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Modelling the Impact of Awareness Programs on the spread of HIV/AIDS

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Abstract: ***The HIV/AIDS epidemic is a global issue. Awareness programs can bring together masses of people and enlighten them on this disease. In this paper, we propose and analyze a mathematical model to study the effect of awareness programs on the transmission of HIV/AIDS. Two steady states, namely the disease free state and the infected state are studied. Stability conditions are obtained for these two steady states. Numerical simulations are carried out to illustrate the results.***

Keywords: AIDS, HIV, Stability analysis.

1. Introduction

The expanded form of AIDS is acquired immunodeficiency syndrome. HIV or the human immunodeficiency virus is the main cause of this deadly disease. HIV attacks the immune cells, namely, the CD4+ T cells of the body [1], [2], due to which the body system gets prone to different kinds of infections. Its victims include individuals from all walks of life. It is one that does not discriminate race, gender or age. If not taken care of, this problem can blossom into a bigger area and can hamper growth, social status and at the same time health of many innocent individuals who are unaware of facts and figures.

Spreading AIDS awareness through correct understanding is very important to reduce the number of people affected by the disease. HIV can only be treated but a complete cure or a vaccine for it is yet to be discovered [3]. Thus efforts to control the epidemic have focused on prevention [4]. By starting an awareness program one can hope to bring together masses of people and give them information on how the disease is contracted by individuals and the ways one can enlist to save themselves from contracting the disease. This knowledge given to the masses will prevent social stigma from becoming rampant and will allow those affected with the disease to live a normal life. Interventions may include general counseling and testing programs [5], [6], partner notification programs [7], and programs aimed at slowing HIV transmission from mother to child [8], [9], from sexual contacts [10]-[14], and from injection drug use [15], [16]. Some other interventions may include programs to ensure the safety of the healthcare system [17]-[20], immigration restrictions [21]-[23], general programs such as registration of HIV-positive individuals [24], care and treatment of individuals with HIV/AIDS [25]-[27].

The HIV/AIDS disease is considered to be an illness that is truly deceiving as the person continues to remain healthy for quite sometime without experiencing any kind of symptoms. Early diagnosis in this disease is very difficult. Immune system cannot be reconstituted even partially when the disease reaches its terminal phase. It is due to this that it is very necessary to create a general awareness about this disease amongst the people [28]. Not only does the person remain infected with the disease but can also infect the rest of the people as well. Hence AIDS awareness amongst people is very necessary.

One of the most challenging things is to live with a disease called HIV/AIDS. Many effective medications are available in the recent times due to which the person can continue to remain healthy even while coping with changes that keep taking place mentally and physically. A lot of support is needed from the immediate family members, relatives and friends since there is constant fear of getting rejected within the social circle. The person needs to get proper medical attention for which he needs to be encouraged constantly. One of the recent and most effective therapies is the Highly Active Antiretroviral Therapy also termed as HAART [29]-[31]. This is a treatment in which multiple drugs are administered to help the patient improve the quality of life. The cycle of AIDS infection gets slowed down with this kind of a therapy.

Prevention needs around the world differ because of economic constraints, differing trends of transmission, and cultural and political factors [4]-[32]. Thus, decisions about HIV prevention and AIDS treatment programs are based on many factors, including program costs and health benefits, available funding, social and ethical issues and political considerations [33]-[35].

A meta-analysis of studies of sexual behavior concluded that HIV counseling and testing was an effective means of inducing HIV-infected individuals to reduce their propensity to transmit the disease, but was ineffective in inducing uninfected individuals to reduce their risk of becoming infected [36]. Interventions may be classified into behavioral and non-behavioral interventions [26]. Behavioral interventions aim to change the risky behavior of infected and at-risk individuals, and include general counseling and testing programs, partner notification programs, and programs aimed at slowing HIV transmission from mothers to their offspring, from sexual contacts, and from injection drug use. Non-behavioral interventions include programs to ensure the safety of the healthcare system, immigration restrictions and quarantine programs, general programs such as registration of HIV-positive individuals, care and treatment of individuals with HIV/AIDS, and possible vaccination if a vaccine against HIV becomes available. A

behavioral intervention may induce behavior change that lasts only for a limited period [37]-[39], or may induce long-term behavior change [40]. Rowley and Anderson [11] found that the faster the spread of the epidemic is, the shorter is the time period over which the impact of limited-term behavioral changes is experienced. Paltiel [41] investigated the cost effectiveness of HIV prevention and treatment interventions at different points during the epidemic life cycle. The analysis suggested that, early in the epidemic, behavioral interventions targeted to infected individuals are the most cost effective, then for some years a combination of interventions targeted to both infected and uninfected individuals is best, and then late in the epidemic life cycle, behavioral interventions targeted to uninfected individuals are the most cost effective.

