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Application of Linear Programming (Assignment Model)

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Abstract: ***The aims of this paper is to clarify the theoretical aspects of the assignment problem and provide customization model that reduces the cost of resource allocation (Source) to a number of points of sale (Destinations) to as minimum as possible. This remarkably will achieve better productivity and the possibility of using it infinancial or administrative units to attain the desired goals of reducing costs and maximizing profits.A comparison with the results obtained using the Quantity System Business software (Win QSB) is also presented***

Keywords: linear programing, Assignment model, hungarian method, cost matrix, profit matrix

1. Introduction

The assignment problem is nothingelse than a balanced transportation problem in which all supplies and demands are equal to 1.

One of the most widely used methods for solving assignment problems is called, the Hungarian method. This method of assignment was developed by the Hungarian mathematician D. Konig in 1955, and is therefore known as Hungarian method of assignment problem.

There are many types of assignment problems, such as:

i. Minimization type of assignment problems.In this case, we need to assign certain jobs to certain workersto minimize the time required to complete the jobs. Here the number of rows *i.e.* jobs are equals to the number of workers*i.e.* columns. The procedure of solving will be discussed in detail in section 2.
ii. Maximization type of assignment problems. Here we have to assign certain jobs to certain facilities to maximize the returns or maximize the effectiveness.
iii. Assignment problems having non-square cost matrix. Here by adding a dummy row or dummy columns as the case may be, we can
iv. convert a non-square matrix into a square matrix and proceed further to solve the problem.
v. Assignment problems with restrictions. Here restrictions such as a job cannot be done on a certain machine or a job cannot be allocated to a certain facility may be specified.

The problem that we are going to present in this paper is of types*(ii),(ii)***.**

This paper has been arranged as follows, in section 2 we present the theoretical framework through which the definition model assignment and assumptions as well as methods of solution. Section 3 is dedicated to apply the suggested solution on Alhram Plaza Centre specializes in the sale of clothing in Tabuk as a model for the problem of assignment after obtaining the data from management of the above-mentioned center. Finally, in section 4 we present a comparison with the results obtained when using the Quantity System Business software (Win QSB), and a brief discussion of the results.

2. The Theoretical Framework

The mathematical assignment model using a linear programming is described as follows:
1-A set of m jobs.
2-A set of n workers.
3-A cost variable Cij of assigning a worker i assigned to do a job j.

The cost matrix:

$$
\begin{array}{cc}
 & \text{workers} \\
 & 1\ 2\ \ldots\ \ldots\ \ldots\ n \\
\textit{jobs}\ \begin{array}{c} 1 \\ 2 \\ \vdots \\ n \end{array} &
\begin{bmatrix} c_{11} & c_{12} & \ldots\ldots\ldots & c_{1n} \\ c_{21} & c_{22} & \ldots\ldots\ldots & c_{2n} \\ . & & & \\ c_{n1} & c_{n2} & \ldots\ldots\ldots & c_{n.n} \end{bmatrix}
\begin{array}{c} 1 \\ 1 \\ \vdots \\ 1 \end{array} \\
 & 1 \quad 1 \quad \cdots \quad 1
\end{array}
$$

Let Xij be the number of unites produced by a worker i when assigned to the job j.
The general formulation of this assignment problem is then.

$$Minimize\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{IJ}$$

$$Subject\ to:$$

$$\sum_{i=1}^{m} X_{IJ} = 1 \qquad i = 1, 2, 3, \ldots\ldots\ldots\ldots m$$

$$\sum_{j=1}^{n} X_{IJ} = 1 \qquad j = 1, 2, 3, \ldots\ldots\ldots\ldots n$$

The steps for solving this assignment problem using the Hungarian method are summarized as follows:

Step I : Given the cost matrix, construct a new matrix by subtracting from each cost the minimum cost in its row. For this new matrix find the minimum cost in each column and subtract from each cost the minimum cost in its column.

Step II: Drawthe least number of horizontal and vertical lines so as to cover all zeros, Let us denote the total number of these lines by N.
i) If the number of these lines is n, then an optimal solution is available among the covered zeros, we therefore proceed to step V.
ii) If the number of lines is less than n then we proceed to step III

Step III : Determine the smallest nonzero cost cell from among the uncrossed cells. Subtract this cost from all the uncrossed elements of the reduced cost matrix, and add the same smallest cost to each element that is covered by two lines. Return to step II.

Step V: we arenow having exactly one zero in each row and each column of the cost matrix. The assignment schedule should be corresponding to the zeros in the optimum (maximal) assignment.

Most of the financial or administrative units in Saudi Arabia do not relay on effective methods during the process of primitive assignment. This eventually would lead to a huge waste of time and increase the overall cost. However, the assignment model in the present study provides an effective solution to the problems that call for the distribution of tasks over the available resources in order to reach an optimal assignment and a manner that achieves optimization of time while reducing costs.

To apply the assignment model, Alhram Plaza Centre specializes in the sale of clothing at Tabukwas chosen. This center has a warehouse in which six workers perform seven jobs (i.e. laniary Section, Baby Section, boys Section, Girls Section, Men Section, Women Section and Shoes Section).

