Comparison of Accuracy of various Impedance Based Fault Location Algorithms on Power Transmission Line

Ankamma Rao J\textsuperscript{1}, Gebreegziabher Hagos\textsuperscript{2}

\textsuperscript{1}Assistant Professor, Electrical & Computer Engineering Department, Samara University, Ethiopia
\textsuperscript{2}Lecturer, Electrical & Computer Engineering Department, Samara University, Ethiopia

Abstract: Transmission line protection is an important issue in power engineering because 85-87\% of power system faults are occurring in transmission lines. This paper presents three developed algorithms: Takagi’s method, Ericsson method, Fault sequence component of current method. MATLAB/Simulink software was used to implement these algorithms. The accuracy of fault location on power transmission line are compared for these three method by varying various parameters like fault type, fault resistance, fault location and fault inception angle, on a given power system model. The simulation results demonstrate the validity of the suitable fault location method in 400KV transmission line.

Keywords: Power system Faults, Fault location, MATLAB, Fault impedance, Fault resistance, Fault inception angle, Accuracy of fault location.

1. Introduction

Electric power systems are designed to ensure a reliable supply of energy with the highest possible continuity. The growing complexity of electrical power system demands performance of protection and control equipment. Faults can occur in any point of the power system, and the most exposed parts are overhead transmission lines. Fault locators are used to pinpoint transmission line faults, and they help in the reduction of maintenance works and quick system recovery from faults. A fault locator is also a very useful tool in evaluating transient faults that could otherwise cause weak spots in transmission and distribution systems, resulting in future problems or faults. During the last 20 years there has been considerable interest in computer relaying of power systems [1] and in the development of microprocessor-based fault locators [10]. The method which uses data from all ends requires synchronized measurement with time stamping and online communication of data to central location [11-13]. This paper describes three impedance based fault location algorithms on 400 KV transmission line.

2. Theory of Impedance Based Fault Location Algorithms

![Figure 1: Fault network diagram](image)

![Figure 2: Pure Fault network](image)

![Figure 3: Incremental positive sequence network diagram](image)

![Figure 4: Negative sequence network diagram](image)

3. Nomenclature

\[ d \] Estimated distance to the fault (units: p.u)
\[ V_{A_P} \] Protective distance relay voltage at the line end A

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To derive the Fault location algorithm, the fault loop composed according to the fault classified type is considered. This loop contains the faulted line segment (between points A and F) and the fault path itself. A generalized model for the fault loop is stated as follows:

\[ V_{A,F} - dZ_L \cdot I_{A,F} - I_F \cdot R_F = 0 \]  \hspace{1cm} (1)

The superimposed circuit (Fig.2) is a current divider of the fault current and thus:

\[ p = \frac{(1-d)Z_A + Z_B}{Z_L} \cdot I_F \]  \hspace{1cm} (2)

Let \( I_p = \frac{\Delta I}{\Delta F} \)  \hspace{1cm} (3)

Where the current distribution factor is

\[ k_F = \frac{(1-d)Z_A + Z_B}{Z_L} \]  \hspace{1cm} (4)

Substitute Equation (3) in Equation (1)

\[ V_{A,F} - dZ_L \cdot I_{A,F} - \frac{\Delta I}{\Delta F} \cdot R_F = 0 \]  \hspace{1cm} (5)

Substitute Equation (5) in Equation (6)

\[ V_{A,F} - dZ_L \cdot I_{A,F} - \frac{\Delta I}{\Delta F} \cdot R_F = 0 \]  \hspace{1cm} (6)

Multiplying Equation (7) by the term \( \frac{\Delta I}{\Delta F} \cdot e^{i\theta} \) and taking imaginary part yields the following formula for the distance to fault:

\[ V_{A,F} \Delta I_A \cdot e^{i\theta} - dZ_L \cdot I_{A,F} \cdot \Delta I_A \cdot e^{i\theta} - \frac{\Delta I}{\Delta F} \cdot e^{i\theta} = 0 \]  \hspace{1cm} (8)

\[ d = \frac{\text{Im}(V_{A,F} \cdot \Delta I_A \cdot e^{i\theta})}{\text{Im}(Z_L \cdot I_{A,F} \cdot \Delta I_A \cdot e^{i\theta})} \]  \hspace{1cm} (9)

