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Thermal Radiation Effects on Hydromagnetic Flow of Nanofluid over a Nonlinearly Stretching Sheet in the Existence of Variable Heat Generation and Viscous Dissipation

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Abstract: ***Very recently, there has been an affordable amount of work carried out by scientists/researchers, on radiative heat transfer in nanofluids due to their abundant applications. Particularly, the conception of radiative heat transfer is extremely employed in nanofluid Solar Collectors. Driven by these, the present work is worries with Radiative heat transfer of Nanofluids over a nonlinearly stretching surface with thermal radiation, in the presence of variable heat generation and viscous dissipation along with variable temperature is investigated for two sorts of nanofluids is created by intermixture solid spherical nanoparticles with water such as Cu-Water nanofluid and Ag-Water nanofluid. Governing equations of the problem are nonlinear partial differential equations which are reduced to nonlinear ordinary differential equations by means of similarity transformations. The resulting boundary value problem is converted into an initial value problem by using an exploitation best shooting method such as Nachtsheim- Swigert shooting iteration scheme for satisfaction of infinity boundary conditions. Then the resultant initial value problem is solved numerically by fourth order Runge-Kutta Integration technique (FORTRAN coding). The effects of the magnetic interaction parameter, heat source/sink parameter, viscous dissipation parameter, radiation parameter, Prandtl number, solid volume fraction parameter and nonlinear stretching parameter on flow and heat transfer characteristics are analyzed. As a verification of the numerical scheme adopted, the numerical results of the problem is compared with that of formerly published work in the absence of nanoparticles, magnetic field, heat source/sink and radiation and found be in are in splendid agreement. Also the numerical values of skin friction at wall and the nondimensional rate of heat transfer are conferred in tabular form.***

Keywords: Nanofluid, Stretching sheet, MHD, radiation

2010 subject Classification: 76D10, 76A02, 76W05, 85A25

1. Introduction

Nanofluids are engineered by suspending nanoparticles with size between 1 – 100nm in traditional heat transfer fluids such as water, oil and ethylene glycol. Typical nanoparticles are metals, oxides, carbides or carbon nanotubes. The shape of the nanoparticles are spherical, rod disks and cylindrical. A awfully bit of guest nanoparticles, when dispersed homogeneously and balanced stably in conventional fluids, can provide remarkable improvements in the thermophysical properties such as thermal conductivity in base fluids. The word Nanofluid is first coined by Choi[1]. After, several talented and studious thermal scientists and engineers has been analyzed that the thermal conductivity of nanofluids are superior as compared to co nventional fluids and identified unusual opportunities to develop next generation coolants such as smart coolants for computer, microchannel heat sinks and safe coolants for nuclear reactor and so on. Due to their abundant applications, Nanofluids heat transfer enhancement and augmented thermal conductivity was studied under various geometrics by Xuan Y and Roetzal W[2], Yimin Xuan and Qiang Li[3],Das et al. [4]Nanofluid flow over a flat plate was studied by Anjali Devi and

Julie Andrews[5]and it had been realized that suspended nanoparticles enhance the heat transfer capacity of the fluids. Thermal conductivity enhancement of nanofluids is extremely helpful in thermal and heat transfer devices. Jung et al. [6] studied the effects of surface charge state of nanoparticles to elucidate the thermal conductivity enhancement of nanofluids. The problem on mathematical modeling of thermal conductivity for nanofluid considering interfacial nano layer is investigated by Rizvi et al. [7].

Flow of a viscous fluid over a stretching sheet is a classical problem in fluid dynamics which has many practical applications in the industrial manufacturing process. Sakiadis[8] was the primary person to debate the laminar boundary layer flow of viscous and incompressible fluid caused by an eternal moving surface and different aspects of the problem has been explored by many researchers within the past decades. But all these studies are classified to linear stretching of the sheet. It means that the stretching is not essentially linear. Therefore, new dimension in the field of stretching sheet has arrived that it will be stretched nonlinearly. Viscous flow over a non-linearly stretching sheet was studied by several researchers like Vajravelu[9],Cortell[10], Talay Akyildiz et al.[11], Rafael Cortell[12]and Swati Mukhopadhyay[13] in the presence of several physical parameters.

Hydromagnetic physical phenomenon flow on a continual stretching sheet has attracted considerable attention throughout the last few decades due to its plentiful applications in industrial manufacturing processes. In particular, the metallurgical processes comparable to drawing, annealing and tinning of copper wires involve cooling of continuous strips or filaments by drawing them

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through a quiescent fluid. Hydromagnetic boundary layer flow of a viscous incompressible fluid which is caused by a sheet stretching consistent with a power law velocity distribution within the presence of a magnetic field was investigated by Chaim[14]. Non-linear hydromagnetic flow and heat transfer over a surface stretching with power law velocity was investigated by Anjali Devi and Thiagarajan [15]. Steady nonlinear Hydromagnetic flow and heat transfer over a stretching surface of variable temperature was studied by Anjali Devi and Thiagarajan[16], The studies on nonlinear MHD forced convection boundary layer flow past a stretching porous surface with dissipation effects was conversed by Anjali Devi and Ganga[17].

Heat generation effects in thermal convection, are significant wherever there could exist a high temperature variations between the surface (e.g. space craft body) and therefore the ambient fluid. Effects of radiation and heat source/sink on unsteady MHD boundary layer flow and heat transfer over a shrinking sheet with suction/injection is investigated by Bhattacharyya[18].

More recently researchers have become fascinated by the radiative properties of nanoparticles in liquid suspensions particularly for nanofluid primarily based absorption solar collectors. Absorption of solar collectors are projected for sort of applications like water heating/cooling, but the potency of those collectors is restricted by the absorption properties of the working fluid, which is incredibly poor for conventional fluid utilized in solar collectors. However nanoparticles offer the potential of rising the radiative properties of liquids resulting in a rise within the efficiency of absorption of solar collectors. In addition it has been shown that mixing nanoparticles in an exceedingly liquid (nanofluid) includes a dramatic impact on the liquid thermophysical properties adore thermal conductivity.

