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A Comparative Study of Optimum Solution between Fractional Transportation and Fractional Transhipment Problem

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Abstract: ***In this paper, a comparison of optimum solution between transportation and transhipment problem is discussed whose objective function is fractional and the objective is to minimize the total actual transportation cost to total standard transportation cost. here, the fractional transportation problem is converted to an equivalent fractional transhipment problem and then solved by using the method of fractional transportation problem and concluded that in some situations, fractional transhipment will be less expensive than fractional transportation by means of numerical example.***

Keywords: Fractional Transportation, Fractional Transhipment, Reduction of cost.

1. Introduction

Transportation problem is a special class of linear programming problem which deals with shipping of commodities from certain sources to various destinations. The objective of the transportation problem is to determine the shipping schedule that minimize the total shipping cost or maximize the total profit which satisfies the supply and demand limits [1]. here, we are considering a class of transportation problem called linear fractional transportation problem. This was originally proposed by Swarup [3] and it had an important role in logistics and supply chain management for reducing cost and improving service. The linear fractional programming problems originate from network models consisting of a finite number of nodes and arcs. These type of problems arise when we want to minimize the cost to time or maximize the profit to time ratio, in which fractional objectives include optimization of total actual transportation cost / total standard transportation cost, total return / total investment etc., In a transportation problem shipment of commodity takes place among sources and destinations. but, instead of direct shipments to destinations, the commodity can be transported to a particular destination through one or more intermediate points called transshipment. In brief, we are considering the transportation and transhipment problems in fractional case and comparing with direct shipment and shipment through intermediate points to give the best optimal solution for this problem. The paper is organized as follows : Section 2 details the linear fractional transportation problem with an example. In Section 3, the formulation of fractional transhipment problem is given. In Section 4, the conversion of fractional transportation problem to an equivalent fractional transshipment problem is considered and then solved. Finally, Section 5 gives the conclusion.

2. The Linear Fractional Transportation Problem

Consider the following transportation problem:-

$$Min\ z = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}\, x_{ij}}$$

Subject to

$\sum_{j=1}^{n} x_{ij} = a_i, \sum_{i=1}^{m} x_{ij} = b_j, \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j, x_{ij} \geq 0$, Where ai is ith source, bj is the jth destination, cij is the total actual transportation cost, dij is the total standard transportation cost from ith to jth destination.

In case, the variables u_i', v_j' and u_i'', v_j'' associated with the numerator and denominator of objective, as given in [4], where u_i' and u_i'', i = 1, 2, ……m, are corresponding to supply constraints and v_j', v_j'', j = 1, 2, …n, are corresponding to demand constraints and defined as :

$u_i' + v_j' = c_{ij}$ (i, j) ϵ J
$u_i'' + v_j'' = d_{ij}$, (i, j) ϵ J

Where J is the set of pairs of indices (i, j) of basic variable x_{ij}.

The Reduced costs Δ_{ij}' and Δ_{ij}'' are defined as :

$\Delta_{ij}' = c_{ij} - (u_i' + v_j')$, i = 1, 2, …….m, j = 1, 2, …….n,

$\Delta_{ij}'' = d_{ij} - (u_i'' + v_j'')$, i = 1, 2, …...m, j = 1, 2, ……n,

Further, we define

$U_i(x) = u_i' - z u_i''$, i = 1, 2, …m,

$V_j(x) = v_j' - z v_j''$, j = 1, 2, …..n,

$Z_{ij}(x) = U_i(x) + V_j(x)$, i = 1, 2, ..., m; j = 1, 2, …, n,

$C_{ij}(x) = c_{ij} - z d_{ij}$, i = 1, 2, ..., m; j = 1, 2, ……, n,

$\Delta_{ij}(x) = C_{ij}(x) - Z_{ij}(x)$, i=1, 2, .., m; j = 1, 2, , n

It can be Expressed as $\Delta_{ij} - \Delta_{ij}' - z\Delta_{ij}''$, i = 1, 2, ..., m; j= 1, 2, .., n,