It was assumed that current distribution factor is a real number so \( e^{i\theta} = 1 \)

\[ d = \frac{\text{Im}(V_{A,F} \cdot \Delta I_A)}{\text{Im}(Z_L \cdot I_{A,F} \cdot \Delta I_A)} \]  \hspace{1cm} (10)

B. Eriksson’s Method

To derive the fault location algorithm, the fault loop composed according to the classified fault type is considered. This loop contains the faulted line segment (between points A and F) and the fault path itself. A generalized model for the fault loop is stated as follows:

\[ V_{A,F} - dZ_L \cdot I_{A,F} - I_F \cdot R_F = 0 \]  \hspace{1cm} (11)

The superimposed circuit (Fig.2) is a current divider of the fault current and thus:

\[ I_F = \frac{(1-d)Z_A + Z_B}{Z_L} \cdot I_F \]  \hspace{1cm} (12)

Where \( I_F = I_F^{pre} \) - incremental current determined from the moment of the fault-inception occurrence (thus in the fault interval), and obtained by taking the fault current and subtracting the pre-fault current (present before fault inception). Note that the recordings of the pre-fault current have to be available.

This allows the total fault current to be determined as

\[ I_F = \frac{(1-d)Z_A + Z_B}{Z_L} \cdot \Delta I_A \]  \hspace{1cm} (13)

Substitute Equation (13) in Equation (11) and it gives

\[ V_{A,F} - dZ_L \cdot I_{A,F} - \frac{(1-d)Z_A + Z_B}{Z_L} \cdot \Delta I_A \cdot R_F = 0 \]  \hspace{1cm} (14)

Expanding the Equation (14) results in

\[ V_{A,F} - dZ_L \cdot I_{A,F} - \frac{(1-d)Z_A + Z_B}{Z_L} \cdot \Delta I_A \cdot R_F = 0 \]  \hspace{1cm} (15)

Dividing the Equation (15) by the term \( I_F \) results in

\[ \frac{V_{A,F}}{I_{A,F}} = (1-d) \cdot Z_A + Z_B \]  \hspace{1cm} (16)

Expanding the Equation (16) by the term \( Z_L \)

\[ \frac{V_{A,F}}{Z_L} = (1+Z_A + Z_B) \]  \hspace{1cm} (17)

Equation (18) can be written in the following form

\[ d^2 - d \cdot k_1 + k_2 - k_3 \cdot R_F = 0 \]  \hspace{1cm} (18)

Where

\[ k_1 = \frac{V_{A,F}}{I_{A,F} \cdot Z_L} \]  \hspace{1cm} (19)

\[ k_2 = \frac{V_{A,F}}{I_{A,F} \cdot Z_L} \]  \hspace{1cm} (20)

\[ k_3 = \frac{Z_A + Z_B}{Z_L} \]  \hspace{1cm} (21)

For eliminating \( R_F \) multiply Equation (19) by \( k_2^2 \)

\[ d^2 \cdot k_2^2 - d \cdot k_2^* \cdot k_1 + k_3^2 \cdot R_F = 0 \]  \hspace{1cm} (22)

\[ R_F \]  \hspace{1cm} (23)

The solution to the Equation (24) is given by

\[ d = \frac{B_1 - \text{sqrt}(B_1^2 - 4 \cdot B_2 \cdot B_3)}{2 \cdot B_2} \]  \hspace{1cm} (25)

