

# Distributed Space – Time Coding Using Multiple Symbol Differential Detection

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**Abstract:** Differential distributed space-time coding (DDSTC) technique has been considered for relay networks to provide both diversity gain and high throughput in the absence of channel state information. Conventional differential detection (CDD) two-symbol non-coherent detection over slow-fading channels has been examined. It is shown that the performance of CDD severely degrades in fast-fading channels and an irreducible error floor exists at high signal-to-noise ratio region. To overcome the error floor experienced with fast-fading, a nearly optimal “multiple-symbol” differential detection (MSDD) is developed. The MSDD algorithm jointly processes a larger window of received signals for detection and significantly improves the performance of D-DSTC in fast-fading channels. The error performance of the MSDD algorithm is illustrated with simulation results under different fading scenarios.

**Keywords:** Channel state information, Differential distributed space-time coding, Space time coding, Multiple symbol differential detection, relay network..

## 1. Introduction

In distributed space-time coding (DSTC) networks, relays cooperate to combine the received symbols, multiply the results by fixed or variable factors and forward new signals to the destination so that a space-time code can be constructed at the destination. When no channel state information (CSI) is available at the relays and destination, differential DSTC (DDSTC) scheme needs only the second-order statistics of the channels at the relays. The performance of the CDD is around 3-4 dB worse than the performance of its coherent version in D-DSTC networks.

In practice, the high speed of mobile users leads to time selective channels. Thus, the common assumption used in differential detection, namely approximate equality of two consecutive channel uses, is violated. Examining the performance of a D-DSTC system shows that the two-symbol differential detection suffers from a severe performance degradation and a high error floor in fast-fading channels.

To overcome the limitations of two-symbol detection in fast fading channels, in this project, a near optimal multiple-symbol differential detection for the D-DSTC system is developed. Multiple-symbol differential detection (MSDD), first proposed for point-to-point communications in [1], jointly processes a larger window of the received symbols for detection. As the complexity of MSDD increases exponentially with the window size, a multiple-symbol differential sphere detection (MSDSD) algorithm is used to reduce the complexity of multiple-symbol detection. Later, the algorithm was extended to unitary space-time codes for MIMO systems.

## 2. System Model

The wireless relay network under consideration is shown in Fig. 3.1. It has one source, R relays and one destination. Source communicates with Destination via the relays. Each node has a single antenna, and the communication between nodes is half duplex (i.e., each node is able to only send or

receive in any given time). Individual channels from Source to the *i*th relay (S<sub>Ri</sub>) and from the *i*th relay to Destination (R<sub>iD</sub>) are Rayleigh flat-fading and spatially uncorrelated. It is assumed that the variances of all the channels are equal, i.e., channels are symmetric. Information bits are converted to symbols using a modulation technique such as PSK or QAM at Source.

The transmission process is divided into two phases and sending a codeword (or matrix) from Source to Destination in two phases is referred to as one transmission block indexed by *k*. Information symbols are encoded into codeword  $V[k] \in V$ . Before transmission, the codeword is differentially encoded as

$$s[k] = V[k]s[k-1], s[0] = [1 \ 0 \ \dots \ 0]^t \quad (1)$$

Obviously, the length of vector  $s[k]$  is *R*.

In phase I, vector  $\sqrt{P_0}s[k]$  is transmitted by Source to all the relays, where  $P_0$  is the average source power per transmission. The transmitted vector is affected by S<sub>Ri</sub>,  $i = 1, \dots, R$ , channel coefficients which are assumed to be quasistatic during each block but change continuously from block to block. The coefficients of S<sub>Ri</sub>,  $i = 1, \dots, R$ , channels during the *k*-th block are represented by  $q_i[k]$ . Also, the auto-correlation value between two channel coefficients, which are *n* blocks apart, follows the Jakes' fading model.