In this paper, we formulate and analyze an ordinary differential equation model to study the effect of awareness programs on the spread of HIV/AIDS, in a variable population with immigration. The total population is divided into three classes, the unaware susceptible class, the aware susceptible class and the infected class. Individuals in both the susceptible classes can become infected through blood transfusion or sexual contact with an infected individual, or from an infected mother to child. But the probability of contracting infection is less for individuals in the aware class than those in the unaware class. We assume that a person once infected with the disease will not recover from the disease and hence cannot return to the uninfected class. In section 3, we determine the steady states and in section 4, we analyze the local and the global stability of the uninfected and the infected states [42]. A numerical simulation is carried out in section 4 to validate our results.

2. The Primary Model

Consider the following systems of ODE.

$$\frac{dX}{dt} = A - \beta XY - \lambda XM - dX$$

$$\frac{dX^*}{dt} = \lambda XM - dX^* - \beta_1 \beta X^* Y \qquad (2.1)$$

$$\frac{dY}{dt} = \beta XY + \beta_1 \beta X^* Y - \alpha Y - dY$$

$$\frac{dM}{dt} = k\alpha Y - \mu_0 M$$

In the above model, X, X^*, Y are the populations of the unaware susceptible, the aware susceptible and the infected respectively at time t. M is the cumulative density of awareness programs in a region at time t. A is the rate of immigration of susceptible. β is the contact rate of unaware susceptible with infected, λ is the dissemination rate of awareness among the susceptibles and d is the natural death rate. β_1 denotes the reduced probability of contracting infection and α denotes the disease induced death rate. k is the proportionality constant which governs the implementation of awareness programs and μ_0 is the depletion rate of awareness programs due to ineffectiveness, dearth of funds, social and political barriers, etc.

Let N be the total population at time t.

Then $N = X + X^* + Y$. Using this in *(2.1)*, we get

$$\frac{dY}{dt} = \beta\left(N - (1-\beta_1)X^* - Y\right)Y - (\alpha + d)Y$$

$$\frac{dX^*}{dt} = \lambda(N - X^* - Y)M - dX^* - \beta_1 \beta X^* Y \qquad (2.2)$$

$$\frac{dN}{dt} = A - dN - \alpha Y$$

$$\frac{dM}{dt} = k\alpha Y - \mu_0 M$$

3. Determination of Steady States

The above system *(2.2)* has two possible steady states

(i) the disease free steady state $E_1(0, 0N, 0)$ where $N = \frac{A}{d}$

and

(ii) the endemic steady state $E_2(\overline{Y}, \overline{X^*}, \overline{N}, \overline{M})$ where

$$\overline{X^*} = \frac{A - \alpha\overline{Y} - d\overline{Y}}{d(1-\beta_1)} - \frac{(\alpha + d)}{\beta(1-\beta_1)}$$

$$\overline{N} = \frac{A - d\overline{Y}}{d}$$

$$\overline{M} = \frac{k\alpha\overline{Y}}{\mu_0} \text{ and}$$

$\overline{Y}$ satisfies the equation

$$A_1\overline{Y}^2 + A_2\overline{Y} + A_3 = 0 \qquad (3.1)$$

where $A_1 = \frac{k\lambda}{\mu_0}\left[\frac{\alpha + d}{(1-\beta_1)d} - \frac{\alpha}{d} - 1\right] + \frac{\beta\beta_1(\alpha + d)}{(1-\beta_1)d}$

$$A_2 = \frac{k\lambda}{\mu_0}\left[\frac{A}{d} - \frac{A}{(1-\beta_1)d} + \frac{(\alpha + d)}{\beta(1-\beta_1)}\right] - \beta\beta_1\left[\frac{A}{(1-\beta_1)d} - \frac{(\alpha + d)}{\beta(1-\beta_1)} + \frac{\alpha + d}{1-\beta_1}\right]$$