Where

\[ B_1 = \text{imag}(k_1) \]  \hspace{1cm} (26)

\[ B_1 = \text{imag}(k_1) \]  \hspace{1cm} (27)

\[ B_0 = \text{imag}(k_2 \cdot k_3^*) \]  \hspace{1cm} (28)

\[ B_0 = \text{imag}(k_2 \cdot k_3^*) \]  \hspace{1cm} (29)
The Equation (18) shows the relationship between $V_{A\,F\,P}, I_{A\,P}, \Delta I_{A\,P}$ and $Z_A$ in determining fault location. The absence of line charging capacitance in the equation for fault location, introduces error in estimated fault location when applied for EHV transmission lines. Another limitation of this method is exclusion of the effect of multiple power corridors normally present in any power network. This drawback when coupled with high resistance fault and fault nearer the receiving end (measurements at the sending end) results in erroneous solution.

C. Fault Sequence Component of Current Method

To derive the Fault location algorithm, the fault loop composed according to the fault classified type is considered. This loop contains the faulted line segment (between points AA and F) and the fault path itself. A generalized model for the fault loop is stated as follows

$$V_{A\,P} - \Delta Z_{UL} \times I_{A\,P} - I_F \times R_F = 0$$ (26)

Where

$$I_F = \alpha_{F1}\times I_{F1} + \alpha_{F2}\times I_{F2} + \alpha_{F0}\times I_{F0}$$ (27)

Fault loop voltages and current can be expressed interns of the local measurements and with using coefficients gathered in Table 1

$$V_{A\,P} = \alpha_1 V_{A1} + \alpha_2 V_{A2} + \alpha_3 V_{A0}$$ (28)

$$I_{A\,P} = \alpha_{F1} I_{F1} + \alpha_{F2} I_{F2} + \alpha_{F0} I_{F0}$$ (29)

Table 1: Coefficients for determining signals defined in Equations (28) and (29)

<table>
<thead>
<tr>
<th>Fault Type</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AG</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>BG</td>
<td>$\alpha^2$</td>
<td>$\alpha$</td>
<td>1</td>
</tr>
<tr>
<td>CG</td>
<td>$\alpha$</td>
<td>$\alpha^2$</td>
<td>1</td>
</tr>
<tr>
<td>AB, ABG, ABC, ABCG</td>
<td>$1 - \alpha^2$</td>
<td>$1 - \alpha$</td>
<td>0</td>
</tr>
<tr>
<td>BC, BCG</td>
<td>$\alpha^2 - \alpha$</td>
<td>$\alpha - \alpha^2$</td>
<td>0</td>
</tr>
<tr>
<td>CA, CAG</td>
<td>$\alpha - 1$</td>
<td>$\alpha^2 - 1$</td>
<td>0</td>
</tr>
</tbody>
</table>

$\alpha = \exp(j2\pi/3)$

The total fault current ($I_F$) is expressed as weighted sum of it’s the symmetrical components ($I_{F1}$, $I_{F2}$, $I_{F0}$), which can be determined with use of fault current distribution factors:

$$I_{F0} = \frac{\Delta I_{A1}}{k_{F1}}$$ (36)

Voltage drop across the fault path (as shown in the third term in Equation (26) is expressed using sequence components of total fault current ($I_{F0}$, $I_{F1}$, $I_{F2}$). Determining this voltage drop requires establishing the weighting coefficients. These coefficients can accordingly be determined by taking the boundary conditions for particular fault type. However, there is some freedom for that. Thus, it is proposed firstly to utilize this freedom for avoiding zero sequence quantities. This is well known that the zero sequence impedance of a line is considered as unreliable parameter. This is so due to dependence of this impedance upon the resistivity of a soil, which is changeable and influenced by weather conditions. Moreover, as a result of influence of overhead ground wires, the zero sequence impedance is not constant along the line length. Thus, it is highly desirable to avoid completely the usage of zero sequence quantities when determining the voltage drop across the fault path. This can be accomplished by setting $I_{F0} = 0$ as shown in Table 2, where the alternative sets of the weighting coefficients are gathered. Secondly, the freedom in establishing the weighting coefficients can be utilized for determining the preference for using particular quantities. The negative sequence (Table 2) or the positive sequence (Table 2) can be preferred