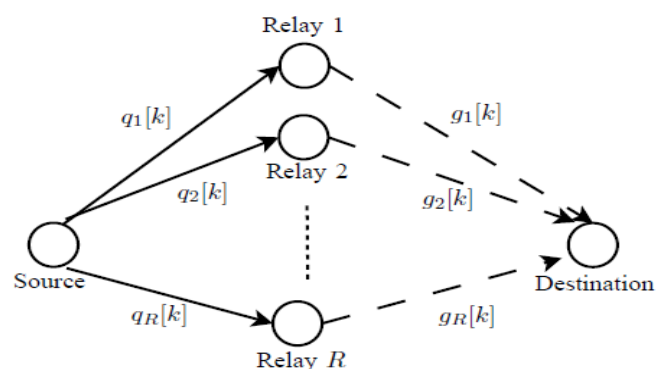


Figure 1: The wireless relay network

The received vector at the  $i$ th relay is  $r_i[k] = \sqrt{(P_0R)} q_i[k]s[k] + u_i[k]$  where  $u_i[k] \in CN(0, N_0IR)$  is the noise vector at the  $i$ th relay. The received vector at the  $i$ th relay is linearly combined with its conjugate as

$$x_i[k] = c_i (A_i r_i[k] + B_i r_i^*[k]) \quad (2)$$

where  $A_i$  and  $B_i$  are the combining matrices and determined based on the space-time code that is used for the network. Usually, one matrix is chosen as a unitary matrix and the other one is set to zero. Also,  $c_i$  is the amplification factor at the relay that can be either fixed or varying. A variable  $c_i$  needs the instantaneous CSI. For D-DSTC, in the absence of CSI, the variance of SR channels (here normalized to one) is utilized to define a fixed amplification factor as

$$c_i = \sqrt{P_i / (P_0 + N_0)} \quad (3)$$

where  $P_i$  is the average power per symbol of the  $i$ th relay. For a given total power in the network,  $P$ , for symmetric Source-Relay (SR) and Relay-Destination (RD) channels,  $P_0 = P/2$  and  $P_i = P/(2R)$  form the optimum power allocation between Source and the relays to minimize the pair wise-error probability (PEP). Hence, the amplification factor  $c = c_i = \sqrt{P/(R(P + 2N_0))}$ ,  $i = 1, \dots, R$ , is chosen for all the relays.

Again, using the quasi-static assumption, the coefficients of  $R_iD$ ,  $i = 1, \dots, R$ , channel during the  $k$ -th block are represented by  $g_i[k] \in CN(0, 1)$ . The corresponding received vector at Destination is

$$y[k] = \sum_{i=1}^R g_i[k] x_i[k] + z_i[k] \quad (4)$$

where  $z_i[k] \in CN(0, N_0IR)$  is the noise vector at Destination. Therefore,

$$Y[k] = c \sqrt{(P_0R)} S[k] h[k] + w[k] \quad (5)$$

where  $S[k]$ ,  $h[k]$  and  $w[k]$  are the distributed space-time code, the equivalent cascaded channel vector and the equivalent noise vector, respectively, defined as

$$S[k] = [\hat{A}_1 \hat{s}_1 \dots \hat{A}_R \hat{s}_R] = V[k] S[k-1] \quad (6)$$

$$h[k] = [h_1[k] \dots h_R[k]]^t$$

### 3. Two-Symbol Differential Detection

Coherent detection of transmitted codeword is possible with the knowledge of both SR and RD channels. In the absence of channel information, in the conventional D-DSTC system, it is assumed that the channels are fixed for two consecutive block uses. In this case  $h[k] \approx h[k-1]$  and then

$$y[k] = V[k] y[k-1] + w[k] - V[k] w[k-1] \quad (7)$$

Given  $y[k]$  and  $y[k-1]$ , differential non-coherent detection is applied to detect the transmitted codeword as

$$V[k] = \arg \min \|y[k] - V[k] y[k-1]\| \quad (8)$$

Comparing (7) and (8) reveals that the equivalent noise power is enhanced by a factor of two, which explains why

the differential non-coherent detection performs approximately  $10 \log_{10} 2 \approx 3$  dB worse than coherent detection in slow fading channels.

However, slow-fading assumption requires a coherence interval of  $3R$  for both SR and RD channels which would be violated for fast-fading channels. Instead, consider that the channel vector  $h[k]$  change by  $\Delta h$  during two consecutive block uses, i.e.,  $h[k] = h[k-1] + \Delta h$ . Therefore, one has

$$y[k] = V[k] y[k-1] + \hat{w}[k], \quad (9)$$

$$\hat{w}[k] = w[k] - V[k] w[k-1] + c \sqrt{(P_0R)} S[k] \Delta h \quad (10)$$

As it can be seen, the equivalent noise power is enhanced by an additional factor which is related to the transmitted power and also the amount of channel variation. This means that a higher degradation in the performance of two-symbol differential detection would be seen in fast-fading channels. Specially, in fast-fading channels, the equivalent noise is dominated by the last term of (10), and an error floor appears at high signal-to-noise ratio.