Steady two-dimensional Marangoni boundary layer flow past a semi-infinite flat in an exceedingly water-based nanofluid containing totally different kind of nanoparticles, namely, copper (Cu), aluminium oxide (Al_2O_3), and titanium dioxide (TiO_2) with radiation effects is analyzed by Hamid et al. [19] . The thermal radiation and viscous dissipation effects on the laminar boundary layer concerning a flat plate in a uniform stream of fluid (Blasius flow) and about a moving plate in a quiescent ambient fluid (Sakiadis flow) both under convective boundary condition is conferred by Olanrewaju et al. [20].

Steady two dimensional flow of an electrically conducting nanofluid flow over a vertical convectively heated permeable stretching surface with variable stream conditions in presence of a uniform transverse magnetic field and internal heat source/sink was analysed by Kundu et al. [21].Unsteady MHD free convection boundary layer flow of nanofluid along a stretching sheet with thermal radiation and viscous dissipation effects was carried out by Md Shakhaoath khan et al. [22]. Very recently, Eldabe al. [23] has analyzed the e consequences of magnetic field and heat generation on viscous flow over a nonlinearly stretching surface in a nanofluid. Effects of thermal radiation on the steady laminar magnetohydrodynamic boundary layer flow of a nanofluid over an exponentially stretching sheet are studied theoretically by Loganathan and vimala[24]

To the best of the authors' knowledge no attempt has been made so far on radiation effects on hydromagnetic boundary layer flow of an incompressible, viscous nanofluid over a nonlinearly stretching surface in the presence of variable heat generation and viscous dissipation with variable temperature this led to present study.

2. Mathematical formulation of the problem

Steady, two dimensional, nonlinear, hydromagnetic laminar boundary layer flow of an incompressible, viscous nanofluid that could be a water based nanofluid containing different types of spherical shaped nanoparticles, specifically copper (Cu) and Silver (Ag) over a nonlinearly moving stretching surface with radiation effect, in the presence of variable magnetic field and variable heat generation and viscous dissipation with variable temperature is taken into account. The x - axis runs on the stretching surface, varying nonlinearly with distance from the slit. A schematic diagram of the physical model and coordinate system is shown in Figure (1). The Thermophysical properties of water and metallic nanoparticles (Copper and Silver) are shown in Table 1.

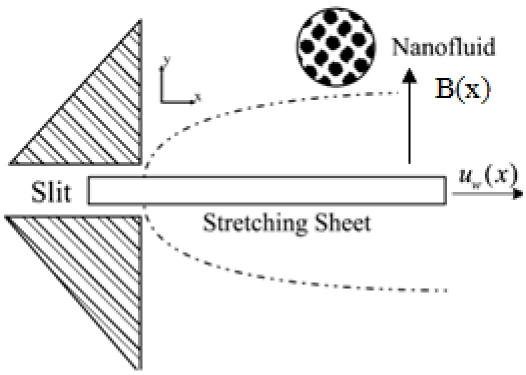

Figure 1: Physical model of the problem

The following assumptions are made in the analysis

- The sheet is stretched with the velocity $u_w = cx^n$, Where c is a positive integer and n is that the nonlinear stretching parameter.
- The variable magnetic field B(x) is applied in the transverse direction.
- The induced magnetic field is negligible in comparision to that of applied magnetic field. Since the induced magnetic field is neglected and B_0 is independent of time, $\mathrm{curl}\vec{E} = 0$. Also, $\mathrm{div}\vec{E} = 0$ in the absence of surface charge density. Hence $\vec{E} = 0$.

2.1 Governing equation of the flow

From the above assumptions, the steady governing equations of the hydromagnetic flow field are often written in the dimensional form as

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Mass conservation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Conservation of momentum

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu_{nf}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B(x)^2}{\rho_{nf}}u \tag{2}$$

Energy conservation

$$\left(\rho C_p\right)_{nf}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k_{nf}\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}+ Q'(T-T_\infty)+\mu_{nf}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

Where u, v are the velocity components in the x and y directions respectively, ρ_{nf} is the effective density of the nanofluid, $(C_p)_{nf}$ is the specific heat capacity of the nanofluid , σ is the electrical conductivity of the fluid, q_r is the radiative heat flux, μ_{nf} is the dynamic viscosity of the nanofluid, $Q'(x) = Q_0 x^{n-1}$ is the dimensional variable heat generation, $B(x) = B_0\, x^{(n-1)/2}$ is the variable magnetic field (Afzal[25]) , T_w is temperature at the wall, T_∞ is the temperature outside the dynamic region,k_{nf} and ν_{nf} are the thermal conductivity and the kinematic viscosity of the nanofluids respectively.

The boundary conditions subjected to the velocity and temperature are given by

$$u = u_w(x) = cx^n,\ v = 0,\ T = T_w(x) = T_\infty + b\,x^m \ at\ y = 0 \tag{4}$$

$$u = 0,\ T \to T_\infty\ as\ y \to \infty \tag{5}$$

where n and m are nonlinear stretching parameter and the surface temperature parameter respectively.

Following Rosseland's approximation [Rosseland[26], Brewster[27]] the radiative heat flux q_r is modeled as

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{6}$$

where σ^* is the Stefan-Boltzman constant and k^* is the mean absorption coefficient. It should be, provide that by using the Rosseland's approximation, the present analysis is limited to optically thick medium Assuming that the temperature differences among the flow are sufficiently small, T^4 [Raptis [28]]is also expressed as a linear function $T^4 \equiv 4T_\infty^3 T-3T_\infty^4$ and

$$\frac{\partial q_r}{\partial y}=-\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2} \tag{7}$$

The expressions for the physical quantities ρ_{nf}, μ_{nf}, k_{nf}, ν_{nf}, α_{nf} and $\left(\rho c_p\right)_{nf}$ are given through the following lines[Ahmad et al. [29]].