The optimum condition for linear fractional transportation problem state that a basic feasible solution is optimal if $\Delta_{ij} \geq 0$, i = 1, 2, ….., m; j = 1, 2, ……, n,

2.2 Numerical Example

$$\text{Min Z} = \frac{9x_{11} + 3x_{12} + 5x_{21} + 2x_{22} + 2x_{31} + 1x_{32}}{5x_{11} + 6x_{12} + 2x_{21} + 3x_{22} + 4x_{31} + 6x_{32}}$$

Subject to :

$x_{11} + x_{12} = 50$

$x_{21} + x_{22} = 40$
$x_{31} + x_{32} = 60$
$x_{11} + x_{21} + x_{31} = 70$
$x_{12} + x_{22} + x_{32} = 80$ and
$x_{ij} \geq 0$ for i = 1, 2, 3, j = 1, 2, 3, 4

As per [2] **Table 1**

	D_1	D_2	
S_1	5 / 30 / 9	6 / 20 / 3	50
S_2	2 / 40 / 5	3 / / 2	40
	4	6	

where the entries at the top left and bottom right corners of each cell represents d_{ij} and c_{ij}, we obtained the feasible solution and the values of x_{ij} are shown in the small rackets in table 1 and

$$\text{Min Z} = \frac{9 \times 30 + 3 \times 20 + 5 \times 40 + 1 \times 60}{5 \times 30 + 6 \times 20 + 2 \times 40 + 6 \times 60} = 0.830$$

According to following Table (II):-**Table II**

	D_1	D_2	
S_1	5 / (30 –θ) / 9	6 / (20 +θ) / 3	$u_1^{'} = 0$, $u_1^{''} = 0$
S_2	2 / 40 / 5	3, 0 / 3, 2	$u_2^{'} = -4$, $u_2^{''} = -3$
S_3	4, -1 / (θ) / -5, 2	6 / (60-θ) / 1	$u_3^{'} = -2$, $u_3^{''} = 0$
	$v_1^{'} = 9$, $v_1^{''} = 5$	$v_2^{'} = 3$, $v_2^{''} = 6$	

Calculating Δ_{ij} for all empty cells, we have

$\Delta_{22} = 3 - (0.830) (0) = 3$

$\Delta_{31} = -5 - (0.830)(-1) = - 4.17$

Clearly, Δ_{31} is the negative, therefore x_{31} will enter into the basis. We take x_{31} = θ (see table II) then θ = 30, continuing in this way, we get the final transportation table III is:- **Table III**

	D_1	D_2	
S_1	5, 0 / 3, 9	6 / 50 / 3	$u_1^{'} = 0$, $u_1^{''} = 0$
S_2	2 / 10 / 5	3 / 30 / 2	$u_2^{'} = -1$, $u_2^{''} = -3$
S_3	4 / 60 / 2	6, 1 / 2, 1	$u_3^{'} = -4$, $u_3^{''} = -1$
	$v_1^{'} = 9$, $v_1^{''} = 5$	$v_2^{'} = 3$, $v_2^{''} = 6$	

$$\text{Min Z} = \frac{3 \text{ x } 50 + 5 \text{ x } 10 + 2 \text{ x } 30 + 2 \text{ x } 60}{6 \text{ x } 50 + 2 \text{ x } 10 + 3 \text{ x } 30 + 4 \text{ x } 60} = 0.584$$

Calculating Δ_{ij} for all empty cells, we have

$\Delta_{11} = 3 - (0.584) = 3$

$\Delta_{32} = 2 - (0.584) = 1.416$

Since all $\Delta_{ij} \geq 0$. The solution is Optimal and Min Z = 0.584

3. The Fractional Transhipment Problem

Consider the Fractional Transhipment Problem as indicated below:-

$$Min\ z = \frac{\sum_{i=1}^{m+n} \sum_{j=1}^{m+n} c_{ij} x_{ij}}{\sum_{i=1}^{m+n} \sum_{j=1}^{m+n} d_{ij} x_{ij}}$$