### 4. Multiple-Symbol Detection

Two-symbol differential detection suffers from large performance degradation in fast-fading channels. To overcome such a limitation, multiple-symbol differential detection scheme is developed that takes a window of the received symbols at the destination for detecting the transmitted signals.

$$y[k] = c \sqrt{(P_0R)} S[k] h[k] + w[k] = c \sqrt{(P_0R)} S[k] G[k] q[k] + w[k] \quad (11)$$

where  $G[k] = \text{diag}\{g_1[k], \dots, g_R[k]\}$  and  $q[k] = [q_1[k], \dots, q_R[k]]^t$ .

Let the  $N$  received symbols be collected in vector  $y = [y^t[1], y^t[2], \dots, y^t[N]]^t$  which can be written as

$$\bar{y} = c \sqrt{(P_0R)} S' h' + w' = c \sqrt{(P_0R)} S' G' q' + w' \quad (12)$$

$$S' = \text{diag}\{S[1], \dots, S[N]\},$$

$$h' = [h^t[1], \dots, h^t[N]]^t,$$

$$G' = \text{diag}\{G[1], \dots, G[N]\},$$

$$q' = [q^t[1], \dots, q^t[N]]^t,$$

$$w' = [w^t[1], \dots, w^t[N]]^t$$

It should be mentioned that  $N$  transmitted symbols collected in unitary block diagonal matrix  $S$  correspond to  $N-1$  data symbols collected in  $V = \text{diag}\{V[1], \dots, V[N-1]\}$  such that

$$S[n+1] = V[n] S[n], \quad n = 1, \dots, N-1 \quad (13)$$

and  $S[N] = IR$  is set as the reference symbol.

Therefore, conditioned on both  $S'$  and  $G'$ ,  $y'$  is a circularly symmetric complex Gaussian vector.

The maximum likelihood (ML) detection of  $N$  transmitted symbols collected in  $S$  or the corresponding  $N-1$  data symbols collected in  $V$  would be given as:

$$\hat{V} = \arg \max_{V \in \mathcal{V}^{N-1}} \left\{ \frac{1}{\pi^N \det\{\Sigma_{\bar{y}}\}} \exp\left(-\bar{y}^H \Sigma_{\bar{y}}^{-1} \bar{y}\right) \right\} \quad (14)$$

where

$$\hat{\mathbf{V}} = \text{diag} \{ \hat{\mathbf{V}}[1], \dots, \hat{\mathbf{V}}[N] \}.$$

The minimization can be solved using the sphere decoding algorithm to obtain  $N - 1$  data symbols with low complexity.

### 5. Simulation Results

In this section a relay network with one source,  $R = 2$  relays and one destination is simulated in different fading scenarios while both two-symbol and multiple-symbol detection schemes are applied. The Alamouti space-time code is chosen for the network. The combining matrices at the relays are designed as

$$\mathbf{A}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{B}_1 = \mathbf{0}, \mathbf{A}_2 = \mathbf{0}, \mathbf{B}_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

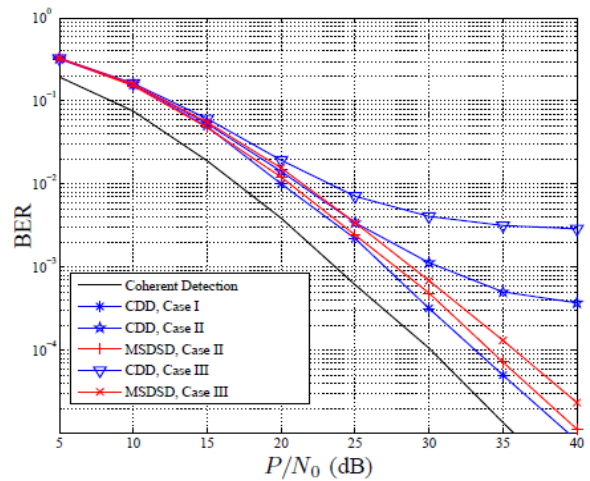
Also, the set of unitary codewords are designed as

$$\mathcal{U} = \left\{ \frac{1}{\sqrt{2}} \begin{bmatrix} u_1 & -u_2^* \\ u_2 & u_1^* \end{bmatrix} \mid u_i \in M\text{-PSK}, i = 1, 2 \right\}.$$