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s\ ,\ \mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}}$$

$$k_{nf}=k_f\left[\frac{k_s+2k_f-2\phi\left(k_f-k_s\right)}{k_s+2k_f+\phi\left(k_f-k_s\right)}\right],$$

$$\nu_{nf}=\frac{\mu_{nf}}{\rho_{nf}},\ \alpha_{nf}=\frac{k_{nf}}{\left(\rho c_p\right)_{nf}}$$

$$\left(\rho c_p\right)_{nf}=(1-\phi)\left(\rho c_p\right)_f+\phi\left(\rho c_p\right)_s \tag{8}$$

where ϕ is the solid volume fraction of nanoparticles . Here the subscript nf denotes the thermo physical properties of nanofluid, f represents the base fluid and s represents the two nano spherical solid particles Cu and Ag. k_{nf} is the thermal conductivity of the nanofluid, k_f and k_s are the thermal conductivities of the base fluid and nanoparticles respectively.

The following similarity transformations [Ferky M Hady et al. [30], Eldabe et al. [23]] are introduced to resolve the equations (1) to (3) with the boundary conditions (4) and (5).

$$\psi=\sqrt{\frac{2c\nu_f}{(n+1)}}x^{\frac{n+1}{2}}f(\eta),\ \eta=y\sqrt{\frac{c(n+1)}{2\nu_f}}x^{\frac{n-1}{2}}$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{9}$$

Where η is the similarity space variable and $f(\eta)$ is the dimensionless stream function.

Using the Stream function

$$u=\frac{\partial\psi}{\partial y}\ \text{and}\ v=-\frac{\partial\psi}{\partial x},$$

The horizontal and vertical velocity components are expressed as follows

$$u=cx^n f'(\eta)\ ,$$

$$v=-\sqrt{\frac{c(n+1)\nu_f}{2}}x^{\frac{n-1}{2}}\left[f'(\eta)\ \eta\ \frac{n-1}{n+1}+f(\eta)\right] \tag{10}$$

Equation of continuity (1) is automatically satisfied. Using the transformations (9) and the equation (7), the nonlinear partial differential momentum equation (2) and energy equation (3) with boundary conditions (4) and (5) are reduced to the following nonlinear ordinary differential equations

$$f'''+(1-\phi)^{2.5}\left[\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)\left(ff''-\frac{2n}{n+1}\left(f'\right)^2\right) -M^2 f'\right]=0 \tag{11}$$

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$$\left(1+\frac{4}{3N_R}\right)\theta''+\Pr\left(\frac{k_f}{k_{nf}}\right)\left[\begin{array}{c}\left(1-\phi+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)\left(f\theta'-\frac{2m}{n+1}f'\theta\right)+\\ Q\theta+\frac{E_c}{(1-\phi)^{2.5}}(f'')^2\end{array}\right]=0 \quad (12)$$

For simplification, m = 2n is chosen in Equation (12), which becomes

$$\left(1+\frac{4}{3N_R}\right)\theta''+\Pr\left(\frac{k_f}{k_{nf}}\right)\left[\begin{array}{c}\left(1-\phi+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)\left(f\theta'-\frac{4n}{n+1}f'\theta\right)+\\ Q\theta+\frac{E_c}{(1-\phi)^{2.5}}(f'')^2\end{array}\right]=0 \quad (13)$$

The Transformed boundary conditions become

$$f(0)=0,\ f'(0)=1,\ \theta(0)=1 \text{ at } \eta=0,$$
$$f'(\infty)=0,\ \theta(\infty)\to 0 \text{ as } \eta\to\infty \quad (14)$$

where prime indicates differentiation with respect to η.

The nondimensional constants occurring in Equation (11) and (13) are the magnetic interaction parameter M^2, the radiation parameter N_R, the heat source (or sink) parameter Q, the Prandtl number Pr and the viscous dissipation parameter (Eckert number) Ec. They are respectively defined as

$$M^2=\frac{2\sigma B_0^2}{c(n+1)\rho_f} \qquad N_R=\frac{k_{nf}k^*}{4\sigma^* T_\infty^3},$$

$$\Pr=\frac{\nu_f(\rho c_p)_f}{k_f},\ Q=\frac{2Q_0}{(n+1)c(\rho c_p)_f}, \quad (15)$$

$$\text{and} \qquad Ec=\frac{(u_w)^2}{(c_p)_f(T_w-T_\infty)}$$

Table 1: Thermo-physical properties of fluid and nanoparticles At 25°C

Physical properties	Water fluid	Cu	Ag
C_P	4179	385	235
ρ	997.1	8933	10500
K	0.613	400	429
α	1.47	1163.1	1738.6

2.2 Skin friction and Nusselt number

For the type of boundary layer flow under consideration, the physical parameters of engineering interest are the skin friction coefficient and Nusselt number. Knowing the velocity field, the shearing stress at the wall can be calculated, which is in the non dimensional form (skin friction coefficient) is given by

The shearing stress at the wall is obtained utilizing

$$\tau=\mu\left(\frac{\partial u}{\partial y}\right)_{\text{at the wall}}$$

Then skin friction coefficient C_f at the wall is derived as follows

$$C_f(\mathrm{Re}_x)^{1/2}=\frac{\sqrt{n+1}}{(1-\phi)^{2.5}}f''(0) \text{ where}$$

$$\mathrm{Re}_x=\frac{cx^{n+1}}{\nu_f} \quad (16)$$

The non dimensional rate of heat transfer (Nusselt number) is defined as

$$(Nu_x)_{nf}=-\frac{k_{nf}}{k_f}\frac{x\left(\frac{\partial T}{\partial y}\right)_{y=0}}{(T_w-T_\infty)} \quad (17)$$

