

Comparison of Certain Types of Continuous Sampling Plans (CSPs) and its Operating Procedures – A Review

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Abstract: Sampling is widely used in manufacturing industries and government agencies for controlling the quality of shipments of components, supplies, raw materials and final products. Continuous sampling plans are useful when the formation of lots for sampling inspection is impracticable or artificial. The application of continuous sampling plan is continuous flow of units from which the production process and units are offered for inspection item by item in the order of their production. Dodge (1943) has introduced the concept of continuous sampling plan and provides mathematical rationale and rules of operation for continuous sampling plan of type CSP-1. In this paper provides various kinds of continuous sampling plans and its operating procedures are compared. Continuous sampling plan is simplest and most commonly used single level Continuous sampling plan. The other continuous sampling plans namely CSP-2, CSP-3, CSP-4, CSP-5, two level CSP, tightened two level CSP, multiple level CSP, tightened multiple level CSP, CSP-F, CSP-T, CSP-T with GERT, three level CSP, modified tightened three level CSP, and CSP-C (Markov chain modal) contain some differences due to the implementation and the theoretical foundation among them.

Keywords: Continuous sampling plan, Mathematical Rationale, Sampling Procedure, Operating procedure, Theoretic foundation.

1. Introduction

Lot acceptance sampling plans (LASP) are used during production to test units submitted for evaluation against certain hypotheses. From a manufacturing perspective, LASP's provide a check on a company's quality control processes. Most LASP samples of a product are carried out in lots. In standard sampling a hypotheses of the sample makes up the criteria by which the process is judged. These units are then accepted or rejected on the basis of the set forth hypothesis. If a process has been tested adequately, then the lot or unit is accepted and passed on to the retailer or customer. If however, quality control is not sufficient, sampling will prevent unacceptable products from leaving the manufacturer. Accepting or rejecting a lot or unit is synonymous with not rejecting or rejecting the null hypothesis in the hypothesis test. Because grouping into lots is not always advantageous, continuous sampling as outlined below, takes a slightly different approach to quality control in manufacturing.

Dodge (1943) has introduced the concept of continuous sampling and provides the mathematical rationale and rules of operation for first continuous sampling plan designated as CSP-1 plan. It is used where product flow is continuous and not easily grouped in lots. Two parameters exist for continuous sampling. One is the frequency (f) and the second is the clearing number (i). The frequency (f) is defined by a number such as 1/2, 1/3, or 1/X. The clearing number (i) is a number such as 30 or 60. A company checks all of its products until 100% of i number of units are inspected and found to be defect free. After 100% of i number of units are found to be defect free 100% inspection is ceased, and one out of every X number of units is checked. The sampling continues until a defect is found. After finding a defect the cycle repeats it until 100% of i number of units has been

found to be defect free. At this point the sample 1/X will begin again. Using Continuous Sampling Carrying out a continuous sampling plan is simple and can be carried out in 3 steps.

1. Inspect all i data.
2. If no defects are found, randomly sample fraction f of data and check again for defects.
3. Whenever a defect is found, correct the flaw and repeat step 1. There are two main parameters to consider when executing a continuous sample.

All other relevant measurements for continuous sampling plan can be derived from these two parameters of sampling frequency for short-run CSP – 1 plan

i = Clearance number for short-run CSP – 1 plan

p = Incoming quality level

pa = probability of accepting incoming unit

$q_i = 1 - p_i$

N = Lot size

2. Operating Procedure of CSP-1

The first continuous sampling plan of Dodge (1943) has two procedures, namely procedure A and procedure B. Procedure A presumes a continuous flow individual units and procedure B is applicable to a product of continuous flow of sub-lots or batches of articles.

2.1 Procedure-A

- At the outset, inspect 100% of the unit consecutively as produced and continue such inspection until i units in succession are found clear of defects.
- When i units in succession are found clear of defects, discontinue 100% inspection and inspect only a fraction f

of the units selecting individual sample units one at a time from the flows of product in such a manner it assures an unbiased sample.

- If a sample unit is found defective, revert immediately to a 100% inspection of succeeding units and continue until again i units in succession are found clear of defects as in paragraph (a).
- Correct or replace all defective units found with good units.

