

Modeling of Water Absorption Process in the Woods

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Abstract: Wood is a material whose role has always been crucial in the history of mankind. The development of technology today allows studying the physical and chemical properties of wood. Among the factors that affect the physical, moisture is certainly the most important. Indeed, changes in humidity have an influence on the density, the dimensions, mechanical resistance and resistance to attack by fungi. Also, for a good use of wood, a perfect knowledge of the moisture content with the microstructure of wood is needed. The purpose of this article is to describe the three-way process of absorption of water, when the relative moisture content equal to 76% and temperature of 22 °C ± 2 °C.

Keywords: Modelisation, Transfer, Diffusion, Wood

1. Introduction

Wood is a hygroscopic material. It has the ability to absorb and release water and naturally to reach a state that corresponds to equilibrium with its surroundings. The method used is to couple the experimental study with the theoretical study. The kinetics of water absorption and profiles of consultations were determined

2. Theoretical Part

The sample is considered a parallelepiped whose edges are parallel to the main axes of diffusion. We applied a mathematical model based on a numerical finite difference method and an analytical method to simulate the moisture absorption kinetics in the wood. The coordinate axes ox, oy and oz are directed along the three main directions Released: Longitudinal, radial and tangential, the general equation of diffusion in three dimensions with constant diffusivity is:

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x} \left(D_L \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial y} \left(D_R \frac{\partial C}{\partial y} \right) + \frac{\partial}{\partial z} \left(D_T \frac{\partial C}{\partial z} \right)$$

Where C = C (x, y, z, t) is the concentration of water diffusing, with DL, DR and DT diffusion coefficients in the longitudinal, radial and tangential.

2.1 Analytical Model

When the principal diffusivities are constant, the equation of transfer within the timber is:

$$\frac{\partial C}{\partial t} = D_L \frac{\partial^2 C}{\partial x^2} + D_R \frac{\partial^2 C}{\partial y^2} + D_T \frac{\partial^2 C}{\partial z^2}$$

Initial conditions and the edges:

$$\begin{aligned} t=0 & \quad 0 < x < a & \quad 0 < y < b \\ & \quad 0 < z < c & \quad C = C(x, y, z, t) = 0 \\ t > 0 & \quad x=0 \quad x=a, y=0 \quad y=b, \\ & \quad z=0 \quad z=c & \quad C(x, y, z, t) = C_{\text{eq}} \end{aligned}$$

Where 0 and a, b, c denote the abscissa of the two faces of the surface.

C (x, t) is the concentration of liquid in the plate at the time t and at the abscissa x.

Ceq: is the concentration obtained at equilibrium.

The kinetics of the transfer of three-way material is obtained as the product of the analytical solutions obtained for each direction of transfer.

$$\frac{M_{\infty} - M_t}{M_{\infty}} = f(t, a) \cdot f(t, b) \cdot f(t, c)$$

$$\frac{M_{\infty} - M_t}{M_{\infty}} = \frac{512}{\pi^6} \sum_{i=0}^{\infty} \frac{1}{(2i+1)} \exp \left[-\frac{(2i+1)^2 \Pi^2 D_L t}{a^2} \right]$$

$$\sum_{k=0}^{\infty} \frac{1}{(2k+1)} \exp \left[-\frac{(2k+1)^2 \Pi^2 D_R t}{b^2} \right]$$

$$\sum_{j=0}^{\infty} \frac{1}{(2j+1)} \exp \left[-\frac{(2j+1)^2 \Pi^2 D_T t}{c^2} \right]$$

A parallelepiped with sides a, b, c and main diffusivities DL, DR and DT. The three factors, for example (f (t, a), f (t, b), f (t, c)) can also be expressed by means of a series involving ierfc function:

$$f(t, a) = 1 - \frac{2}{a} \sqrt{D_L t} \left[\pi^{-\frac{1}{2}} + 2 \sum_{n=1}^{\infty} (-1)^n \text{ierfc} \left(\frac{nL}{(D_L t)^{\frac{1}{2}}} \right) \right]$$

For long transfer time (Mt/ M_∞ > 0.6-0.7) the first term in the power series is predominant. Therefore

$$\frac{M_{\infty} - M_t}{M_{\infty}} = \frac{512}{\pi^6} \cdot \exp \left[-\left(\frac{D_L}{a^2} + \frac{D_R}{b^2} + \frac{D_T}{c^2} \right) \frac{\pi^2}{4} t \right]$$

For short time all the terms containing ierfc remain negligible. Therefore:

$$\frac{M_{\infty} - M_t}{M_t} = (1 - A\sqrt{t})(1 - B\sqrt{t})(1 - C\sqrt{t})$$

Along With:

$$A = \frac{2}{a} \sqrt{\frac{D_L}{\pi}} \quad B = \frac{2}{b} \sqrt{\frac{D_R}{\pi}} \quad C = \frac{2}{c} \sqrt{\frac{D_T}{\pi}}$$