Using Equation (17), The Nusselt number can be calculated as

$$\frac{(Nu_x)_{nf}}{(\mathrm{Re}_x)^{1/2}}=-\frac{k_{nf}}{k_f}\sqrt{n+1}\,\theta'(0)$$

3. Numerical Solutions of the problem

The system of non-linear boundary layer equations (11) and (13) together with the boundary conditions (14) are solved numerically using most efficient Nachtsheim- Swigert shooting iteration scheme for satisfaction of asymptotic boundary conditions along with fourth order Runge-Kutta Integration method (FORTRAN coding is utilized). The crux of the problem is that proper initial guesses for the values of $F''(0)$ and $\theta'(0)$ are to be made to initiate the shooting process. The success of procedure depends very much on how good this guess is. For different values of M^2,N_R, n, Q,ϕ and Ec completely different initial guesses were made taking into account of the convergence. Convergence is taken up to the order of 10^{-5}. Numerical results of dimensionless velocity and temperature distribution are found and are given graphically. Also from the process of numerical computation, the skin friction coefficient and the rate of heat transfer are sorted out and their numerical values are presented in tabular form.

4. Results and Discussion

The numerical and graphical results for the Cu-water nanofluid and Ag-water nanofluid which is formed by spherical shaped copper nanoparticles and silver nanoparticles respectively with mixing water are obtained in this work. The solid volume fraction of the nanoparticles is investigated in the range of 0≤ϕ≤0.2 and the value of the Prandtl number for the base fluid (water) is kept to be as a constant of Pr = 6.2 at 25°C.

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The accuracy of the present method is testified by comparing authors' results with those of Cortell (12) and Eldabe et al.(23) for the nondimensional rate of heat transfer $-\theta'(0)$ in the absence of nanoparticles ($\phi = 0$), Magnetic interaction parameter and variable heat source/sink parameter and without thermal radiation parameter ($N_R \rightarrow \infty$) which are vividly shown in Table 2. It is clearly note that authors' results are in very good agreement with that of Cortell and Eldabe et al.

Table 2: Comparisons of - $\theta'(0)$ to previously published data at Pr =1.0 and Pr = 5.0, $\phi = 0.0$, $M^2 = 0.0$, Q = 0.0, $N_R \rightarrow \infty$ for different values of Ec and n

		Pr = 1.0		
Ec	n	Cortell (12)	Eldabe et al (23)	Present work
0	0.75	1.25267	1.25270	1.25271
0	1.5	1.43939	1.43937	1.43937
0	7.0	1.69929	1.69930	1.69932
0	10.0	1.72893	1.72894	1.72896
0.1	0.75	1.21998	1.21994	1.21995
0.1	1.5	1.40508	1.40515	1.40516
0.1	7.0	1.66250	1.66257	1.66260
0.1	10.0	1.69182	1.69188	1.69190
		Pr = 5.0		
0	0.75	3.12497	3.12478	3.12535
0	1.5	3.56773	3.56795	3.56796
0	7.0	4.18537	4.18545	4.18547
0	10.0	4.25597	4.25596	4.25598
0.1	0.75	3.01698	3.01698	3.01704
0.1	1.5	3.45572	3.45587	3.45591
0.1	7.0	4.06572	4.06575	4.06576
0.1	10.0	4.13529	4.13529	4.13528

Figs 2 – 11 depict the dimensionless velocity and temperature distribution of Cu – Water nanofluid and Ag-Water nanofluid for different values of physical parameters M^2, Ec, n, Q, N_R and ϕ .

Fig.2 shows the influence of magnetic interaction parameter on dimensionless velocity distribution. Increasing values of magnetic interaction parameter decelerates the velocity. This is often due proven fact that the variation in M^2 leads to the variation of the Lorentz force due to magnetic field and the Lorenz force produces more resistance to the transport phenomena. Fig 3 elucidates the dimensionless temperature profiles for numerous values of magnetic interaction parameter in the presence of radiation. Temperature is accelerated owing to increase value of magnetic field. It is further noted from the Fig.2 & Fig.3,the same values of magnetic interaction parameter, the thickness of the thermal boundary layer is slightly superior to those of momentum boundary layer.

Fig 4 and Fig 5 represents the velocity and temperature distribution of Cu – Water nanofluid and Ag – Water nanofluid for several values of non linear stretching parameter. Increase in n which reduce both dimensionless velocity and temperature within the boundary layer. From fig.4 the nonlinear stretching parameter has less significant effect on velocity.