2.2 Procedure-B

- At the outset, start inspection 100% of the unit in a sub lot and continue such inspection unit i inspected units in succession are found clear of defects.
- When i units in succession are found clear of defects, discontinue sub-inspection and inspect only a fraction f of the units from each of the sub lots, selecting the sample units in such a way as to represent the sub lot.
- If a sample units is found defective, start a 100% inspection of the remainder of the sub lot and continue the 100% inspection until again i inspected units in succession are found clear of defects, as in paragraph (a) extending such inspection into succeeding sub lots if necessary.
- If 100% inspection extends into one or more succeeding sub lots and if the units inspected exceeds a fraction f of the units in the sub lot, accept without further inspection and if it is less than f inspect additional units from the same sub lot to make up the sample equal to a fraction f of the number of units in the sub lot.
- Correct or replace all defective units found with good units.

According to Derman et.al (1959), while on a partial inspection “selecting individual sample units one at a time from the flow of units in such a manner as to assume an unbiased sample”. Implies the following three interpretations:

1. Systematic sampling - Sample every K^{th} item
2. Probability sampling - Sample every item with probability f .
3. Random sampling - Sample only a fraction f of the units at random from a segment $1/f$.

The CSP-1 plan is defined by two parameters, f and i which can be changed at will and in general i is an integer and f is a fraction, $0 < f < 1$. The average fraction of total production, $P_a(p)$, accepted or passed on a sampling basis is given

$$P_a(p) = \frac{q^i}{f + (1-f)q^i}$$

Specify f = sampling frequency, i = clearing interval

- Begin 100 % inspection.
- After i units in succession have been found without a defective, start sampling procedure.
- Randomly inspect a fraction of the units.

These formulas apply when defective units found are corrected or replaced by good units. Dodge (1943) has pointed that the replacement of i by $9i-1$ yields the formulas for the non-rectifying assumption.

Dodge(1943) derived AOQL under constant P model as

$$AOQL = P_L = P_m [1 - f\{f + (1-f)(1 - P_m)\}^i]$$

Where P_M is the solution of the equation

$$(i + 1)P_M - 1 = \left(\frac{1}{f} - 1\right) (1 - P_M)^{i+1}$$

Dodge (1943) followed the power series approach to derive performance measures whereas Lieberman and Solomon (1955) followed Markov-Chain methods. Roberts (1955) defined the states for CSP-1 and derived AOQ by solving the resulting Markov-Chain of CSP-1 for equilibrium probabilities of the states.

Dodge and Torrey (1951) proposed two additional continuous sampling plans, CSP-2 and CSP-3, which are modifications of CSP-1 plans.

2.3 Conditions for application

- There is continuous flow of units from the production process and units are offered for inspection one by one in the order of production.
- The process is producing or capable of production materials whose process quality level is stable.
- Sample space, equipments, and work force are provided at or near the site of inspection to permit rapid 100% inspection when required.
- The inspection is relatively easy and quick, eg. or Visual inspection or automatic inspection.
- The inspection is non-destructive since the procedure incorporates 100% screening.
- The sampling procedures can apply to defective units, defect (individual or classes).

Continuous sampling plans are applicable to situations where there is continuous flow of products and these products are submitted for inspection in the order of production.

3. Operating Procedure of CSP-2

- At the outset, inspect 100% of the units consecutively as produced and continue such inspection until i units in succession are found clear of defects.
- When I units in succession are found clear of defects discontinue 100% inspection and inspect only a fraction of the units, selecting individual sampling units one at a time from the flow of product in such a manner it assures an unbiased sample.
- When a defect is found continue the sampling but keep count the no. of units inspected after finding the defect. If a defect is found in the next k or less units inspection is reverted to 100% inspection and if no defect is found in the next k sample units continue the sampling units till the next defect is found, then repeat the same procedure starting from the beginning of the same paragraph.
- CSP-2 plan has been designated with three parameter I , f and k the minimum number of conforming units required between any two non-conforming units. Here we consider $k=i$ so that the number of parameters is reduced to two. The Average fraction of total production accepted on a sampling basis is

$$p_a(p) = \frac{q^i(2 - q^k)}{f(1 - q^k)(1 - q^i) + q^i(2 - q^k)}$$

Where $k = i$

$$p_a(p) = \frac{q^i(2 - q^i)}{f(1 - q^i)(1 - q^i) + q^i(2 - q^i)}$$

The average outgoing quality is

$$AOQ = \frac{p(1 - f)q^i(2 - q^k)}{f(1 - q^k)q^i + q^i(2 - q^k)}$$

Where $k = i$

The procedure is as follows

- Specify f = sampling frequency
- i = clearing interval
- Begin 100% inspection
- After i units in succession have been found without a defective, start sampling procedure.
- Randomly inspect a fraction of the units.
- When a defective is found, continue sampling for k successive sample units.