Subject to

$\sum_{j=1}^{m+n} x_{ij} - t_i = a_i, x_{1i} + x_{2i} + \cdots + x_{1-i,i} + x_{i+1,i} + \cdots + x_{m+n,i} = t_i$, (i = 1, 2, ……..m)

$\sum_{i=1}^{m+n} x_{ij} - t_j = b_j, (j = m+1, \ldots \ldots . . m+n)$

$x_{j1} + x_{j2} + \cdots + x_{j,j-1} + x_{j,j+1} + \cdots + x_{j,m+n} = t_j$ (j = m + 1, ……., m + n)

$$\sum_{i=1}^{m} a_i = \sum_{j=m+1}^{m+n} b_j$$

Where a_i is i^{th} source, b_j is the j^{th} destination, c_{ij} is the total actual transportation cost, d_{ij} is the total standard transportation cost from i^{th} to j^{th} destination, x_{ij} is the amount of goods shipped from the i^{th} terminal (T_i) to the j^{th} terminal (T_j) and x_{ij} will be equal to zero because no units will be shipped from a terminal to itself. Now, assume that at m terminals (T_1, T_2, ..., T_m), the total out shipment exceeds the total in shipment by amounts equal to a_1, a_2, ..., a_m respectively and at the remaining n terminals (T_{m+1}, T_{m+2}, ..., T_{m+n}), the total in shipment exceeds the total out shipment by amounts b_{m+1}, b_{m+2}, ..., b_{m+n} respectively. If the total in shipment at terminals T_1, T_2, ..., T_m be t_1, t_2, ..., t_m respectively and the total out shipment at the terminals T_{m+1}, T_{m+2}, ..., T_{m+n} be t_{m+1}, t_{m+2}, ..., t_{m+n} respectively.

4. Conversion of the Fractional Transportation Problem as Fractional Transhipment Problem

By using the fractional transportation technique to solve the fractional transshipment model, we have to determine the unit cost of shipping the commodities through the transient nodes. In general, the shipping cost from one location to itself should be zero and the shipping cost from the source S_i to the destination D_j should be the same as the shipping cost from D_j to S_i, but that may change depending on the problem. However, the unit shipping cost from a source to another source or from a destination to another destination is in general not given in the original transportation problem. So, to formulate the problem as a transhipment problem, we assume that the commodities can pass through any one of the nodes in the network before they finally reach their destinations. We suppose that the cost is the same for shipments in opposite directions and unit cost of shipments among the sources is 4/3 while among destination is 3/2. Here, the buffer Stock is Rs.150. The transhipment problem [5] is thus changed into the following transportation Problem, as per tables (I to V) worked out below:-

Table I

	S_1	S_2	S_3	D_1	D_2	
S_1	0 / 0	3 / 4	3 / 4	5 / 9	6 / 3	200
S_2	3 / 4	0 / 0	3 / 4	2 / 5	3 / 2	190

S_3	3 4	3 4	0 0	4 2	6 1	210
D_1	5 9	2 5	4 2	0 0	2 3	150
D_2	6 3	3 2	6 1	2 3	0 0	150
	150	150	150	220	230	

Table II

	S1	S2	S3	D1	D2	
S1	0 0	3 4	3 ϵ_1 4	5 9	6 200 3	200
S2	3 4	0 150 0	3 4	2 40 5	3 2	190
S3	3 4	3 4	0 0	4 180 2	6 30 1	210
D1	5 ϵ_2 9	2 5	4 150 2	0 0	2 3	150
D2	6 150 3	3 2	6 1	2 3	0 0	150
	150	150	150	220	230	

To resolve degeneracy, the quantity ϵ_1, ϵ_2 are allocated to unoccupied cells.