Therefore:

$$\frac{M_{\infty} - M_t}{M_t} = (A + B + C)t^{0.5} - (BC + CA + AB)t + ABC t^{1.5}$$

If wearing abscissa $t_{1/2}$ and ordinate M_t / M_{∞} , a straight line is obtained, the tangent at the origin has the equation:

$$\frac{M_t}{M_{\infty}} = \sqrt{t} \cdot \frac{2}{\sqrt{\pi}} \left(\frac{\sqrt{D_L}}{a} + \frac{\sqrt{D_R}}{b} + \frac{\sqrt{D_T}}{c} \right)$$

2.2 Numerical Model

When the diffusion coefficients are constant and when the initial distribution of the diffusing substance is uniform in the sample interior, an analytical solution exists. But often the initial distribution is not uniform and it is useful to construct a numerical model.

Along each direction of diffusion, the cuboids sides L, T, R, is divided into slices of equal thickness, ΔL in the longitudinal direction, ΔT the tangential direction and ΔR the radial direction. Positions in the timber are associated with three variables i, j, k are defined by the relations:

$$X = i \cdot \Delta L \text{ avec } 0 < i < 2NL \quad L = 2NL \cdot \Delta L$$

$$Y = j \cdot \Delta R \text{ avec } 0 < j < 2NR \quad R = 2NR \cdot \Delta R$$

$$Z = k \cdot \Delta T \text{ avec } 0 < k < 2NT \quad T = 2NT \cdot \Delta T$$

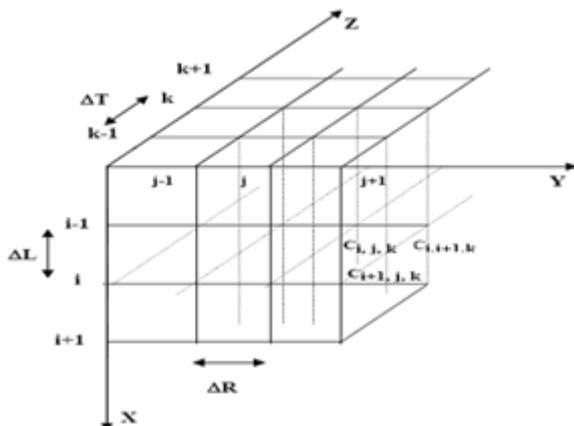


Figure1: Diagram in space for numerical analysis Pour un système tridimensionnel

The time is divided into equal intervals: Δt

It is necessary to perform numerical analysis by distinguishing more positions in the sample:

- The interior of the sample, where the moisture is transferred only by diffusion.

- The faces, where there's transfer across the interface in one direction and diffusive transport in the other two directions.
- The edges, for which there's transfer across the interface in both directions.
- Corners, which are in direct contact with the surrounding steam along the three directions.

Concentration inside: $1 < i < N_L$; $1 < j < N_R$; $1 < k < N_T$

It performs the mass balance of the product within a volume element of parallelepiped dimension ΔL , ΔR , ΔT centered at the point node i, j, k during the time increment Δt , considering the substance to within the solid and the boundary conditions, the new concentration within, at the node i, j, k is:

$$C_{N_{L,j,k}} = C_{i,j,k} + \frac{1}{M_T} [C_{i-1,j,k} - 2C_{i,j,k} + C_{i+1,j,k}] + \frac{1}{M_R} [C_{i,j-1,k} - 2C_{i,j,k} + C_{i,j+1,k}] + \frac{1}{M_L} [C_{i,j,k-1} - 2C_{i,j,k} + C_{i,j,k+1}]$$

Where M_L , M_R and M_T are dimensionless numbers:

$$M_R = \frac{(\Delta L)^2}{\Delta t \cdot D_R} \quad M_L = \frac{(\Delta L)^2}{\Delta t \cdot D_L} \quad M_T = \frac{(\Delta T)^2}{\Delta t \cdot D_T}$$

Concentration at the surface:

The mass transfer coefficient h at the surface is infinite, then:

$$C_{eq} = C_s$$

Localized mass amount of moisture in the solid:

Local moisture mass in the sample is determined at every instant by integrating the concentration in space. The calculation is done for each axis. Along the Z axis (k), for each pair (i, j) such that:

$1 < i < N_L$ and $1 < j < N_R$

$$MK_{i,j} = \frac{3}{4} C_{i,j,0} + \frac{9}{4} C_{i,j,1} + 2 \sum_{k=2}^{N_T-1} C_{i,j,k} + C_{i,j,N_T}$$

Along the axis Y (j) for each integer i such that $0 < i < N_L$

$$M_j = \left[\begin{array}{c} \frac{3}{4} MJ_0 + \frac{9}{4} MJ_1 + \\ N_L - 1 \\ 2 \sum_{i=2} MJ_i + MJ_{N_L} \end{array} \right] \Delta L \cdot \Delta T \cdot \Delta R$$

$$MJ_i = \frac{3}{4} MK_{i,0} + \frac{9}{4} MK_{i,L} + 2 \sum_{k=2}^{N_R-1} Mk_{i,j} + Mk_{i,N_R}$$

