00	0.00
0.02	0.02	0.03	A3	0.0	0.00	-	0.00	0.00	0.00	0.00
0.25	0.40	0.61	A4	1.00	1.00	1.00	-	1.00	1.00	0.57
0.14	0.22	0.37	A5	1.00	1.00	1.00	0.41	-	0.41	0.24

Table 24: Fuzzy synthetic ,degree of possibility and priority vector of alternatives under C11

S_i			$V(M_i \geq M_j)$							
Ls	Ms	Us	Obj	A1	A2	A3	A4	A5	MV	WV
0.12	0.24	0.42	A1	-	1.00	1.00	1.00	0.47	0.47	0.29
0.11	0.17	0.29	A2	0.71	-	1.00	1.00	0.14	0.14	0.08
0.02	0.02	0.03	A3	0.0	0.00	-	0.00	0.00	0.00	0.00
0.08	0.14	0.25	A4	0.57	0.83	1.00	-	0.00	0.00	0.00
0.25	0.43	0.72	A5	1.00	1.00	1.00	1.00	-	1.00	0.62

Table 19: Fuzzy synthetic ,degree of possibility and priority vector of alternatives under C6

S_i			$V(M_i \geq M_j)$							
Ls	Ms	Us	Obj	A1	A2	A3	A4	A5	MV	WV
0.17	0.30	0.54	A1	-	1.00	1.00	0.90	1.00	1.00	0.36
0.10	0.16	0.25	A2	0.36	-	1.00	0.28	0.94	0.28	0.10
0.02	0.02	0.04	A3	0.0	0.00	-	0.00	0.00	0.00	0.00
0.18	0.34	0.62	A4	1.00	1.00	1.00	-	1.00	1.00	0.36
0.09	0.17	0.34	A5	0.55	1.00	1.00	0.47	-	0.47	0.17

Table 25: Fuzzy synthetic ,degree of possibility and priority vector of alternatives under C12

S_i			$V(M_i \geq M_j)$							
Ls	Ms	Us	Obj	A1	A2	A3	A4	A5	MV	WV
0.10	0.15	0.23	A1	-	0.12	1.00	0.74	0.27	0.12	0.05
0.20	0.33	0.57	A2	1.00	-	1.00	1.00	1.00	1.00	0.40
0.02	0.02	0.03	A3	0.0	0.00	-	0.00	0.00	0.00	0.00
0.11	0.19	0.32	A4	1.00	0.46	1.00	-	0.58	0.46	0.18
0.17	0.30	0.49	A5	1.00	0.91	1.00	1.00	-	0.91	0.37

Table 26: Fuzzy synthetic ,degree of possibility and priority vector of alternatives under C13

S_i			$V(M_i \geq M_j)$							
Ls	Ms	Us	Obj	A1	A2	A3	A4	A5	MV	WV
0.24	0.42	0.24	A1	-	1.00	0.96	1.00	0.75	0.75	0.25
0.03	0.04	0.03	A2	0.00	-	0.00	0.00	0.00	0.00	0.00
0.26	0.47	0.26	A3	1.0	1.00	-	1.00	0.81	0.81	0.27
0.15	0.30	0.15	A4	0.65	1.00	0.61	-	0.43	0.43	0.14
0.32	0.61	0.32	A5	1.00	1.00	1.00	1.00	-	1.00	0.33

Table 27: Fuzzy synthetic ,degree of possibility and priority vector of alternatives under C14

S_i			$V(M_i \geq M_j)$							
Ls	Ms	Us	Obj	A1	A2	A3	A4	A5	MV	WV
0.21	0.33	0.49	A1	-	1.00	0.90	1.00	1.00	0.90	0.36
0.02	0.03	0.05	A2	0.00	-	0.00	0.35	0.00	0.00	0.00
0.22	0.36	0.58	A3	1.0	1.00	-	1.00	1.00	1.00	0.40
0.03	0.07	0.14	A4	0.00	1.53	0.00	-	0.11	0.00	0.00
0.18	0.37	0.77	A5	1.00	1.00	1.00	1.00	1.00	1.00	0.50