Min Z =

$$\frac{3 \times 200+5 \times 40+2 \times 180+1 \times 30+2 \times 150+0 \times 150+3 \times 150+4 \times \epsilon_1+9 \times \epsilon_2}{6 \times 200+2 \times 40+4 \times 180+6 \times 30+4 \times 150+0 \times 150+6 \times 150+3 \times \epsilon_1+5 \times \epsilon_2} = 0.527$$

Table III

	S1	S2	S3	D1	D2	
S1	0 -4 θ -1 0	3 1 5 4	3 ϵ_1-θ 4	5 1 5 9	6 200 3	$u_1' = 0$ $u_1'' = 0$
S2	3 1 -8 4	0 150 0	3 2 -1 4	2 40 5	3 -1 -2 2	$u_2' = 1$ $u_2'' = -2$
S3	3 -1 -5 4	3 1 7 4	0 -3 -2 0	4 180 2	6 30 1	$u_3' = -2$ $u_3'' = 0$
D1	5 ϵ^1 9	2 -1 8 5	4 150+θ 2	0 -5 -2 0	2 -5 2 3	$u_4' = -2$ $u_4'' = 1$
D2	6 150 3	3 -1 11 2	6 1 5 1	2 2 3 3	0 -8 5 0	$u_5' = -8$ $u_5'' = 2$
	$v_1' = 11$ $v_1'' = 4$	$v_2' = -1$ $v_2'' = 2$	$v_3' = 4$ $v_3'' = 3$	$v_4' = 4$ $v_4'' = 4$	$v_5' = 3$ $v_5'' = 6$	

When calculating Δ_{ij}, we have,

Δ_{11} = -11 -(0.527) (-4)= -8.892	Δ_{33} = -2 - (0.527) (-3) = -0.419
Δ_{12} = 5 - (0.527) (1) = 4.473	Δ_{42} = 8 - (0.527) (-1) = 8.527
Δ_{14} = 5 - (0.527) (1) = 4.473	Δ_{44} = -2 - (0.527) (-5) = 0.635
Δ_{21} = -8 - (0.527) (1) = -8.527	Δ_{45} = 2 - (0.527) (-5) = 4.635
Δ_{23} = -1 - (0.527) (2) = -2.054	Δ_{52} = 11 -(0.527) (-1)= 11.527
Δ_{25} = -2 - (0.527) (-1) = -1.473	Δ_{53} = 5 - (0.527) (1) = 4.473
Δ_{31} = -5 - (0.527) (-1) = -4.473	Δ_{54} = 7 - (0.527) (-4) = 9.108
Δ_{32} = 7 - (0.527) (1) = 6.473	Δ_{55} = 5 - (0.527) (-8) = 9.216

Clearly, Δ_{11} enters the basis. Proceeding in this way, we get the following table (IV)

Table IV

	S1	S2	S3	D1	D2	
S1	0 ϵ''+θ 0	3 1 5 4	3 -5 -2 4	5 1 5 9	6 200 - θ 3	$u_1' = 0$ $u_1'' = 0$
S2	3 5 3 4	0 150 0	3 -3 -3 4	2 40 5	3 -1 -2 2	$u_2' = 1$ $u_2'' = -2$
S3	3 3 6 4	3 1 7 4	0 -8 -4 0	4 180 - θ 2	6 30 + θ 1	$u_3' = -2$ $u_3'' = 0$
D1	5 9 13 9	2 4 10 5	4 150+ϵ_1- θ 2	0 -3 ϵ^1 +θ -8 0	2 0 4 3	$u_4' = -4$ $u_4'' = -4$
D2	6 150- θ 3	3 -5 0 2	6 -8 + θ -8 1	2 -8 -4 3	0 -12 -6 0	$u_5' = 3$ $u_5'' = 6$
	$v_1' = 0$ $v_1'' = 0$	$v_2' = -1$ $v_2'' = 2$	$v_3' = 6$ $v_3'' = 8$	$v_4' = 4$ $v_4'' = 4$	$v_5' = 3$ $v_5'' = 6$	

$$\text{Min Z} = \frac{3 \times 200+5 \times 40+2 \times 180+1 \times 30+2 \times 150+0 \times 150+3 \times 150}{6 \times 200+2 \times 40+4 \times 180+6 \times 30+4 \times 150+0 \times 150+6 \times 150} = 0.527$$