$$A_3 = \left[\frac{\alpha + d}{\beta(1-\beta_1)} - \frac{A}{(1-\beta_1)d}\right]d = \frac{\alpha + d}{\beta(1-\beta_1)}(1 - R_0)d$$

where $R_0 = \frac{\beta A}{(\alpha + d)d}$

Solving *(3.1)*, we get

$$\overline{Y} = \frac{-A_2 \pm \sqrt{A_2^2 - 4A_1A_3}}{2A_1}$$

4. Stability Analysis

Lemma 1. *The bounded set*

$$S = \left\{(Y, X^*, N, M) \in R_+^4 : 0 \le Y, X^* \le N \le \frac{A}{d}, 0 \le M \le \frac{k\alpha A}{\mu_0 d}\right\}$$

is positively invariant with respect to system (2.2).

The proof of the lemma is given in Appendix A.

Theorem 1. *The non-infected state* $E_1\left(0,0,\frac{A}{d},0\right)$ *is locally asymptotically stable when* $R_0 < 1$.

The proof of the theorem is given in Appendix B.

Theorem 2. *The infected state* $E_2\left(\overline{Y},\overline{X^*},\overline{N},\overline{M}\right)$ *is locally asymptotically stable if* $R_0 \geq 0$ *provided* $A_{12}^2 < A_{11}A_{22}, A_{13}^2 < A_{11}A_{33}, A_{14}^2 < A_{11}A_{44}, A_{23}^2 < A_{22}A_{33}, A_{24}^2 < A_{22}A_{44}$

The proof of the theorem is given in Appendix C.

Remark 1. The equilibrium number of infected person decreases with an increase in the value of the dissemination rate of awareness programs, the implementation rate of awareness programs and the disease induced death rate respectively.

Theorem 3 *The infected state* $E_2\left(\overline{Y},\overline{X^*},\overline{N},\overline{M}\right)$ *is globally asymptotically stable if* $R_0 > 0$ *provided* $a_{12}^2 < a_{11}a_{22}, a_{13}^2 < a_{11}a_{33}, a_{14}^2 < a_{11}a_{44}, a_{23}^2 < a_{22}a_{33}, a_{24}^2 < a_{22}a_{44}$

The proof of the theorem is given in Appendix *D*.

5. Numerical Simulations

We choose the following parameters in the model
$A = 400, \beta = 0.003, \beta_1 = 0.4, \lambda = 0.8,$
$\alpha = 0.75, d = 0.5, \mu_0 = 0.95, k = 0.5$
With the above values of parameters, the positive equilibrium E_2 exists and it is given by
$\overline{Y} = 2.2077, \overline{X^*} = 796.6885, \overline{N} = 796.6885, \overline{M} = 0.8714$
Also, with the above parameters, $R_0 = 1.92 > 1$Hence the conditions in the above theorems are satisfied.

5.1. Behavior of infected population for different values of β

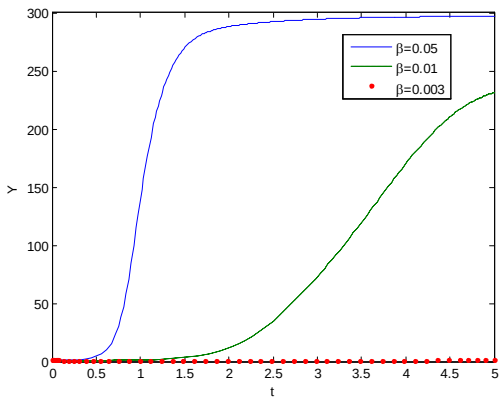

Figure 1

It is noted here that the infected population increases as the contact rate of unaware susceptible with infected population increases.

5.2. Behavior of infective population for different values of λ

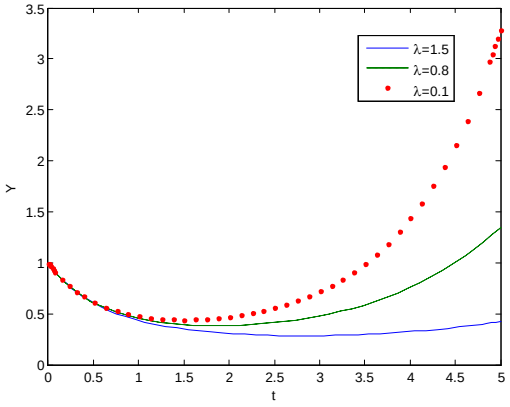

Figure 2

It is noted here that the infected population decreases as the dissemination of awareness increases.