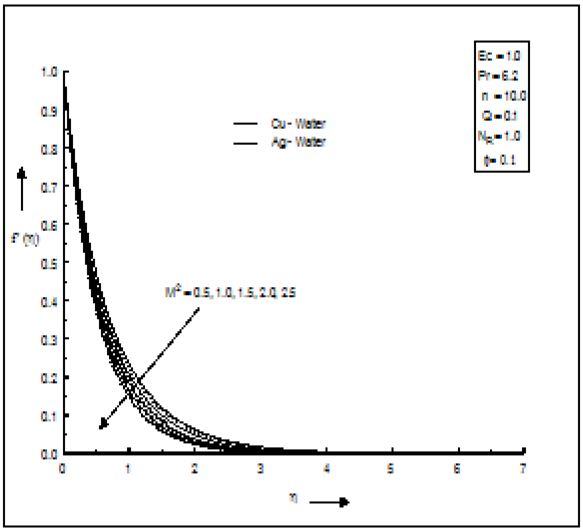

Figure 2: Effect of magnetic field on dimensionless velocity

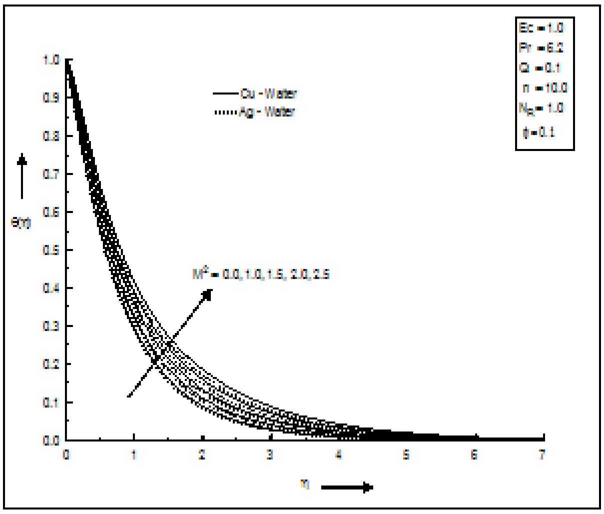

Figure 3: Dimensionless temperature distribution for various values of M^2

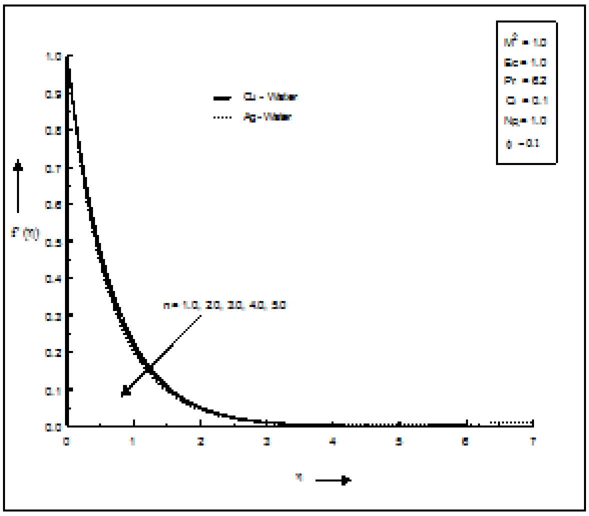

Figure 4: Dimensionless **velocity for different n**

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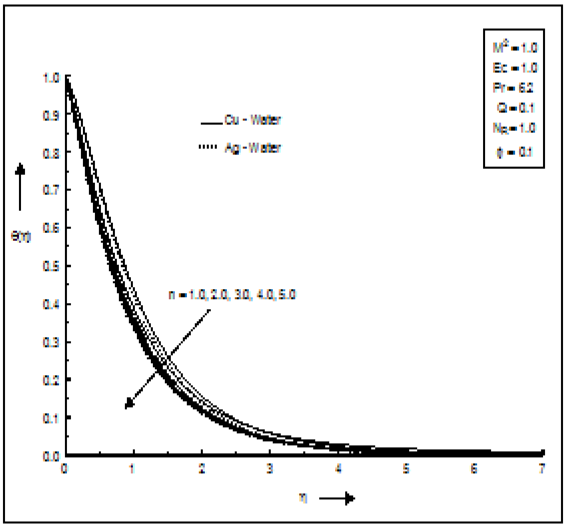

Figure 5: Effect of n over dimensionless temperature distribution

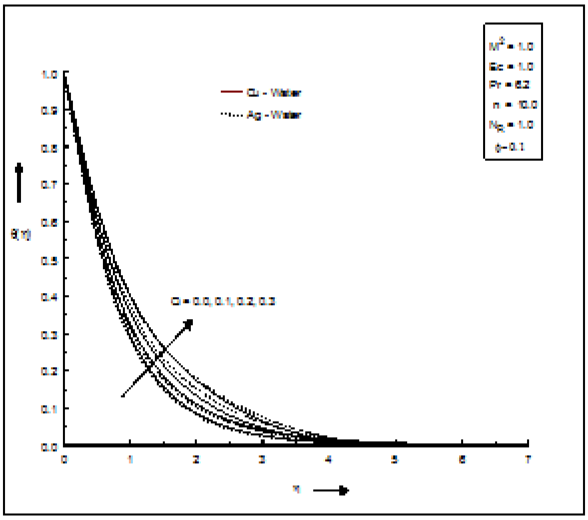

Figure 6: Temperature profiles for different values of heat generation parameter

Fig 6 and Fig 7 shows the influence of heat source and heat sink parameter on the temperature distribution. In the case of heat generation parameter, it may be seen that heat energy is generated in thermal physical phenomenon that cause the temperature to strengthen with a rise in the values of heat source parameter. This will increases thermal boundary layer thickness and from fig.7 heat sink will increase additional heat off from the nonlinearly stretching sheet which reduces the thermal boundary layer thickness. This reduces the temperature of the nonlinearly stretching sheet. Fig 8 displays the result of Eckert number on dimensionless temperature distribution. It is implies that the Eckert number enhances the temperature. Also we have a tendency to note that each value of Ec, the thickness of the thermal boundary layer in Cu – water nanofluid is greater than Ag – water nanofluid.