For example, considering AG fault one has:

$$\begin{bmatrix} I_{F0} \\ I_{F1} \\ I_{F2} \end{bmatrix} = \begin{bmatrix} \frac{1}{\alpha} \\ 1 \\ \frac{1}{\alpha^2} \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\alpha} \times I_{FA} \\ I_{FA} \\ 0 \end{bmatrix}$$ (30)

Thus, symmetrical components of a fault current are:

$$I_{F0} = I_{F1} = I_{F2} = \frac{1}{\alpha} \times I_{FA} = I_F$$ (31)

It follows from Equation (31) that the total faults current ($I_F = I_{F0} + I_{F1} + I_{F2}$) can be expressed in the following alternative ways, depending on which symmetrical component is preferred:

$$I_F = 3 \times I_{F1}$$ (32)

$$I_F = 3 \times I_{F2}$$ (33)

$$I_F = 3 \times I_{F0}$$ (34)

$$I_F = 1.5 \times I_{F1} + 1.5 \times I_{F2}$$ (35)

Table 2: Alternative sets of weighting coefficients

<table>
<thead>
<tr>
<th>Fault Type</th>
<th>$\alpha_{F1}$</th>
<th>$\alpha_{F2}$</th>
<th>$\alpha_{F0}$</th>
<th>$\alpha_{F1}$</th>
<th>$\alpha_{F2}$</th>
<th>$\alpha_{F0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AG</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>BG</td>
<td>0</td>
<td>$-1.5 + j1.5\sqrt{3}$</td>
<td>0</td>
<td>$-1.5 - j1.5\sqrt{3}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>CG</td>
<td>0</td>
<td>$-1.5 - j1.5\sqrt{3}$</td>
<td>0</td>
<td>$-1.5 + j1.5\sqrt{3}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>AB</td>
<td>0</td>
<td>$1.5 - j0.5\sqrt{3}$</td>
<td>0</td>
<td>$1.5 + j0.5\sqrt{3}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>BC</td>
<td>0</td>
<td>$j\sqrt{3}$</td>
<td>0</td>
<td>$-j\sqrt{3}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>CA</td>
<td>0</td>
<td>$-1.5 - j0.5\sqrt{3}$</td>
<td>0</td>
<td>$-1.5 + j0.5\sqrt{3}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>ABG</td>
<td>1.5 + j0.5\sqrt{3}</td>
<td>1.5 - j0.5\sqrt{3}</td>
<td>0</td>
<td>1.5 + j0.5\sqrt{3}</td>
<td>1.5 - j0.5\sqrt{3}</td>
<td>0</td>
</tr>
<tr>
<td>BCG</td>
<td>$-j\sqrt{3}$</td>
<td>0</td>
<td>$j\sqrt{3}$</td>
<td>0</td>
<td>$-j\sqrt{3}$</td>
<td>0</td>
</tr>
<tr>
<td>CAG</td>
<td>1.5 - j0.5\sqrt{3}</td>
<td>1.5 + j0.5\sqrt{3}</td>
<td>0</td>
<td>1.5 - j0.5\sqrt{3}</td>
<td>1.5 + j0.5\sqrt{3}</td>
<td>0</td>
</tr>
<tr>
<td>ABC, ABCG</td>
<td>1.5 + j0.5\sqrt{3}</td>
<td>1.5 - j0.5\sqrt{3}</td>
<td>0</td>
<td>1.5 + j0.5\sqrt{3}</td>
<td>1.5 - j0.5\sqrt{3}</td>
<td>0</td>
</tr>
</tbody>
</table>
Taking into account a set of weighting coefficients that for zero sequence: \( \alpha_{00} = 0 \) and expressing the symmetrical components of total fault current with use of fault current distribution factors and one obtains:

\[
I_p = \alpha_{p1} \frac{\Delta I_{\pm1}}{k_{p1}} + \alpha_{p2} \frac{I_{\pm2}}{k_{p2}}
\]  