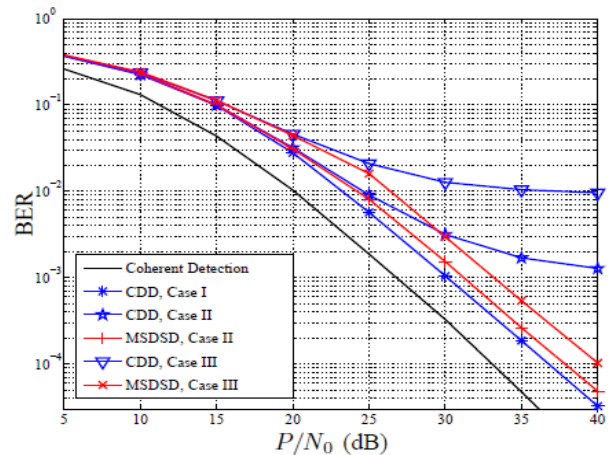
The amplification factor at the relays is fixed to  $c = \sqrt{(P_i / (P_0 + N_0))}$  to normalize the average relay power to  $P_i$ . The power allocation between Source and the relays is such that  $P_0 = P/2$  and  $P_i = P/4$ ,  $i = 1, 2$ , where  $P$  is the total power in the network.

This simulation method has been developed to generate time-correlated fix-to-mobile channel coefficients. The amount of time correlation between the coefficients is determined by the normalized Doppler frequency which is actually a function of the velocity of the mobile user. It is assumed that relays are fixed and then based on the mobility of Source and Destination different cases can be considered. In Case I, it is assumed that the mobility of Source and Destination is low such that all the channels are slow fading or approximately static and the normalized Doppler frequencies of SR and RD channels are set to .001. In Case II, Source and Destination are moving and Source has a slightly higher mobility than Destination such that the normalized Doppler frequency of SR and RD channels are  $f_{sr} = .006$ ,  $f_{rd} = .004$ . In Case III, it is assumed that Source and Destination are moving faster and Destination has a slightly higher mobility than Source such that  $f_{sr} = .009$ ,  $f_{rd} = .01$ .

To evaluate the BER of the system, in each case, binary data is converted to BPSK/QPSK constellation and then to unitary codewords. Next, the codewords are encoded differentially. At Destination, two consecutive received codewords are used to estimate the transmitted symbols using the two-symbol differential detection. The simulation is run for various values of the total power in the network. The practical values of the BER are computed for all cases and plotted versus  $P/N_0$ .



**Figure 2:** CDD and MSDSD techniques for D-DSTC relaying with two relays in different fading scenarios using Alamouti code and QPSK



**Figure 3:** CDD and MSDSD techniques for D-DSTC relaying with two relays in different fading scenarios using Alamouti code and BPSK

For CDD in Case I, with slow-fading channels, the error probability is monotonically decreasing with  $P/N_0$  and the desired cooperative diversity is achieved for the D-DSTC system. However, in CDD Case II and III, with fairly fast-fading channels, the BER plot gradually deviates from the results in Case I. Given the poor performance of the CDD in Case II and III, MSDSD-DSTC algorithm is applied to Case II and Case III. The BER results of MSDSD-DSTC algorithm are also plotted. Since the best performance is achieved in the slowfading environment, the BER plot of Case I can be used as a benchmark to see the effectiveness of MSDSD-DSTC algorithm.

### 6. Conclusion

This paper has shown that using two-symbol detection for differential distributed space-time coding in relay networks fails to provide a satisfactory performance in fast-fading channels. A near optimal multiple-symbol differential detection algorithm was then developed that can be implemented using sphere decoding with low complexity. Simulation results has illustrated that the multiple-symbol detection significantly improves performance of the differential distributed space-time coding systems in fast-fading channels.

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## Author Profile

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