5.3 Behavior of the aware population for different values of λ

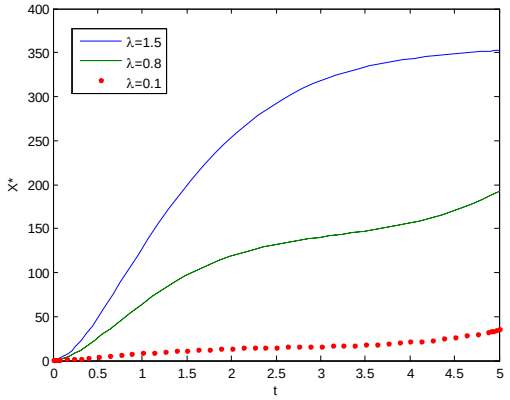

Figure 3

It is noted here that the aware population increases as the dissemination of awareness increases.

5.4. Behavior of the cumulative density of awareness programs for different values of α

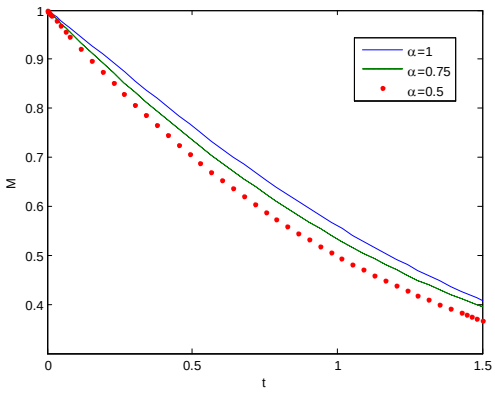

Figure 4

It is noted here that the density of awareness programmes increases as the disease induced death rate increases.

6. Conclusions

In this paper, we have studied the effect of awareness programs on the transmission of HIV/AIDS. The total population is divided into three classes, the unaware susceptible class, the aware susceptible class and the infective class. We have a system of four ODEs dealing with the infected population, the aware uninfected population, the total population and the cumulative density of awareness programs in a region. Existence of equilibrium states and the conditions for local and global stability have been obtained. The basic reproduction number R_0 is obtained and it determines the dynamics of the model. It is seen that the uninfected state is stable when $R_0 < 0$ and the infected state is stable when $R_0 \geq 0$. The density of awareness programs increases with an increase in the disease induced death rate. Also, the proportion of infected individuals decreases with an increase in the number of awareness programs.

Appendices

Appendix A: Proof of Lemma 1

From the third equation of (*2.2*),

$$A - dN - \alpha Y = 0$$

$$\Rightarrow N \leq \frac{A}{d}$$

From the fourth equation of (*2.2*),

$$M = \frac{k\alpha Y}{\mu_0} \leq \frac{k\alpha A}{\mu_0 d}$$

Hence the bounded set

$$S = \left\{ \left(Y, X^*, N, M\right) \in R_+^4 : 0 \leq Y, X^* \leq N \leq \frac{A}{d}, 0 \leq M \leq \frac{k\alpha A}{\mu_0 d} \right\}$$

attracts all solutions initiating in the interior of the positive orthant.

Appendix B: Proof of Theorem 1

The Jacobian matrix corresponding to the system (*2.2*) at the disease free equilibrium state $E_1\left(0, 0, \frac{A}{d}, 0\right)$ is

$$J\left(E_1\right) = \begin{bmatrix} \frac{\beta A}{d} - (\alpha + d) & 0 & 0 & 0 \\ 0 & -d & 0 & \frac{\lambda A}{d} \\ -\alpha & 0 & -d & 0 \\ k\alpha & 0 & 0 & -\mu_0 \end{bmatrix}$$

Hence the non-infected state E_1 is locally asymptotically stable when

$\frac{\beta A}{d} - (\alpha + d) \leq 0$ i.e. $R_0 < 1$.