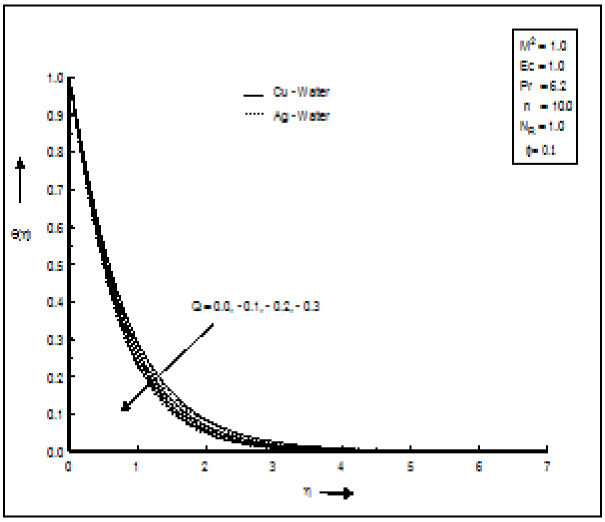

Figure 7: Effect of heat sink parameter over dimensionless temperature distribution

Figures 9 and 10 illustrate the effect of nanoparticles volume fraction ϕ on the nanofluid velocity and temperature profile respectively. It is clear that, as the nanoparticles volume fraction increases, the nanofluid velocity decreases and also the temperature increase. Additionally from fig.10, once the amount of nanoparticles will increase, the thermal conductivity enhances then the thermal boundary layer thickness accelerates

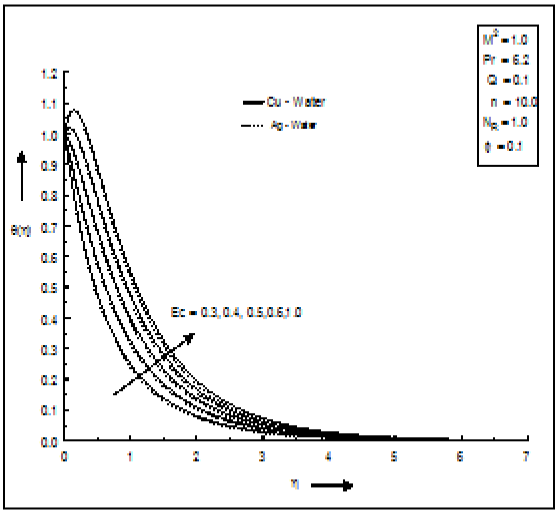

Figure 8: Variation in Ec over Temperature profiles

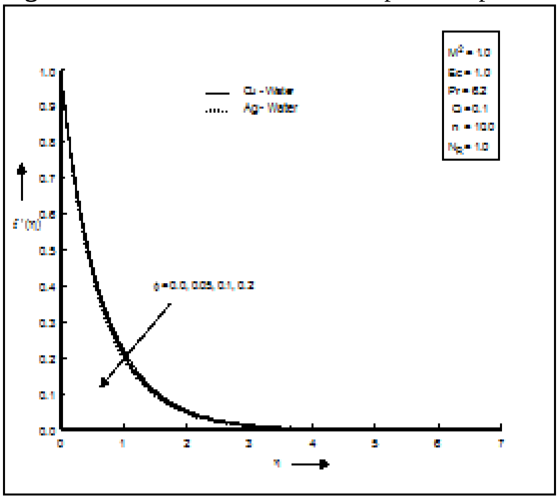

Figure 9: Velocity profiles for various values of ϕ

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In Fig 11, the impact of radiation parameter on dimensionless temperature distribution is shown. It is evident that the effect of Radiation parameter is to lower the temperature for its ascending value with higher temperature at the wall. It is clearly seen that the Radiation parameter decreases the thermal boundary layer thickness. The radiation ought to be at its minimum in order to facilitate the cooling process. Also it absolutely was seen that the amendment in temperature profiles in the case of Ag – water nanofluid was slightly less significant than for a Cu – water nanofluid.

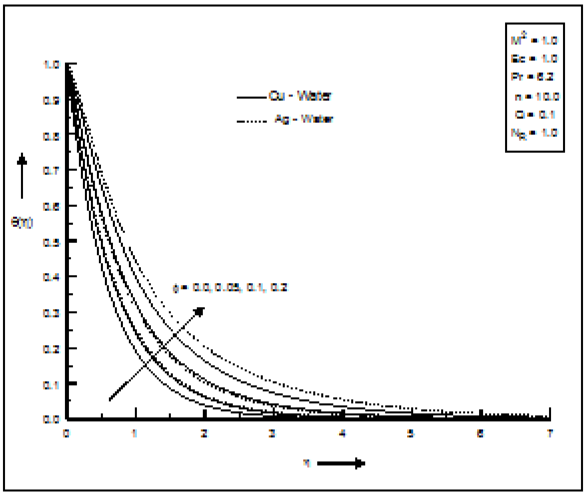

Figure 10: Temperature profiles for different values of ϕ

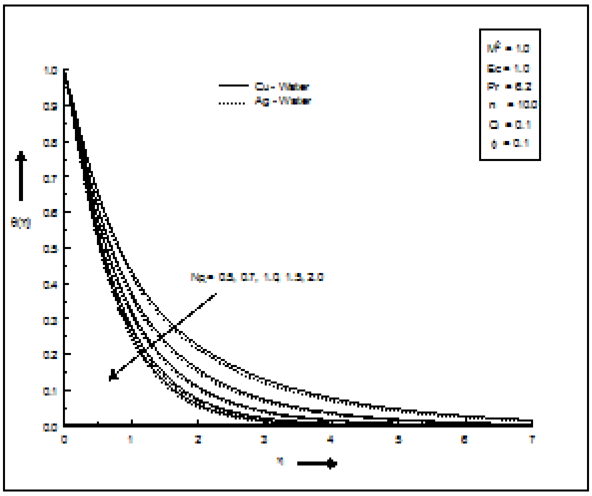

Figure 11: Effect of radiation parameter on θ(η)

The results for the skin friction co efficient and non dimensional rate of heat transfer for different values of magnetic interaction parameter and nonlinearly stretching parameter are given in Table 3. From these results, it is ascertained that | f′′(0) | increases with a rise in the magnetic interaction parameter and non linear stretching parameter. Ag nanoparticles are the highest skin friction co efficient, followed by Cu nanoparticles. And the nondimensional rate of heat transfer suppressed due to effect of M^2 and the increasing values of nonlinearly stretching parameter raises the nondimensional rate of heat transfer in Cu-Water nanofluid and Ag-Water nanofluid.