If no defective is found in the k samples, continue sampling on a normal basis. If a defective is found in the k samples, revert 100% inspection immediately.

4. Operating Procedure of CSP-3

- Dodge and Torrey (1951) derived performances measure u , v , f AOQ(P) and $P_a(p)$ for CSP-2 and CSP-3 plans. CSP-3 plan is a simple improvement of CSP-2 plans where the inspection of four during sampling is restored to when one of the four is found defective. It provides extra production against "spotty quality".
- Operating procedure of CSP-3 is same as CSP-2 except, in addition, being step (3) as follows:
- When a defective is found, inspect the next four units. If a defect is found in the four units, revert to 100 % inspection of succeeding units as in step (1). If no defect is found in the four units continue sampling, but keep count of the number of units inspected after finding the defect.

Dodge and Torrey (1951) proposed the following performance measures and derived them by following approach.

The Average fraction of total production accepted on a sampling basis is

$$p_a(p) = \frac{q^i [1 + q^4(1 - q^k)]}{f(1 - q^{k+4})(1 - q^i) + q(1 + q^{4i}(1 - q^k)) + 4fpq^i}$$

Where $k = i$

The average outgoing quality is

$$AOQ = \frac{p(1 - f)q^i [1 + q^4(1 - q^k)]}{f(1 - q^{k+4})(1 - q^i) + q[1 + q(1 - q^k)] + 4fpq^i}$$

Where $k = i$

The procedure is as follows

- Specify f = sampling frequency
- i = clearing interval
- Begin 100% inspection
- After i units in succession have been found without a defective, start sampling procedure.

- Randomly inspect a fraction of the units.
- When a defective is found, continue sampling for k successive sample units.

If no defective is found in the k samples, continue sampling on a normal basis. If a defective is found in the k samples, revert 100% inspection immediately.

5. Operating Procedure of CSP-4

The practical application of CSP-4 is a production process which alternates between producing all defective items during partial inspection and producing all non-defective items during 100% inspection, will not represent the least favourable case. In fact, for CSP-4 Derman plan required during sampling inspection i consecutive units to be conforming in order that the reduced clearance number.

x can be used during 100% inspection and hence the same clearance number i is used in both 100% inspection and sampling inspection. The modification proposed here is that during sampling inspection k consecutive units need to be found conforming in order to quality for reduced 100% inspection with clearance number x and the CSP-5 plan incorporating this modification is referred to in this plan.

The CSP-5 plan with its parameters are i , f , x , and k . Where $k = i$ this sampling plan becomes Aashim plan. But CSP-4 plan with its parameters are i , f , x , and k . Where $k = (i - k + 1)/k$ sampling plan becomes Derman, Johns and Lieberman (1959) the average fraction of total production, $P_a(p)$, accepted or passed on a sampling basis is given by

$$p_a(p) = \frac{q^i}{q^i + f[1 - q^i + q^{(i-k+1)/k}(q^i - q^x)]}$$

Where $k = (i - k + 1)/k$ The average outgoing quality is given by

$$AOQ(p) = \frac{p(1 - f)q^i}{q^i + f[1 - q^i + q^{(i-k+1)/k}(q^i - q^x)]}$$

Where $k = (i - k + 1)/k$ The procedure is as follows

- Specify f = sampling frequency
- i = clearing interval
- x = clearing interval ($< i$)
- Begin 100% inspection
- After i units in succession have been found without a defective, start sampling procedure
- Randomly inspect a fraction of the units.
- When a defective is found, continue sampling for k successive sample units.

If no defective is found in the k samples, continue sampling on a normal basis. If a defective is found in the k samples, revert 100% inspection immediately.

$$AOQL = \frac{(c_4 + 2) - 2\sqrt{c_4 + 1}}{c_4^2},$$

$$c_4 \neq 0; \frac{1}{4}, c_4 = 0 \text{ where } c_4 = (i - k + 1)/k$$

6. Operating Procedure of CSP-5

- At the outset, inspect 100% of the units consecutively as produced and continue such inspection until 'i' units in succession are found clear of defects.
- When i units in succession are found clear of defects, discontinue 100% inspection and inspect only a fraction f of the units selecting individual sample units one at a time from the flow of product in such a manner as to assure an unbiased sample.
- Continue sampling inspection at rate of until a nonconforming unit is found.
- If a nonconforming unit is encountered during sampling inspection
 - i) After at least i consecutive sample conforming units proceed to 100 inspection with clearance number x, which is less than i.
 - ii) Before reaching i consecutive sample conforming units, proceeding to 100% inspection with clearance number i, replace all non-conforming units with conforming units.

