

Robust Sliding Mode Control Using Fuzzy Controller

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Abstract: In this paper, a fuzzy sliding mode controller is designed for a nonlinear control system. Instead of fuzzifying the error and the error change, we fuzzify the sliding surface. Sliding surface is formed from error and error change, i.e. we are converting a two variable problem into a single variable problem. Now, the basis of rules formation is the sliding surface and the output is deduced by the proper organization of inference. This fuzzy sliding mode controller is based on Variable Structure Control (VSC) theory that introduces the boundary layer in the switching surface and also the control law is continuously approximated in this layer which guarantees the stability and robustness. By using Lyapunov's theory, we ensure the stability of this controller.

Keywords: Sliding mode control, fuzzy logic controller, Lyapunov Theory, variable structure control, robust control.

1. Introduction

Sometimes we choose a simplified representation for the dynamics of a nonlinear system for ease of mathematics and usually because of some unknown plant parameters, there exists model imprecision. This affects the nonlinear control system in a very drastic manner. Robust control is one of the most effective techniques to deal with these model uncertainties.

To control complex systems or imperfectly modeled systems using fuzzy logic, a lot of efforts have been made in the past and these have been fruitful in many areas. However the designing of fuzzy logic controller greatly depends upon the expert's knowledge or trial and errors. Furthermore, fuzzy controller does not guarantee the stability and the robustness due to the linguistic expressions of the fuzzy control [4].

Sliding mode is an important robust control approach. It is a distinct type of Variable Structure Control (VSC) in which the control law is continuously changed during the control process according to some well-defined rule which depends on the states of the system [5]. Here the control law is a switching control. Sliding mode control will force the state trajectory of the system to reach, and subsequently remain on the predetermined surface. After reaching sliding surface, the system becomes completely inconsiderate to parameter variations and external disturbances [3]. So, robustness is achieved but this method uses drastic changes of the control input which produces the phenomenon of chattering. The chattering may excite the high frequency components of the system that is neglected while modelling. In order to avoid the problem of chattering, the boundary layer is introduced and the control input is approximated in this boundary layer.

So, the fuzzy controller is designed to deal with this chattering problem. In many fuzzy control systems, the rules are generated by using the error and the error change, but here we fuzzify the sliding surface i.e. ultimately we are reducing the order of the system [2]. The sliding surface, denoted by s , is a combination of the error and the change in error. This controller can get rid of the chattering problem and guarantees the stability and the robustness.

2. Sliding Mode Control

Let the n th order system as follows

$$\dot{x}^{(n)}(t) = f(x, t) + b(x, t)u(t) + d(x, t) \quad (1)$$

where $x^T = [x \ \dot{x} \ \dots \ x^{(n-1)}]$ is the state matrix of the system, $b(x, t)$ and $f(x, t)$ are the functions which are determining the dynamics of the system, $d(x, t)$ is the disturbance, and $u(t)$ is the control input signal.

Defining the error as

$$e(t) = x(t) - x_o(t) \quad (2)$$

where $x_o(t)$ is the desirable state. Now, defining the sliding surface for the system as

$$S(x, t) = \dot{e}(t) + \lambda e(t) \quad (3)$$

which is converting a higher order nonlinear function into a first order linear function which consists error and the error change with a slope of λ . The trajectory of the system is kept onto this sliding surface with the help of a control law. And this control law can be found out by Lyapunov's direct method.

Assume Lyapunov's function as

$$V(s) = \frac{1}{2} S^2(x, t) \quad (4)$$

where $V(s)$ is a positive definite function for $\forall S(x, t) > 0$. The stability condition can be given as

$$\dot{V}(s) = S\dot{S} \leq -\eta |S(x, t)| \quad (5)$$

where η is a strictly positive real constant. On substituting equation (1) into equation (5) and after doing some simplifications, we can get,

$$S(f + b + u + d - \ddot{x}_o + \lambda \dot{e}) \leq -\eta |S| \quad (6)$$

So, the reaching condition can be satisfied by choosing a control input as

$$u = \frac{-f + \ddot{x}_o - \lambda \dot{e}}{b} - K \text{sgn}(S) \quad (7)$$

K is a bounded strictly positive real constant which is depending on the external disturbances and the estimated system parameters. The $\text{sgn}()$ function denotes the signum function which is a sensing function which senses the condition at which the trajectory meets with the sliding surface. $\text{sgn}()$ can be given as

$$\text{sgn}(s) = \begin{cases} 1 & \text{for } s > 0 \\ 0 & \text{for } s = 0 \\ -1 & \text{for } s < 0 \end{cases} \quad (8)$$