Table 3: Skin friction co efficient and nondimensional Heat transfer rate for different values of M^2 and n when N_R = 1.0, Ec = 1.0 ϕ = 0.1, Q = 0.1 and Pr = 6.2

n	M^2	Cu - water		Ag - Water	
		$-\frac{\sqrt{n+1}}{(1-\phi)^{2.5}} f''(0)$	$-\frac{k_{nf}}{k_f}\sqrt{n+1}\theta'(0)$	$-\frac{\sqrt{n+1}}{(1-\phi)^{2.5}} f''(0)$	$-\frac{k_{nf}}{k_f}\sqrt{n+1}\theta'(0)$
10.0	0.0	6.26119	5.53456	6.52939	5.56902
	1.0	7.32813	3.79481	7.55891	3.88866
	2.0	8.25241	2.22612	8.45842	2.36634
	3.0	9.08123	0.76319	9.26910	0.93810
0.0	2.0	1.85068	-167942	1.87079	-1.65085
1.0		3.14271	-0.12647	3.20711	-0.08425
3.0		4.77646	0.80504	4.88846	0.88652
5.0		5.97941	1.33883	6.12454	1.44213
7.0		6.97801	1.54594	7.15004	1.86072
10.0		8.25241	2.22612	8.45842	2.36634

Table 4: The nondimensional Heat transfer rate for different values of N_R, Ec and Q when ϕ = 0.1, n = 10.0 and Pr = 6.2

M^2	N_R	Q	Ec	Cu - water		Ag – Water	
				$\theta'(0)$	$-\frac{k_{nf}}{k_f}\sqrt{n+1}\theta'(0)$	$\theta'(0)$	$-\frac{k_{nf}}{k_f}\sqrt{n+1}\theta'(0)$
1.0	1.0	0.0	1.0	-0.93945	4.15258	-0.95899	4.23574
		0.1		-0.85851	3.79481	-0.88041	3.88866
		0.2		-0.77336	3.41843	-0.77746	3.48977
		0.3		-0.67333	2.97627	-0.69343	3.06279
		-0.1		-1.01279	4.47676	-1.03061	4.55208
		-0.2		-1.08129	4.77955	-1.09769	4.84836
		-0.3		-1.146617	5.06633	-1.16133	5.12945

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2.0	0.5 1.0 1.5 2.0	0.1	1.0	-0.44144 -0.50362 -0.51027 -0.51268	1.95126 2.22612 2.25551 2.26616	-0.46850 -0.53573 -0.54609 -0.54648	2.06931 2.36625 2.40759 2.41374
1.0	1.0	0.1	0.1 0.3 0.5 1.0	-2.11048 -1.83225 -1.55414 -0.85851	9.32880 8.09896 6.86965 3.79481	-2.17149 -1.88457 -1.59775 -0.88041	9.59112 8.32384 7.05707 3.88866

Table 4 shows the nondimensional rate of heat transfer for various values of physical parameters like radiation parameter, heat source/sink parameter and Eckert number. It is evident that the nondimensional rate of heat transfer decreases with increasing values of heat source parameter Q. In distinction, the heat sink parameter (Q<0) accelerates the nondimensional rate of heat transfer. When the radiation parameter will increase, the non dimensional rate of heat transfer is not decreasing whereas the Eckert number increases opposite trend are occurred.

5. Conclusion

In this work , steady, two dimensional, hydromagnetic laminar boundary layer flow of a viscous, incompressible Cu- Water nanofluid and Ag- Water nanofluid past a nonlinearly stretching surface with radiation effects within the presence of variable magnetic field and heat source/sink and viscous dissipation effects with variable temperature are investigated. The governing partial differential equations were converted to ordinary differential equation by employing a appropriate similarity transformation and were then solved numerically using most efficient Nachtsheim-Swigert shooting iteration scheme for satisfaction of asymptotic boundary conditions along with fourth order Runge-Kutta Integration method (FORTRAN coding). The results are presented for various values of physical parameters such as magnetic interaction parameter M^2, nonlinearly stretching parameter n, heat source/sink parameter Q, viscous dissipation parameter Ec, solid volume fraction parameter ϕ and thermal radiation parameter N_R.

The following conclusions are drawn as a result of the discussion of this work.

- The increase of the nonlinearly stretching parameter and magnetic interaction parameter leads to the decrease the dimensionless surface velocity, these yields a rise within the skin friction co efficient at the surface.
- An increment in the solid volume fraction ϕ yields a decrease in the nanofluids velocity in Cu – water nanofluid and Ag – water nanofluid.
- A rise in magnetic interaction parameter, Eckert number and solid volume fraction and heat source parameter enhances the temperature in the boundary layer region whereas nonlinear stretching parameter, heat sink parameter and the radiation parameter reduces the temperature.
- The influence of nonlinearly stretching parameter, heat sink parameter and radiation parameter is to extend the rate of heat transfer, in the mean while, the heat source parameter, magnetic interaction parameter and Eckert number decelerates the nondimensional rate of heat transfer.
- The Ag nanoparticles evidenced to own the best skin friction co efficient for this problem than the Cu nanoparticles
- When radiation parameter increases, the nondimensional rate of heat transfer of Ag -water nanofluid higher than Cu - water nanofluid

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References

[1] Choi.S., Enhancing thermal conductivity of fluids with nanoparticles. In : Sidiner, DA, Wang HP (eds) Developments and applications of non-Newtonian flows, *ASMEFED*, 231/MD, 99-105, (1995),.

[2] Xuan.Y., and Roetzel. W.,Conceptions for heat transfer correlation of nanofluids. International Journal of Heat and Mass Transfer, 43, pp. 3701-3707, (2000)..

[3] Yimin Xuan and Qiang Li, Heat transfer enhancement of nanofluids,*International journal of heat & fluid flow*,28,58-64, (2000).