Along the axis X (i)

3. Experimental Part

3.1 Experimental procedure:

- Species: Three samples of Moroccan origin *Epicia* are used in the study.
- Sample 1 dimensions: (L = 1.964cm³, R = 1.957cm³, T = 6.126cm³)
- Sample 2: (L = 2.056cm³, R = 2.056cm³, T = 6.029cm³)
- Sample 3: (L = 2.043cm³, R = 2.043cm³, T = 6.112cm³)

The experimental device is:

- A balance (of 10⁻³g sensitivity) to track the evolution of the mass of the sample over time by gaining weight at successive time intervals, the balance is equipped with a frame with all its sides closed to prevent the disruption caused measures the movement of air in the laboratory.
- An oven to dry the sample at 102 °C and having the measurement of the sample in the anhydrous state.
- Desiccators
- Crystallizer
- Thermometer

3.2 The experimental Contacts

Sample preparation does not require deep pockets. Indeed, the samples were cut in the tangential and radial directions of the longitudinal timber. The solution is distributed with excess salt (NaCl, 76% RH) in the crystallizer that is available in the dryer. The solution may be put directly into the bottom of the desiccators on a height of a few centimeters in order to have a maximum evaporation surface. The sample weight change is measured at well-defined intervals.

4. Results and Discussion

Three types of results are interest in this paper:

- The determination of parameters such as the main diffusivity and moisture content at equilibrium.
- The validity of the analysis and the numerical model is tested by comparing the absorption kinetics obtained by experiments and calculations.
- Determination of concentration profiles.

Measurement Parameters

Transfer to a dimension along a main direction of diffusion and the four sides are protected by a waterproof film. Each main diffusivity is calculated from the straight line obtained by plotting the quantity (Qt) of the moisture carried by the corresponding axis in function of the square root of time, diffusivity is easily calculated when the moisture content of the equilibrium (Q_∞).

Along With:

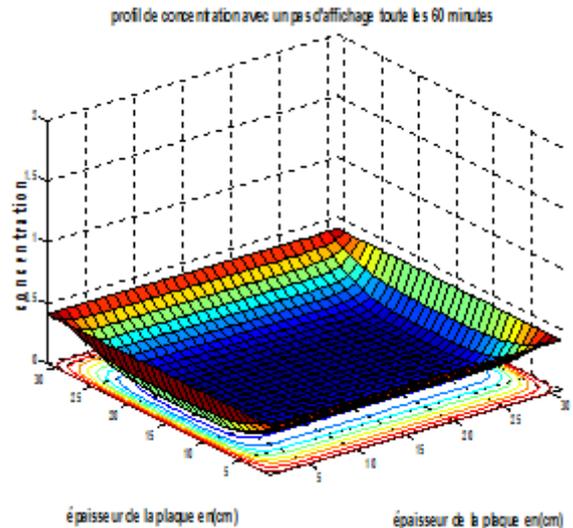
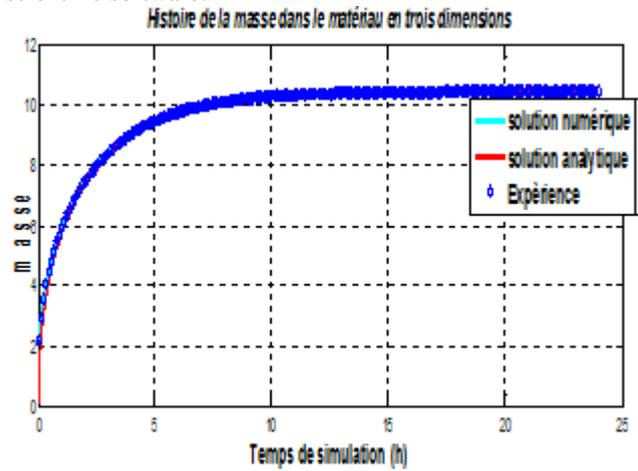
$$\frac{Q_t}{Q_\infty} = \frac{4}{L} \left(\frac{D.t}{\Pi} \right)^{0.5}$$

Where L is the thickness of the plate.

Table 1: Parameter Values

Principal axis	Diffusivity (cm ² /s)
Longitudinal	1,97.10 ⁻⁵
Tangentiel	1,53.10 ⁻⁶
Radial	1,49 .10 ⁻⁶

The following results are obtained by calculations on scientific software.



- The validity of the digital model is evaluated by comparing the absorption kinetics obtained either by testing or by calculation using the above parameters.
- In these conditions, the analytical solution and the numerical model give the same curves.
- A good agreement is shown between the theoretical and experimental kinetics, proving the validity of the model.

5. Conclusion

According to this study, we conclude that:

- This model simulates in a few hours on transfers of up in reality several months or more.
- The validation of analytical and numerical model was made by comparing the experimental results and the theoretical results.
- The resulting profile provides better information on concentrations of moisture in the wood.

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