Table 28: Fuzzy synthetic ,degree of possibility and priority vector of alternatives under C15

S_i			$V(M_i \geq M_j)$							
Ls	Ms	Us	Obj	A1	A2	A3	A4	A5	MV	WV
0.05	0.16	0.39	A1	-	1.00	0.99	0.99	0.46	0.46	0.17
0.03	0.06	0.20	A2	0.60	-	0.56	0.57	0.13	0.13	0.05
0.07	0.17	0.61	A3	1.0	1.00	-	1.00	0.62	0.62	0.22
0.06	0.17	0.52	A4	1.00	1.00	1.00	-	0.57	0.57	0.20
0.15	0.45	1.17	A5	1.00	1.00	1.00	1.00	-	1.00	0.36

Table 29: Fuzzy synthetic ,degree of possibility and priority vector of alternatives under C16

S_i			$V(M_i \geq M_j)$							
Ls	Ms	Us	Obj	A1	A2	A3	A4	A5	MV	WV
0.04	0.06	0.21	A1	-	1.00	0.20	0.43	0.29	0.20	0.07
0.03	0.06	0.11	A2	0.96	-	0.00	0.15	0.00	0.00	0.00
0.13	0.38	0.93	A3	1.0	1.00	-	1.00	1.00	1.00	0.36
0.08	0.23	0.56	A4	1.00	1.00	0.74	-	0.91	0.74	0.27
0.12	0.27	0.70	A5	1.00	1.00	0.85	1.00	-	0.85	0.30

Table 30: Fuzzy synthetic ,degree of possibility and priority vector of alternatives under C17

S_i			$V(M_i \geq M_j)$							
Ls	Ms	Us	Obj	A1	A2	A3	A4	A5	MV	WV
0.09	0.21	0.45	A1	-	1.00	1.00	.48	0.88	0.48	0.22
0.03	0.05	0.13	A2	0.19	-	1.00	0.00	0.12	0.12	0.05
0.02	0.05	0.08	A3	0.0	1.00	-	0.00	0.00	0.00	0.00
0.22	0.45	0.90	A4	1.00	1.00	1.00	-	1.00	1.00	0.45
0.11	0.25	0.56	A5	1.00	1.00	1.00	0.63	-	0.63	0.28

Table 31: Fuzzy synthetic, degree of possibility and priority vector of alternatives under C18

S_i			$V(M_i \geq M_j)$							
Ls	Ms	Us	Obj	A1	A2	A3	A4	A5	MV	WV
0.04	0.10	0.26	A1	-	1.00	1.00	0.31	0.11	0.11	0.06
0.03	0.10	0.21	A2	0.99	-	1.00	0.17	0.00	0.00	0.00
0.02	0.03	0.09	A3	0.4	0.48	-	0.00	0.00	0.00	0.00
0.16	0.33	0.68	A4	1.00	1.00	1.00	-	0.79	0.79	0.42
0.22	0.45	0.84	A5	1.00	1.00	1.00	1.00	-	1.00	0.53

Table 33: Fuzzy synthetic, degree of possibility and priority vector of alternatives under C20

S_i			$V(M_i \geq M_j)$							
Ls	Ms	Us	Obj	A1	A2	A3	A4	A5	MV	WV
0.17	0.29	0.50	A1	-	1.00	1.00	1.00	0.74	0.74	0.32
0.16	0.25	0.40	A2	0.86	-	1.00	1.00	0.58	0.58	0.25
0.02	0.02	0.05	A3	0.0	0.00	-	0.35	0.00	0.00	0.00
0.03	0.06	0.13	A4	0.00	0.00	1.00	-	0.00	0.00	0.00
0.22	0.38	0.61	A5	1.00	1.00	1.00	1.00	-	1.00	0.43