Appendix C: Proof of Theorem 2

From the first equation of system (*2.2*),

$$Y = N - (1 - \beta_1) X^* - \frac{(\alpha + d)}{\beta}$$

$Y > 0$ when $\frac{\beta A}{d} - (\alpha + d) \geq 0$

That is, the infected equilibrium state $E_2\left(\overline{Y}, \overline{X^*}, \overline{N}, \overline{M}\right)$ exists only when $R_0 \geq 1$

where $R_0 = \frac{\beta A}{d(\alpha + d)}$

The Jacobian matrix of the system (*2.2*) at the infected equilibrium state E_2 is

$$J\left(E_2\right) = \begin{bmatrix} \beta\overline{N} - \beta(1-\beta_1)\overline{X^*} - 2\beta\overline{Y} - (\alpha+d) & -\beta(1-\beta_1)\overline{Y} & \beta\overline{Y} & 0 \\ -\lambda\overline{M} - \beta_1\beta\overline{X^*} & -(\lambda\overline{M} + d) - \beta_1\beta\overline{Y} & \lambda\overline{M} & \lambda\left(\overline{N} - \overline{X^*} - \overline{Y}\right) \\ -\alpha & 0 & -d & 0 \\ k\alpha & 0 & 0 & -\mu_0 \end{bmatrix}$$

Now,

$$\beta\overline{N} - \beta(1-\beta_1)\overline{X^*} - 2\beta\overline{Y} - (\alpha+d) = \beta\left[\frac{A - \alpha\overline{Y}}{d} - (1-\beta_1)\left\{\frac{A - \alpha\overline{Y} - d\overline{Y}}{d(1-\beta_1)} - \frac{\alpha + d}{\beta(1-\beta_1)}\right\} - 2\overline{Y}\right] - (\alpha+d)$$
$$= -\beta\overline{Y}$$

Therefore

$$J\left(E_2\right) = \begin{bmatrix} -\beta\overline{Y} & -\beta(1-\beta_1)\overline{Y} & \beta\overline{Y} & 0 \\ -\lambda\overline{M} - \beta_1\beta\overline{X^*} & -(\lambda\overline{M} + d) - \beta_1\beta\overline{Y} & \lambda\overline{M} & \lambda\left(\overline{N} - \overline{X^*} - \overline{Y}\right) \\ -\alpha & 0 & -d & 0 \\ k\alpha & 0 & 0 & -\mu_0 \end{bmatrix}$$

The character equation is

$$f(\gamma) = \begin{pmatrix} -a_1 - \gamma & a_2 & a_3 & 0 \\ -a_4 & -a_5 - \gamma & a_6 & a_7 \\ -a_8 & 0 & a_9 - \gamma & 0 \\ a_{10} & 0 & 0 & -a_{11} - \gamma \end{pmatrix} = 0$$

where $a_1 = \beta\overline{Y}$; $a_2 = -\beta(1-\beta_1)\overline{Y}$; $a_3 = \beta\overline{Y}$;

$a_4 = \lambda\overline{M} - \beta_1\beta\overline{X^*}$; $a_5 = -\left(\lambda\overline{M} + d\right) - \beta_1\beta\overline{Y}$; $a_6 = \lambda\overline{M}$;

$a_7 = \lambda\left(\overline{N} - \overline{X^*} - \overline{Y}\right)$; $a_8 = \alpha$; $a_9 = d$; $a_{10} = k\alpha$; $a_{11} = \mu_0$

Thus the characteristic equation is

$$\gamma^4 + A_3\gamma^3 + A_2\gamma^2 + A_1\gamma + A_0 = 0$$

where $A_3 = a_1 + a_5 + a_9 + a_{11}$

$A_2 = a_1a_5 + a_1a_9 + a_1a_{11} + a_5a_9 + a_5a_{11} + a_9a_{11} + a_2a_4 + a_3a_8$

$A_1 = a_1a_5a_9 + a_1a_5a_{11} + a_1a_9a_{11} + a_5a_9a_{11} - a_2a_4a_9 - a_2a_4a_{11} - a_2a_6a_8 + a_2a_7a_{10} + a_3a_5a_8 + a_3a_8a_{11}$

$A_0 = a_1a_5a_9a_{11} - a_2a_4a_9a_{11} - a_2a_6a_8a_{11} + a_2a_7a_9a_{10} + a_3a_5a_8a_{11}$