(39)

Considering that for the fault current distribution factors for positive- and negative-sequence, with respect to their magnitude and angle, we have

\[
k_{p1} = k_{p2} = |k_p| e^{\gamma}
\]  

(40)

\[
\gamma = \text{angle}(k_{pL}) = \text{angle}(k_{pF})
\]  

(41)

The Equation (39) can be rewritten as

\[
I_p = \frac{\alpha_{p1} \Delta I_{\pm1} + \alpha_{p2} I_{\pm2}}{|k_p| e^{\gamma}}
\]  

(42)

Substitute Equation (42) in the basic Equation (26)

\[
V_{A,F} - dZ_{1L} \ast I_{A,F} - \frac{\alpha_{p1} \Delta I_{\pm1} + \alpha_{p2} I_{\pm2}}{|k_p| e^{\gamma}} R_F = 0
\]  

(43)

Multiplying the Equation (43) by the term \((e^{\gamma})(\alpha_{p1} \Delta I_{\pm1} + \alpha_{p2} I_{\pm2})^*\) yields

\[
V_{A,F} \ast (\alpha_{p1} \Delta I_{\pm1} + \alpha_{p2} I_{\pm2})^* \ast e^{\gamma} - dZ_{1L} \ast I_{A,F} \ast (\alpha_{p1} \Delta I_{\pm1} + \alpha_{p2} I_{\pm2})^* \ast e^{\gamma} = 0
\]  

(44)

Eliminating the term \(R_F\) by taking imaginary parts of the Equation (18) and then rearranging, the resultant formula for the sought distance to fault \(d\) (p.u.) is obtained as follows:

\[
d = \frac{|Im[V_{A,F} \ast (\alpha_{p1} \Delta I_{\pm1} + \alpha_{p2} I_{\pm2})^* \ast e^{\gamma}]|}{|Im[Z_{1L} \ast I_{A,F} \ast (\alpha_{p1} \Delta I_{\pm1} + \alpha_{p2} I_{\pm2})^* \ast e^{\gamma}]|}
\]  

(45)

\[
d = \frac{|Im[V_{A,F} \ast \alpha_{p1} \Delta I_{\pm1}^* + \alpha_{p2} I_{\pm2}^* | e^{\gamma}]|}{|Im[Z_{1L} \ast I_{A,F} \ast \alpha_{p1} \Delta I_{\pm1}^* + \alpha_{p2} I_{\pm2}^* \ast e^{\gamma}]|}
\]  

(46)

In formula (45), the angle of the current distribution factor (for the positive or negative-sequence) is involved. It is proposed to assume that this angle equals zero \(\gamma = 0\), i.e., that the fault current distribution factor is a real number. In practice, this assumption is not completely fulfilled and thus there is a certain error due to this.

### 4. Power System Model

The SimPowerSystem which is an extension to the simulink of MATLAB software was used to simulate the double end fed power system. The 100 km, 400 kV transmission line was modeled using distributed parameter model as shown in Fig. 5.

**Figure 5:** Power System model

The transmission line parameters are as follows:
- Positive Sequence Resistance, \( R_1 \) : 0.0275 \( \Omega \)/km
- Zero Sequence Resistance, \( R_0 \) : 0.275 \( \Omega \)/km
- Zero Sequence Mutual Resistance, \( R_{0m} \) : 0.21 \( \Omega \)/km
- Positive Sequence Inductance, \( L_1 \) : 0.00102 H/km
- Zero Sequence Inductance, \( L_0 \) : 0.003268 H/km
- Positive Sequence Capacitance, \( C_1 \) : 13 \( \epsilon_{00}^{\circ} \) F/km

### 5. Simulation Results

The simulation is carried out for these algorithms by varying various fault parameters like fault inception angle, fault resistance, fault type, fault location. The various measurements processed for various types of faults during implementation of algorithm are shown in table 4. The accuracy of fault location of these three algorithms are compared and shown in Table 3.