3.6. Overall priorities of alternatives

Among numerous compensatory decision rules used in the context of decision making, the most popular are additive decision rules. Weighted linear combination is used in the calculation of overall priorities by multiplying the weight of the elements that comprise each set by the weight of the elements that comprise their subset, level by level, the composite measure of all elements in the hierarchy can be computed. The estimated vector of the relative weights of the elements in a level say, P^{th} level with respect to an element in the previous $(P-1)^{th}$ level of hierarchy may be denoted w' . For a hierarchy, the composite vector of the weights of the elements at the P^{th} level denoted by w is computed as shown in equation 20.

$$w = B_1 \times B_2 \times \dots \times B_{P-1} \times w' \quad (20)$$

where B_k is the matrix of estimated weights of k^{th} level, $k=2,3,\dots,P-1$. A final appraisal score e_i for each alternative i is computed by weighted linear combination of i^{th} alternative with respect to j^{th} criteria as constructed in equation 21.

$$e_i = \sum_{j=1}^n w_j \times r_{ij} \quad (21)$$

Table 34: Performance matrix

D1				D2					D3			D4		D5			D6				Score	Rank
0.17				0.32					0.00			0.00		0.50			0.35					
C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20			
0.50	0.50	0.00	0.00	0.37	0.27	0.37	0.00	0.00	0.50	0.50	0.00	1.00	0.00	0.00	1.00	0.00	1.00	0.00	0.00			
A1	0.28	0.29	0.17	0.07	0.19	0.36	0.16	0.24	0.17	0.21	0.29	0.05	0.25	0.36	0.17	0.07	0.22	0.06	0.00	0.32	0.177	4
A2	0.00	0.00	0.00	0.00	0.00	0.10	0.00	0.12	0.00	0.16	0.08	0.40	0.00	0.00	0.05	0.00	0.05	0.00	0.00	0.25	0.009	5
A3	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.27	0.40	0.22	0.36	0.00	0.00	0.30	0.00	0.184	3
A4	0.21	0.36	0.18	0.49	0.57	0.36	0.51	0.41	0.44	0.00	0.00	0.18	0.14	0.00	0.20	0.27	0.45	0.42	0.00	0.00	0.488	2
A5	0.48	0.36	0.65	0.44	0.24	0.17	0.33	0.23	0.39	0.64	0.62	0.37	0.33	0.24	0.36	0.30	0.28	0.53	0.70	0.43	0.490	1

Thus, Table 34 describes the performance matrix of all hierarchy and estimates the global score of alternatives. The final score ranking alternatives as $A_5 \succ A_4 \succ A_3 \succ A_1 \succ A_2$.

4. Conclusions

Because traditional engineering economic models do not take care of the inherent strategic benefits of planning and design of projects, a multi-attribute decision-making method should be used to justify them. In order to achieve an optimum decision, business professionals should consider both the performance features and cost figures of each planning and design alternative of project. This study developed a fuzzy AHP framework to select the best planning and design alternative to delete the weakness of objects and magnifies its effectiveness. While fuzzy AHP requires cumbersome computations, it is a more systematic method than the others, and it is more capable of capturing a human's appraisal of ambiguity when complex multi-attribute decision-making problems are considered. This is true because pairwise comparisons provide a flexible and realistic way to accommodate real-life data. The financial side of the framework is based on fuzzy multi-attribute analysis. The results of fuzzy multi-attribute analysis are included into fuzzy AHP analysis. Using fuzzy AHP concept in multi-attribute analysis investment decisions in fuzzy environment results a very effective way to evaluate alternatives because it delete the ineffective objects from the competitions with delete the low importance for the other hierarchies that depend on it. Using the very same developed framework, a subjective comparison, such as the comparison of diverse projects, has been conducted and demonstrated to readers. For further research, the authors suggest the other multi-attribute approaches such as fuzzy TOPSIS and fuzzy outranking methods to be used with fuzzy multi-attribute analysis for planning and design selection.

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