By using the transformations

$Y_1 = y - \overline{Y}$

$X_1^* = x + \overline{X^*}$

$N_1 = n + \overline{N}$

$M_1 = m + \overline{M}$

We linearize the system,

$$\begin{bmatrix} \dot{y} \\ \dot{x} \\ \dot{n} \\ \dot{m} \end{bmatrix} = J(E_2) \begin{bmatrix} y \\ x \\ n \\ m \end{bmatrix}$$

and get

$\dot{y} = -\beta\overline{Y}y - \beta(1-\beta_1)\overline{Y}x + \beta\overline{Y}n$

$\dot{x} = -\left(\lambda\overline{M} + \beta_1\beta\overline{X^*}\right)y - \left(\lambda\overline{M} + d + \beta_1\beta\overline{Y}\right)m$

$\dot{n} = -(\alpha y + dn)$

$\dot{m} = k\alpha y - \mu_0 m$

Consider the following positive definite function,

$$V = \frac{1}{2}\left[\frac{y^2}{Y} + C_1x^2 + C_2m^2\right]$$

Then

$$\begin{aligned}\dot{V} = &-\frac{1}{3}\beta y^2 + \left[-\beta(1-\beta_1) - C_1\left(\lambda\overline{M} + \beta_1\beta\overline{X^*}\right)\right]yx \\ &-\frac{1}{3}C_1\left(\lambda\overline{M} + d + \beta_1\beta\overline{Y}\right)x^2 \\ &-\frac{1}{3}\beta y^2 + (\beta - C_2\alpha)yn - \frac{1}{2}C_2dn^2 \\ &-\frac{1}{3}\beta y^2 + C_3k\alpha ym - \frac{1}{2}C_3\mu_0m^2 \\ &-\frac{1}{3}C_1\left(\lambda\overline{M} + d + \beta_1\beta\overline{Y}\right)x^2 - C_1\lambda\overline{M}xn \\ &-\frac{1}{2}C_2dn^2 - \frac{1}{3}C_1\left(\lambda\overline{M} + d + \beta_1\beta\overline{Y}\right)x^2 \\ &+C_1\lambda\left(\overline{N} - \overline{X^*} - \overline{Y}\right)xm + \frac{1}{2}C_3\mu_0m^2\end{aligned}$$

Taking $C_1 = p_1$, $C_2 = \frac{2}{3}p_2$, $C_3 = \frac{2}{3}p_3$, we get

$$\begin{aligned}\dot{V} = &-\frac{1}{3}\beta y^2 + \left[-\beta(1-\beta_1) - p_1\left(\lambda\overline{M} + \beta_1\beta\overline{X^*}\right)\right]yx \\ &-\frac{1}{3}p_1\left(\lambda\overline{M} + d + \beta_1\beta\overline{Y}\right)x^2 - \frac{1}{3}\beta y^2 \\ &+\left(\beta - \frac{2}{3}p_2\alpha\right)yn - \frac{1}{3}p_2dn^2 - \frac{1}{3}\beta y^2 \\ &+\frac{2}{3}p_3k\alpha ym - \frac{1}{3}p_3\mu_0m^2 \\ &-\frac{1}{3}p_1\left(\lambda\overline{M} + d + \beta_1\beta\overline{Y}\right)x^2 + -\left(p_1\lambda\overline{M}\right)xn \\ &-\frac{1}{3}p_2dn^2 - \frac{1}{3}p_1\left(\lambda\overline{M} + d + \beta_1\beta\overline{Y}\right)x^2 \\ &+p_1\lambda\left(\overline{N} - \overline{X^*} - \overline{Y}\right)xm - \left(-\frac{1}{3}p_3\mu_0\right)m^2\end{aligned}$$

so that

$$\begin{aligned}\dot{V} = &-\frac{1}{3}A_{11}y^2 + A_{12}yx - \frac{1}{3}A_{22}x^2 \\ &-\frac{1}{3}A_{11}y^2 + A_{13}yn - \frac{1}{3}A_{33}n^2 \\ &-\frac{1}{3}A_{11}y^2 + A_{14}ym - \frac{1}{3}A_{44}m^2 \\ &-\frac{1}{3}A_{22}x^2 + A_{23}xn - \frac{1}{3}A_{33}n^2 \\ &-\frac{1}{3}A_{22}x^2 + A_{24}xm - \frac{1}{3}A_{44}m^2\end{aligned}$$

where $A_{11} = \beta$, $A_{22} = p_1\left(\lambda\overline{M} + d + \beta_1\beta\overline{Y}\right)$, $A_{33} = p_2d$,