3. Chattering

From (7) we can see that the sign of control input depends on the sign of $sgn(S)$. The sign of the control input will change as the trajectory crosses the line $S = 0$. This will produce a zig-zag motion of the trajectory which is referred to as chattering. As all physical systems are low frequency systems in nature, so we neglect the high frequency components of the systems for the ease of modeling. Chattering generates the high frequency signals into the system which will affect the performance of the system. To avoid chattering, we will design a fuzzy sliding surface through fuzzy logic controller.

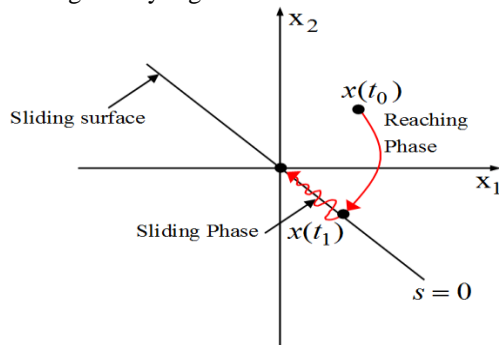


Figure 1: Sliding mode control with chattering

4. Fuzzy Sliding Mode Controller

As mentioned before, we are not fuzzifying the error and the error change as done in conventional fuzzy controller design. Instead, we are fuzzifying the sliding surface which is the combination of the error and the error change. Now, the basis of the rule formation is the sliding surface which ultimately reduces the number of fuzzy rules also.

Firstly, the crisp sliding surface $s = 0$ is extended to the fuzzy sliding surface defined by the expression for the fuzzy controller design as,

$$\tilde{s} \text{ is ZERO} \quad (9)$$

where \tilde{s} : linguistic variable for s and
ZERO: one of the fuzzy set of fuzzy sliding surface.

The following fuzzy sets are introduced in the universe of discourse of s as

$$U(\tilde{s}) = \{NMB, NB, NM, ZR, PM, PB, PMB\} \quad (10)$$

Similarly, the control input is partitioned as

$$U(\tilde{u}) = \{SMALLEST, SMALLER, SMALL, MEDIUM, BIG, BIGGER, BIGGEST\} \quad (11)$$

Fuzzy rules are as follows:

- RULE 1: If s is NMB then u is BIGGEST
- RULE 2: If s is NB then u is BIGGER
- RULE 3: If s is NM then u is BIG
- RULE 4: If s is ZR then u is MEDIUM
- RULE 5: If s is PM then u is SMALL
- RULE 6: If s is PB then u is SMALLER
- RULE 7: If s is PMB then u is SMALLEST

The membership functions of the above fuzzy sets are shown in Figure 2(a) and 2(b):

