

being modelled here are European call options for monthly rainfall. The contracts are based on a single site, the Dagorreti weather in Nairobi, Kenya. And the rainfall is the total daily are recorded by the Kenya Meteorological Authority.

In this article we use daily prices and rainfall with monthly averages. The rainfall data used consist of monthly averages in mm for Nairobi city from January 2002 to December 2012. and the stock market share price is the daily share price for (Kenya Power (KPLC) for the period January 2002 to December 2012

By assuming a constant MPR risk, we estimate the esscher transform of the gamma distribution. In fact, we observe that the esscher distribution also transforms the mean and variance of the distribution.

Theoretical prices of the monthly rainfall options under Q are estimated by using the $MPR \neq 0$ in the simulated rainfall process. This approach advantageous since we can pick any non-zero values for θ .

The monthly values for θ are obtained from the parameters estimates of the monthly rainfall and our choice of interest rate which in our case is $r = 0.11$ which is a bond market rate in Kenya.

We remark that we are not able to compare the theoretical prices with the actual market prices since derivative do not trade in the Kenyan market. We observe from the Table 1.0 that, with increase in the strike price K, there is a corresponding increase in the payoff.

4. Conclusion

In this article, we have presented a method on how to calculate risk-neutral prices of rainfall derivatives. A standard rainfall model is used to simulate the rainfall process. Then the process distribution is shifted by the Esscher transform to obtain neutral prices.

Rainfall derivatives do not trade in the Kenyan market. Therefore the reported prices are actually hypothetical prices since they are not from actual trading. Hopefully, in the near future, when derivative trading get established, similar approaches can be used to investigate the behaviour of rainfall derivatives and the nature of the market price of risk. Our calculation can be used for daily trading to analyse temporal behaviour of market price of risk and spatial behaviour among different regions in the country.

Table 1: EsscherPrices, $t=0.25, r=0.11, S(0)=100$

K/Month	Jan	Feb	Mar	Apr	May	Jun
80	35.724	35.724	52.224	35.724	35.724	35.724
90	30.908	30.908	58.658	33.158	30.908	30.908
100	32.093	32.093	65.093	35.093	32.093	26.093
110	42.277	43.777	75.277	45.277	43.777	24.350

120	45.717	46.467	82.461	50.217	46.467	29.450
130	50.775	50.803	80.646	55.974	50.804	36.635
140	58.247	59.094	77.330	65.557	59.094	39.733
150	69.495	69.717	77.811	74.837	69.638	46.918

K/Month	Jul	Aug	Sept	Oct	Nov	Dec
80	37.974	35.724	35.724	35.724	35.724	35.724
90	38.408	30.927	33.908	33.908	33.158	30.908
100	50.093	26.264	43.343	43.343	35.093	26.103
110	54.282	24.107	47.527	47.527	45.277	25.137
120	58.533	24.531	53.217	53.217	47.999	31.054
130	71.856	38.337	58.275	58.275	53.845	36.770
140	78.098	66.449	73.136	73.136	59.958	40.483
150	79.737	00.000	74.906	74.906	71.487	49.081

References

- [1] Alaton P, Djehiche B, Stillberger D (2002) On Modelling and Pricing Weather Derivatives. *Applied Mathematical Finance* 9: 1-20
- [2] Bolin, L (2002) All credit to South Africa. *Futures and Options World*, July 2002
- [3] Cao M, Li A, Wei J (2004) Precipitation Modelling and Contract Valuation: A frontier in Weather Derivatives. *The Journal of Alternative Investments* 7: 93-99
- [4] Carmona R, Diko P (2005) Pricing Precipitation based Derivatives. *International Journal of Theoretical and Applied Finance* 8: 959-988
- [5] Cooper, I (2001) Talk about the Weather. *Risk Professional*, July / August 2001
- [7] Douglas J (2002b) Containing the Weather. *Futures and Options World*
- [8] Gerber, H.U, Shiu E.S.W (1994) Option Pricing by Esscher Transform. *Transactions of Society of Actuaries*, pp. 99-192.
- [9] Geysler, J.M (2002) Weather derivatives, concept, application and analyses. <http://www.wupacza/academic/ecoagric>
- [10] Geysler J.M, Van de Venter T.M (2001) Hedging maize yield with weather derivatives. <http://www.wupacza/academic/fulltext>
- [11] Hardle, W.K, Ospienko M (2011) Pricing Chinese rain: A multi-site Multi-period Equilibrium Pricing Model for Rainfall Derivatives. *Humboldt-Universitat zu Berlin*
- [12] Lee Y, Oren S (2010) A Multiperiod Equilibrium Pricing Model of Weather Derivatives. *Energy systems* 1: 3-30
- [13] Leobacher G, Ngare, P (2011) On Modelling and Pricing Rainfall Derivatives with Seasonality. *Applied Mathematical Finance* 18: 71-91
- [15] Richards T.J, Manifredo M.R, Sanders D.R (2004) Pricing Weather Derivatives. *Arizona State University*
- [16] Turvey C (2002) Weather Derivatives For Specific Event Risks in Agriculture. *Review of Agricultural Economics* 23: 333-351