Lieberman and Solomon (1955) considered an extension of CSP-1 plan which

- a) Allows for smoother transition between sampling inspection and 100% inspection.
- b) Requires 100% inspection only when the quality submitted is quite inferior and
- c) Allows for a minimum amount of inspection when quality is definitely good.

A generalized CSP-5 plan is described and is referred to here as general CSP-5 plan. It is a Markov-Chain formulation performance measures and procedure for the construction. A special case of the general CSP-5 plan introduced. In situation where there is no advantage to reducing the sampling frequency upon demonstration of good product quality, reduced inspection can be achieved by using a smaller clearance interval. This is the main feature of CSP-5 plan proposed by Aasheim (1972). It is a single level continuous sampling procedure with parameters i, f and x, the reduced clearance number. Aasheim plan required during sampling inspection i consecutive units to be conforming in order that the reduced clearance number.

x can be used during 100% inspection and hence the same clearance number i is used in both 100% inspection and sampling inspection. The modification proposed here is that during sampling inspection k consecutive units need to be found conforming in order to quality for reduced 100% inspection with clearance number x and the CSP-5 plan incorporating this modification is referred to in this plan.

CSP-5 plan with its parameters are i, f, x, and k. Where k = i this sampling plan becomes Aasheim plan. The average fraction of total production, Pa (p), accepted or passed on a sampling basis is given by

$$p_a(p) = \frac{q^i}{q^i + f[1 - q^i + q^k(q^i - q^x)]}$$

Where k=i

The average outgoing quality is given by

$$AOQ(p) = \frac{p(1-f)q^i}{q^i + f[1 - q^i + q^k(q^i - q^x)]}$$

Where k=i

The procedure is as follows

- Specify f= sampling frequency
- i= clearing interval
- x = clearing interval (<i)
- Begin 100% inspection
- After i units in succession have been found without a defective, start sampling procedure
- Randomly inspect a fraction of the units.
- When a defective is found, continue sampling for k successive sample units.

If no defective is found in the k samples, continue sampling on a normal basis. If a defective is found in the k samples, revert 100% inspection immediately.

7. Comparison of CSP-1 Plan With Other CSP Plans

1. In order to compare the performance of CSP-1 and CSP-2 plans, which are optimal for (CSP-1) p = 0.06 and (CSP-2) 0.09 each one ensuring the same AOQL of 0.05.
2. The exact optimum CSP-2 F > F*, the observed difference with optimum CSP-1 is too small to be of practical importance.
3. Find that in one case amount of inspection for other CSP plans are smaller than that of CSP-1 for higher values of p.
4. Dodge and Torrey expected that for same value of AOQL, inspection under CSP-2 will be less than that under CSP-1 if p is less than some values.
5. Note that for optimum CSP-1 and 2 plans the values of i are more or less same and other plans have some differences.
6. The amount of inspection for optimum CSP-2 is not smaller than the corresponding optimum CSP-1 plan the AOQ curve is better for optimum CSP-2 plan and comparing other sampling plans CSP=5 have better AOQ and OC curve.
7. Another important criterion will be to compare the pt (%) for all types of plans to know the protection offered against a sudden deterioration in quality.

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9. Conclusion

Acceptance sampling is the technique which deals with the procedure in which decisions to accept or reject the lots or process are based on the examination of samples. The work presented in this paper mainly relates to the continuous sampling plans namely CSP-1, CSP-2, CSP-3, CSP-4, and CSP-5 contain some differences due to the implementation, operating procedures and the theoretical foundation among them. Note that for optimum CSP-1 and 2 plans the values of i are more or less same and other plans have some

differences. The amount of inspection for optimum CSP-2 is not smaller than the corresponding optimum CSP-1 plan the AOQ curve is better for optimum CSP-2 plan and comparing other sampling plans CSP=5 have better AOQ and OC curve. Another important criterion will be to compare the p_t (%) for all types of plans to know the protection offered against a sudden deterioration in quality. Find that in one case amount of inspection for other CSP plans are smaller than that of CSP-1 for higher values of p .

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