The spreadsheet was designed through distributing workers to all sections.Thereafter, the average number of pieces completed byeach worker in each section for a period of two months is calculated. Where every worker had a chance to work for seven days in each section. Table 1 shows the average number of pieces performed in each section by each worker.

3. The Problem

Table1: The average number of pieces performed in each section of each workers

From/to	Worker(1)	Worker(2)	Worker(3)	Worker(4)	Worker(5)	Worker(6)
Lanjary Section	630	700	580	720	600	500
boys Section	510	690	510	420	410	210
Girls Section	180	180	205	190	280	380
Women Section	195	140	300	280	175	170
Men Section	560	450	480	650	505	490
Shoes Section	210	220	270	205	335	290
Baby section	720	480	650	500	410	500

To solve this assignment problem using the Hungarian method we follow the steps presented previously.
Step 1:Since, the number of rows and columns of the profitmatrix are not equal, a dummy column (Worker7) should be added to obtain the equilibrium (n=m).
Step 2: in order to maximize profitfunction we multiply the profits matrix through by -1 and solve the problem as a minimization problem.
Step3: Select the most negative matrix element and subtract it from all other elements of the matrix.The mostnegative value is -900.
Step 4:To obtain a zero element in each column/row we subtract each element of that column/row from the lowest number present in the column/row.
Step 5: Now, horizontal and vertical lines are drawn. The number of horizontal lines is one and the number of vertical line is three. Since the order of matrix is 7 x 7, therefore, $N \neq n$
Step 6: Now, in the uncrossed cell the least cost is selected. Subtract this cost from all the uncrossed elements of the reduced cost matrix, and add the same smallest cost to each element that is covered by two lines. In our case the least cost is found to be 10.

The resulting matrixhas two horizontal lines and three vertical lines which do not satisfy our condition (N= n).
Step 7:Repeat step (6) until the condition
(N= n)is satisfied.
Finally, we obtained the following optimum solution matrix (Table 2). The totalprofit associated with the optimal solution is shown in Table 3.

Table 2: The optimum solution matrix of pieces performed in each section of each workers

From/to	Worker(1)	Worker(2)	Worker(3)	Worker(4)	Worker(5)	Worker(6)	Worker(7)
Lanjary	105	270	0	15	0	85	255
boys	155	0	×	245	120	295	185
Girls	360	595	180	350	125	0	×
Women	285	575	15	200	170	150	0
Men	90	435	15	0	×	×	170
Shoes	270	495	155	285	0	30	×
Baby	0	165	15	×	15	80	117

Table 3: The total profit associated with the optimal solution

no	From	To	Assignment	Total profit
1	Lanjary Section	Worker 3	1	580
2	boys Sectio	Worker 2	1	690
3	boys Sectio	Worker 6	1	380
4	Women Section	Worker 7	1	000
5	Men Section	Worker 4	1	650
6	Shoes Section	Worker 5	1	335
7	Baby section	Worker 1	1	720
	Total Objective Function Value = 3355			

3.1 Solution by the WIN QSB Package

Since assignment problems are LPs, they can therefore be solved as any LP with the Simplex/Big M method. Also, form a transportation point of view; they canbe solved using the specialized transportation algorithm. For comparison, the Quantity System Business(Win QSB) software was used. The results from the Win QSB were similar to those obtained by the manual solution as follow:

Table 4: The optimum Solution by the WIN QSB package

no	From	To	Assignment	Unit profit	Total profit
1	Assignment 1	Assignment 3	1	580	580
2	Assignment 2	Assignment 2	1	690	690
3	Assignment 3	Assignment 6	1	380	380
4	Assignment 4	Assignment 7	1	000	000
5	Assignment 5	Assignment 4	1	650	650
6	Assignment 6	Assignment 5	1	335	335
7	Assignment 7	Assignment 1	1	720	720
	Total Objective Function Value = 3355				

3.2Discussion and Conclusion

The matrix in Table 2 has three horizontal lines and four vertical lines, therefore it satisfiesthe condition N= n. Consequently an optimal solution is available. The optimal assignments can be made as follows

- Select the first row, Assign worker (3) to Laniary Section. Cross out zeros at the boys – Worker3 cells.
- Consider row 3. Assign Worker(6) to Girls Section. Cancel the zero at the Men - Worker(6) cells.
- Since there is a single zero in the four and six rows, put Worker(7) to Women Section and cross out the zero at GirlsSection- Worker(7), and ShoesSection- Worker(7).
- put Worker(5) to Shoes Section and cross out the zero at the Men - Worker(5) cells.
- Consider row 5. Assign Worker(4) to Girls Section. Cancel the zero at Baby - Worker(4) cells.
- There is only one zero left in each of the second and six rows, we assign Worker(2) to boys Section and Worker(1) to Baby Section.

The total profit associated with the optimal workers to job assignment pattern in Table 3. The manual solution is similar to the results obtained from the Win QSB solution as shown in Table 3.

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