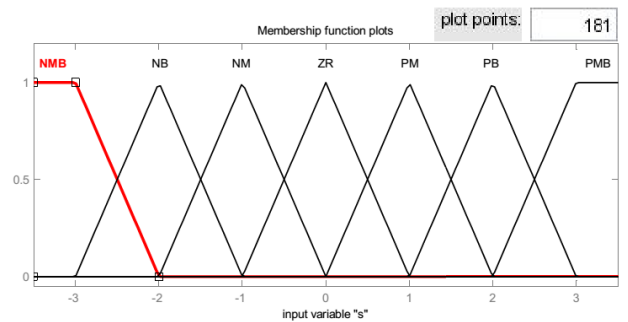


Figure 2(a): Membership functions for the sliding surface, s

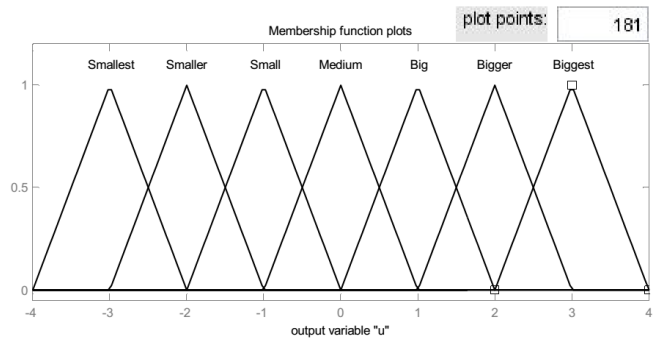


Figure 2(b): Membership functions for control input, u

The result of inference from these seven rules is shown in Figure 3.

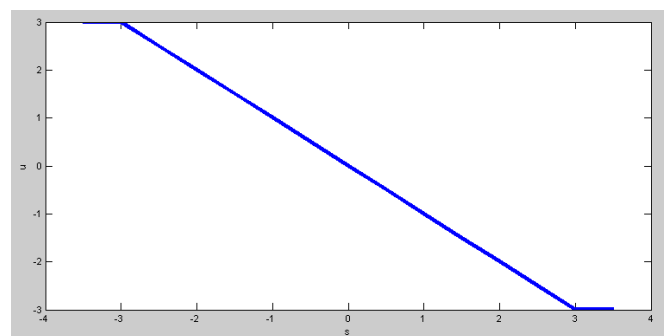


Figure 3: The result of the inference

5. Simulation and Results

The simulations are performed on a nonlinear control system having the transfer function as follows:

$$G(s) = \frac{2502.96}{s^2 - 981} \quad (12)$$

which is an unstable system having its poles on the right side of the s -plane. The above transfer function can be regarded as any real time system such as magnetic levitation system.

We choose the sliding surface as

$$S = \dot{e} + 3e \quad (13)$$

This sliding surface is given to fuzzy controller where the fuzzy rules are made. Rules are based on the sliding surface. Then control input is generated as follows:

$$u = \ddot{x}_d + 3\dot{e} - 99x + 0.2sgn(\dot{e} + 3e) \quad (14)$$

The Simulink model and simulation results are shown in following figures:

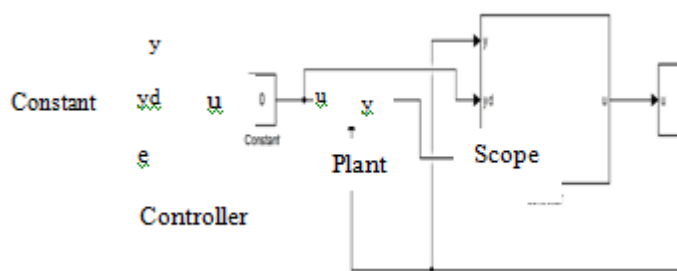


Figure 4: Simulink model for the system

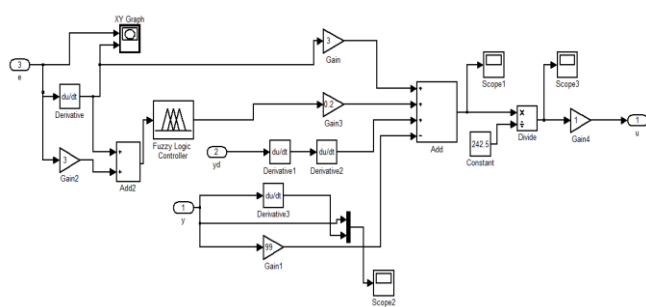


Figure 5: Fuzzy sliding mode controller

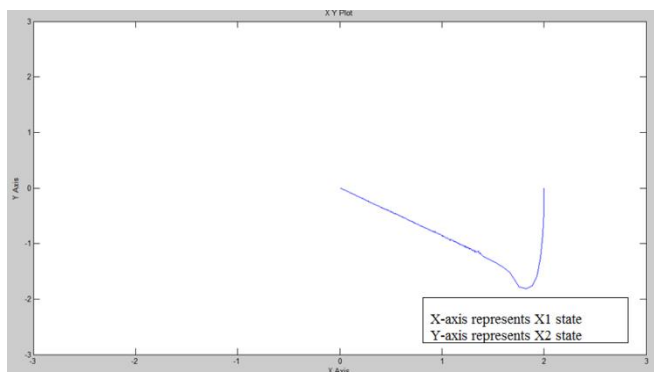


Figure 6: Phase plane trajectory of the system

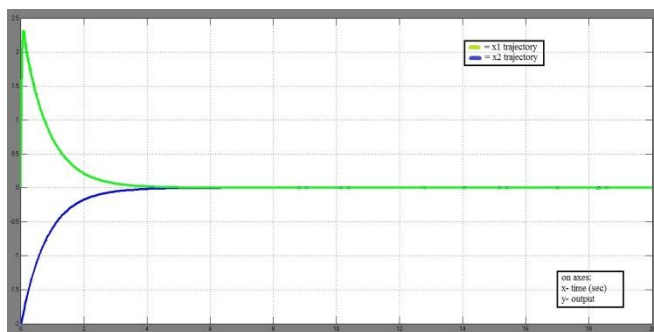


Figure 7: Convergences of state trajectories

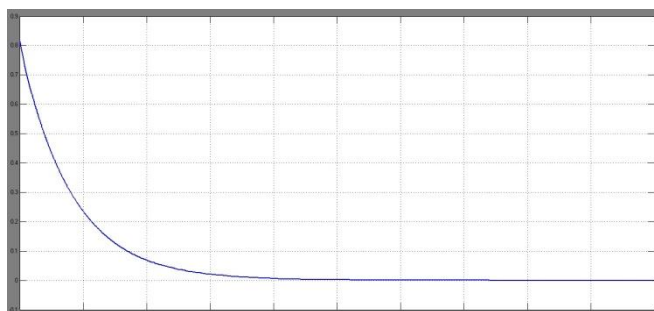


Figure 8: Sliding mode control law, u

Simulation is done on MATLAB using Simulink to verify the controller. The results are shown in the figure 6, figure 7, and figure 8. From figure 6, we can see phase plane trajectory plot starts from the initial point (2, 0), move towards the sliding surface. After reaching the sliding surface, trajectory slides along it to reach equilibrium point (0, 0). From figure 7, we can see both x and y reach 0 after 4.5 seconds approximately. Figure 8 shows the sliding mode control law, u .

6. Conclusion

This paper highlights the basic idea of sliding mode control, chattering phenomenon and methods to avoid chattering. We designed a fuzzy controller in which sliding surface is fuzzified which eliminates the chattering problem. This compresses the two variables problem into a single variable problem i.e. converting a second order nonlinear system into a first order linear system. By this we are also able to reduce the no. of rules in fuzzy controller. Then by applying Lyapunov's theory, we guarantee the stability and robustness.

References

- [1] ChintuGurbani and Dr. Vijay Kumar, "Designing Robust Control by Sliding Mode Control Technique," Advance in Electronic and Electric Engineering ISSN 2231-1297, Volume 3, Number 2, pp. 137-144, 2013.
- [2] Sung-Woo Kim and Ju-Jang Lee, "Design of a Fuzzy Controller with Fuzzy Sliding Surface," Fuzzy Sets and Systems 71, Elsevier Science B.V., pp. 359-367, 1995.
- [3] Sung-Woo Kim and Ju-Jang Lee, "Fuzzy Logic Based Sliding Mode Control," Fifth IFSA World Congress, pp. 822-825, 1993.
- [4] D. Dubois and H. Prade, "Fuzzy Sets and Systems: Theory and Applications" (Academic Press, Orlando, FL, 1980).
- [5] Sung-Woo Kim and Ju-Jang Lee, "Variable structure system and fuzzy sliding surface," Trans. KIEE, 42(1993) 87-96 (in Korean).
- [6] R. Palm, "Sliding Mode Fuzzy Control," IEEE Int. Conf. on Fuzzy Systems, pp. 519-526, 1992

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