The fault location error is calculated as

\[
\text{Error}(\%) = \frac{|\text{Calculated Fault Location} - \text{Actual Fault Location}|}{\text{Total Line Length}} \times 100
\]  

(47)

### Table 3: Comparison of Impedance based algorithms for \( R_2=64 \) ohms and FIA=36° for 400KV Transmission line

<table>
<thead>
<tr>
<th>Fault Type</th>
<th>Actual fault location</th>
<th>Takagi’s Method</th>
<th>Ericsson’s Method</th>
<th>Fault sequence Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>AG</td>
<td>10</td>
<td>9.6817</td>
<td>0.3183%</td>
<td>9.6880</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.312%</td>
<td></td>
<td>6.312%</td>
</tr>
<tr>
<td>BG</td>
<td>20</td>
<td>19.7033</td>
<td>0.2967%</td>
<td>19.7087</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.312%</td>
<td></td>
<td>6.312%</td>
</tr>
<tr>
<td>CG</td>
<td>30</td>
<td>29.36</td>
<td>0.639%</td>
<td>29.363</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.312%</td>
<td></td>
<td>6.312%</td>
</tr>
<tr>
<td>AB</td>
<td>40</td>
<td>40.0723</td>
<td>0.0723%</td>
<td>40.0749</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.312%</td>
<td></td>
<td>6.312%</td>
</tr>
<tr>
<td>BC</td>
<td>50</td>
<td>49.20</td>
<td>0.793%</td>
<td>49.207</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.312%</td>
<td></td>
<td>6.312%</td>
</tr>
<tr>
<td>CA</td>
<td>60</td>
<td>59.18</td>
<td>0.812%</td>
<td>59.185</td>
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<tr>
<td></td>
<td></td>
<td>6.312%</td>
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<td>6.312%</td>
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<tr>
<td>ABG</td>
<td>70</td>
<td>69.23</td>
<td>0.766%</td>
<td>69.227</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.312%</td>
<td></td>
<td>6.312%</td>
</tr>
<tr>
<td>BCG</td>
<td>80</td>
<td>79.36</td>
<td>0.637%</td>
<td>79.351</td>
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<tr>
<td></td>
<td></td>
<td>6.312%</td>
<td></td>
<td>6.312%</td>
</tr>
<tr>
<td>CAG</td>
<td>85</td>
<td>84.364</td>
<td>0.636%</td>
<td>85.032</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.312%</td>
<td></td>
<td>6.312%</td>
</tr>
<tr>
<td>ABC,ABCG</td>
<td>90</td>
<td>89.96</td>
<td>0.039%</td>
<td>89.940</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.312%</td>
<td></td>
<td>6.312%</td>
</tr>
</tbody>
</table>

### Table 4: Measurement processed for various types of faults for 400KV power transmission line for impedance based fault location algorithms

<table>
<thead>
<tr>
<th>FAULT TYPE</th>
<th>( V_{A,F} )</th>
<th>( I_{A,F} )</th>
<th>( \Delta I_{A} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AG</td>
<td>( V_A )</td>
<td>( I_A + K_R I_N )</td>
<td>( \frac{3}{2} (\Delta I_A - I_{5A}) )</td>
</tr>
<tr>
<td>BG</td>
<td>( V_BA )</td>
<td>( I_B + K_R I_N )</td>
<td>( \frac{3}{2} (\Delta I_B - I_{5B}) )</td>
</tr>
</tbody>
</table>
6. Conclusion

In this paper, three impedance based fault location algorithms: Takgi’s Method, Ericsson’s Method, Fault sequence component of current Method was implemented using Matlab Simulink and programing. The accuracy of fault location of these three algorithms are compared by varying various fault parameters like fault inception angle, fault type, fault location, fault resistance. The simulation results show that all ten types of faults are correctly located with fault location error less than 1%.

References