When calculating Δ_{ij}, we have,

Δ_{12}	=	5	-	Δ_{33}	=	4	-
Δ_{13}	=	-2	-	Δ_{41}	=	13	-
Δ_{14}	=	5	-	Δ_{42}	=	10	-
Δ_{21}	=	3	-	Δ_{45}	=	4	-
Δ_{23}	=	-3	-	Δ_{52}	=	0	-
Δ_{25}	=	-2	-	Δ_{53}	=	-8	-
Δ_{31}	=	6	-	Δ_{54}	=	-4	-
Δ_{32}	=	7	-	Δ_{55}	=	-6	-

Clearly, Δ_{53} enters the basis. So θ = 150, Hence the Final Table VI is : **Table V**

	S_1	S_2	S_3	D_1	D_2	
S_1	0 ϵ''+150 0	3 0 3 4	3 3 2 4	5 1 5 9	6 50 3	$u_1' = 0$ $u_1'' = 0$
S_2	3 6 5 4	0 150 0	3 6 3 4	2 1 2 5	3 40 2	$u_2' = -1$ $u_2'' = -3$
S_3	3 3 6 4	3 0 5 4	0 ϵ 0	4 70 2	6 140 1	$u_3' = -2$ $u_3'' = 0$
D_1	5 9 13 9	2 3 8 5	4 8 4 2	0 ϵ^1 +150 0	2 0 4 3	$u_4' = -4$ $u_4'' = -4$
D_2	6 0 4 3	3 -6 2 2	6 150 1	2 -8 0 3	0 -12 -2 0	$u_5' = -1$ $u_5'' = 6$

$v_1^{'} = 0$	$v_2^{'} = 1$	$v_3^{'} = 2$	$v_4^{'} = 4$	$v_5^{'} = 3$
$v_1^{''} = 0$	$v_2^{''} = 3$	$v_3^{''} = 0$	$v_4^{''} = 4$	$v_5^{''} = 6$

$$\text{Min Z} = \frac{3 \times 50 + 2 \times 40 + 2 \times 70 + 1 \times 140 + 1 \times 150 + 0 \times 150}{6 \times 50 + 3 \times 40 + 4 \times 70 + 6 \times 140 + 6 \times 150 + 0 \times 150} = 0.270$$

The Reduced costs are worked out as follows:-

Δ_{12} = 3 – (0.270) (0) = 3	Δ_{41} = 13 – (0.270) (9) = 10.57
Δ_{13} = 2 – (0.270) (3) = 1.19	Δ_{42} = 8 – (0.270) (3) = 7.19
Δ_{14} = 5 – (0.270) (1) = 4.73	Δ_{43} = 4 – (0.270) (8) = 1.84
Δ_{21} = 5 – (0.270) (6) = 3.38	Δ_{45} = 4 – (0.270) (0) = 4
Δ_{23} = 3 – (0.270) (6) = 1.38	Δ_{51} = 4 – (0.270) (0) = 4
Δ_{24} = 2 – (0.270) (1) = 1.73	Δ_{52} = 2 – (0.270) (-6) = 3.62
Δ_{31} = 6 – (0.270) (3) = 5.19	Δ_{54} = 0 – (0.270) (-8) = 2.16
Δ_{32} = 5 – (0.270) (0) = 5	Δ_{55} = -2 – (0.270) (-12) = 1.24

Since all $\Delta_{ij} \geq 0$. The solution is Optimal and Minimum Z = 0.270

5. Conclusion

The main reason to consider transhipment is that commodities are allowed to pass transiently through other sources and destinations before it ultimately reaches its designated destination. It therefore is capable of seeking the minimum-cost route between a source and a destination. Hence, by adopting linear fractional transportation method, we have compared the optimum solution of fractional transportation with fractional transhipment problem and given that in some situations the transportation through intermediate points can be less expensive than direct shipment. This Method would be very beneficial in the working of reduction of cost.

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