[4] Das SK, Putra N, Thiesen P,Roetzel W, Temperature dependence of thermal conductivity enhancement for nanofluids. *Journal of Heat Transfer*, 125,567-574, (2003).

[5] Anjali Devi.S.P and Julie Andrews, Laminar flow of nanofluids over a horizontal flat plate, *IJAMM* , (2011).

[6] Jung-Yeul Jung and Yong Tae Kang, Effect of surface charge state on the thermal conductivity of nanofluids, *Heat Mass Transfer* , 48,713–718, (2012),.

[7] Imbesal Hassan Rizvi, Ayush Jain Subrata Kr.Ghosh, P.S. Mukherjee, Mathematical modeling of thermal conductivity for nanofluid considering interfacial nano-layer, Heat and Mass trarnsfer,49,595-600, (2013).

[8] Sakiadis, B.C., Boundary-layer behavior on continuous solid surfaces: I. Boundary-layer equations for two-dimensional and axisymmetric flow, *AICHE J., 7,26-28*,(1961) .

[9] Vajravelu.K., viscous flow over a non linearly stretching sheet, *Applied Mathematics and Computation*,124, Pages 281–288,(2001).

[10] Cortell.R.,Effects of viscous dissipation on and radiation on the thermal boundary layer over a nonlinearly stretching sheet, Physics Letters A, 372(5), 631-636, (2008).

[11] Akyildiz, F. T., Siginer, D. A., Vajravelu, K., Cannon, J. R., & Van Gorder, R. A., Similarity solutions of the boundary layer equations for a nonlinearly stretching sheet. *Mathematical Methods in the Applied Sciences*,33(5), 601-606 (2010).

[12] Rafael Cortell.,Heat and fluid flow due to non-linearly stretching surfaces *Applied Mathematics and Computation*,217,7564-7572,(2011).

[13] Swati Mukhopadhyay,(2013), Analysis of boundary layer flow over a porous nonlinearly stretching sheet

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with partial slip at the boundary ,*Alexandria Engineering Journal*, *52,563-569*

[14] Chaim.C, Hydromagnetic flow over a surface stretching with a power law velocity, *International* Journal of *Engineering Science*, Vol. *33*, No. *3* , *429*-435,(*1995*).

[15] Anjalidevi, S. P.,Thiyagarajan, M., Nonlinear hydromagnetic flow and heat transfer over a surface stretching with a power-law velocity, *Heat and Mass Transfer*,*38*, *723-726*, (2002).

[16] Anjali Devi, S. P., Thiyagarajan, M., Steady nonlinear hydromagnetic flow and heat transfer over a stretching surface of variable temperature, *Heat and Mass Transfer*, 42, 671-677,(2006).

[17] Anjali Devi, S.P. and Ganga, B.,Nonlinear MHD forced convection boundary layer flow past a stretching surface with dissipation effects, *Nonlinear Analysis* : *Modeling and Control*, 14(3), 303,(2009).

[18] Krishnendu Bhattacharyya, Effects of radiation and heat source/sink on unsteady MHD boundary layer flow and heat transfer over a shrinking sheet with suction/injection,*Front. Chem. Sci. Eng*,5(3),376–384,(2011).

[19] Rohana Abdul Hamid, Norihan md.Aritin, Roslinda mohd Nazal and loan Pop, Radiation effects on marangoni boundary layer flow past a flat plate in Nanofluid, *Proceedings of the Int. Multi conference of Engineers and computer sciences*,*2*,2011,.

[20] Olanrewaju,P.O.,Gbadeyan,J.A., Agboola, O.O.and Abah, S.O., Radiation and viscous dissipation effects for the Blasius and Sakiadis flows with a convective surface boundary condition**,** *International Journal of Advances in Science and Technology*,*2, No.4*,(2011).

[21] Kundu.P.K., Nanofluid flow towards a convectively heated stretching surface with heat source/sink: A lie group analysis, *African Mathematical union and springer-verlag berlin Heidelberg*, (2012).

[22] Md Shakhaoath Khan, Ifsana Karim, Lasker Ershad Ali and Ariful Islam, Unsteady MHD free convection boundary-layer flow of a nanofluid along a stretching sheet with thermal radiation and viscous dissipation effects, *International Nano Letters*, 2,24, (2012).

[23] Eldabe.N.T., Elbashbeshy,E.M.A., Elsaid.E.M., Effects of magnetic field and Heat generation on viscous flow and heat transfer over a nonlinearly stretching surface in a Nanofluid, *Int.Journal of Applied Mathematics*, 28, (2013).

[24] Loganathan. P., vimala., C., MHD boundary layer flow of nanofluid over an exponentially stretching sheet in the presence of radiation, *Heat Trans Asian Res*, 43(4), 321–331, (2014).

[25] Afzal,N., Heat transfer from a stretching surface, Int. J. Heat Mass Transfer, 36,1128-1131, (1993).

[26] Rosseland, S., *Theoretical Astrophysics*, Clarendon Press, Oxford, (1936).

[27] Brewster, M. Q., *Thermal Radiative Transfer and Properties.* John Wiley and sons Inc, (1992).

[28] Raptis. A., Flow of a micripolarfluid past a continuously moving plate by the presence of Radiation, Int. Jour. of Heat & Mass transfer, 41 (18),2865-2866, (1998).

[29] Ahmad, S., Rohini, A.M. and Pop,I., Blasius and Sakiadis problems in nanofluids, *Acta Mechanica*, 218, 195-204, (2011).

[30] Fekry M Hady, Fouad S Ibrahim Sahar M Abdel-Gaied and Mohamed R Eid, Radiation effect on viscous flow of a nanofluid and heat transfer over a nonlinearly stretching Sheet, *Nanoscale Research Letters*, 7,229, (2012).

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