$A_{44} = -p_3\mu_0$, $A_{12} = -\beta(1-\beta_1) - C_1\left(\lambda\overline{M} + \beta_1\beta\overline{X^*}\right)$,

$A_{13} = \beta - \frac{2}{3}p_2\alpha$, $A_{14} = \frac{2}{3}p_3k\alpha$, $A_{23} = -p_1\lambda\overline{M}$,

$A_{24} = p_1\lambda\left(\overline{N} - \overline{X^*} - \overline{Y}\right)$

Sufficient conditions for $\dot{V}$ to be negative definite are that the following inequalities hold

$A_{12}^2 < A_{11}A_{22}$, $A_{13}^2 < A_{11}A_{33}$, $A_{14}^2 < A_{11}A_{44}$, $A_{23}^2 < A_{22}A_{33}$, $A_{24}^2 < A_{22}A_{44}$

Appendix D: Proof of Theorem 3

Let

$$V_1\left(\overline{Y}, \overline{X^*}, \overline{N}, \overline{M}\right) = Y - \overline{Y} - \overline{Y}\ln\left(\frac{Y}{\overline{Y}}\right) + C_1\left(X^* - \overline{X^*} - \overline{X^*}\ln\left(\frac{X^*}{\overline{X^*}}\right)\right) + \frac{C_2}{2}\left(N - \overline{N}\right)^2 + \frac{C_3}{2}\left(M - \overline{M}\right)^2$$

Then

$$\begin{aligned}\dot{V}_1 = &-\frac{1}{2}a_{11}\left(Y - \overline{Y}\right)^2 + a_{12}\left(Y - \overline{Y}\right)\left(X^* - \overline{X^*}\right) - \frac{1}{2}a_{22}\left(X^* - \overline{X^*}\right)^2 \\ &-\frac{1}{2}a_{11}\left(Y - \overline{Y}\right)^2 + a_{13}\left(Y - \overline{Y}\right)\left(N - \overline{N}\right) - \frac{1}{2}a_{33}\left(N - \overline{N}\right)^2 \\ &-\frac{1}{2}a_{11}\left(Y - \overline{Y}\right)^2 + a_{14}\left(Y - \overline{Y}\right)\left(M - \overline{M}\right) - \frac{1}{2}a_{44}\left(M - \overline{M}\right)^2 \\ &-\frac{1}{2}a_{22}\left(X^* - \overline{X^*}\right)^2 + a_{23}\left(X - \overline{X^*}\right)\left(N - \overline{N}\right) - \frac{1}{2}a_{33}\left(N - \overline{N}\right)^2 \\ &-\frac{1}{2}a_{22}\left(X^* - \overline{X^*}\right)^2 + a_{24}\left(X - \overline{X^*}\right)\left(M - \overline{M}\right) - \frac{1}{2}a_{44}\left(M - \overline{M}\right)^2\end{aligned}$$

where

$$a_{11} = \frac{2}{3}\beta; \qquad a_{22} = \frac{2}{3}C_1\frac{\lambda M}{X^*\overline{X^*}}\left(\overline{N}+\overline{Y}\right)$$

$$a_{33} = C_2 d; \qquad a_{44} = C_3\mu_0;$$

$$a_{12} = C_1\frac{\lambda M}{\overline{X^*}} - \beta\left(1-\beta_1\right) - C_1\beta\beta_1;$$

$$a_{13} = -C_2\alpha; \qquad a_{14} = C_3 k\alpha$$

$$a_{23} = C_1\frac{\lambda M}{\overline{X^*}}; \qquad a_{24} = C_1\left[\frac{\lambda N}{X^*} - \lambda - \frac{\lambda Y}{X^*}\right]$$

$\dot{V}_1$ will be negative definite if

$a_{12}^2 < a_{11}a_{22}, a_{13}^2 < a_{11}a_{33}, a_{14}^2 < a_{11}a_{44}, a_{23}^2 < a_{22}a_{33}, a_{24}^2 < a_{22}a_{44}$

These inequalities are satisfied when $R_0 > 1$.

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