Appendix

Proof for proposition 2.6.1

$$\begin{aligned} \text{for } M(u + \theta) &= \int_0^\infty e^{(u+\theta)y} \frac{1}{\beta^\alpha \Gamma_\alpha} y^{\alpha-1} e^{-\frac{y}{\beta}} dy \\ &= \frac{1}{\beta^\alpha \Gamma_\alpha} \int_0^\infty y^{\alpha-1} e^{-\left(\frac{1-\beta(u+\theta)}{\beta}\right)y} dy = \frac{1}{\beta^\alpha \Gamma_\alpha} \int_0^\infty y^{\alpha-1} e^{-\left(\frac{1-\beta(u+\theta)}{\beta}\right)y} dy * \left[\frac{1-\beta(u+\theta)}{\beta} \right]^{\alpha-1} \\ &= \left(\frac{\beta}{1-\beta(u+\theta)} \right)^{\alpha-1} \frac{1}{\beta^\alpha \Gamma_\alpha} \int_0^\infty \left(\frac{1-\beta(u+\theta)y}{\beta} \right)^{\alpha-1} e^{-\left(\frac{1-\beta(u+\theta)}{\beta}\right)y} dy \quad (a) \end{aligned}$$

Now let $y_* = \left(\frac{1-\beta(u+\theta)}{\beta} \right) y$

Therefore $dy = \left(\frac{\beta}{1-\beta(u+\theta)} \right) dy_*$

Equation (a) becomes

$$\left(\frac{\beta}{1-\beta(u+\theta)} \right)^{\alpha-1} \frac{1}{\beta^\alpha \Gamma_\alpha} \int_0^\infty (y_*)^{\alpha-1} e^{-y_*} \left(\frac{\beta}{1-\beta(u+\theta)} \right) dy_* \quad (b)$$

Recall: $\Gamma_\alpha = \int_0^\infty y^{\alpha-1} e^{-y} dy$

$$\begin{aligned} \text{This leads (12) to } &\left(\frac{\beta}{1-\beta(u+\theta)} \right)^{\alpha-1} \frac{1}{\beta^\alpha \Gamma_\alpha} \left(\frac{\beta}{1-\beta(u+\theta)} \right) \Gamma_\alpha \\ &= \left(\frac{1}{1-\beta(u+\theta)} \right)^\alpha \quad (c) \end{aligned}$$

$$\text{And for } M(\theta, 1) = \int_0^\infty e^{\theta y} \frac{1}{\beta^\alpha \Gamma_\alpha} y^{\alpha-1} e^{-\frac{y}{\beta}} dy$$

$$= \frac{1}{\beta^\alpha \Gamma_\alpha} \int_0^\infty y^{\alpha-1} e^{-\frac{(1-\beta\theta)y}{\beta}} dy$$

$$= \frac{1}{\beta^\alpha \Gamma_\alpha} \int_0^\infty y^{\alpha-1} e^{-\frac{(1-\beta\theta)y}{\beta}} dy * \left(\frac{1-\beta\theta}{\beta} \right)^{\alpha-1}$$

$$= \frac{1}{\beta^\alpha \Gamma_\alpha} \left(\frac{\beta}{1-\beta\theta} \right)^{\alpha-1} \int_0^\infty \left(\frac{1-\beta\theta}{\beta} y \right)^{\alpha-1} e^{-\frac{(1-\beta\theta)y}{\beta}} dy$$

Let $y_* = \frac{y(1-\beta\theta)}{\beta}$, then $dy = \left(\frac{\beta}{1-\beta\theta} \right) dy_*$

Which now gives rise to, as above,

$$\begin{aligned} &= \frac{1}{\beta^\alpha \Gamma_\alpha} \left(\frac{\beta}{1-\beta\theta} \right)^{\alpha-1} \int_0^\infty (y_*)^{\alpha-1} e^{-y_*} \left(\frac{\beta}{1-\beta\theta} \right) dy_* \\ &= \left(\frac{\beta}{1-\beta\theta} \right)^{\alpha-1} \frac{1}{\beta^\alpha \Gamma_\alpha} \left(\frac{\beta}{1-\beta\theta} \right) \Gamma_\alpha = \left(\frac{1}{1-\beta\theta} \right)^\alpha \quad (d) \end{aligned}$$

Dividing (c) by (d) completes the proof.