

Modelling and Pricing Rainfall Derivatives to Hedge on Weather Risk in Kenya

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Abstract: *Weather risk is an unmitigated source of financial losses in developing economies. There is need to model this type risk in order to mitigate and reduce losses associated with the weather. In this article, we model the rainfall process at a particular location in Kenya using a Markovian Gamma distribution whose parameters are estimated by way of maximum likelihood. The derivatives prices are estimated by making use of the Esscher transform. The obtained prices are adjusted by calibrating the market price of rainfall risk. The empirical analysis is conducted using Kenyan precipitation and stock market data.*

Keywords: Esscher transform, Weather Derivatives, Markovian distribution, Equivalent Martingale, Market Price of Risk

1. Introduction

Weather conditions greatly affect business revenues in an array of industries. The weather influence presents both challenges that are adverse with huge losses and opportunities for the emergence, development and growth of the financial instruments like weather derivatives. Businesses use weather derivatives to hedge on their risks in order to make trading profits. Weather exposure can be hedged just the same way as currency exposure.

Weather conditions like rainfall, temperature, frost, snow are always unpredictable. Moreover, by being so unpromising and their patterns being abnormal over the decades, many industries become victims of the weather in many ways (Geysler, Van der enter, 2001).

Over the years, the agricultural based businesses have used futures contracts of agricultural commodities to hedge on weather related risks. However, traditional methods cannot cover a number of weather risks. This has given rise to the emergence of a more robust, flexible financial instrument called the weather Derivatives (Geysler, 2002).

A weather derivative is a contract. It stipulates how payment will be settled between the parties involved based on the prevailing meteorological conditions during the contract period (Leobacher and Ngare, 2011). They are usually typically swaps, futures and options based on different underlying weather measures (Alaton, Djehiche and Stillberger, 2002). Weather derivatives, unlike traditional derivatives, have no underlying tradable instrument or stock. Therefore, they cannot be used to hedge price risk since the weather itself cannot be priced. Instead, they hedge against volumetric or quantity risk associated with weather conditions.

The weather derivatives market is well developed in the United States of America (USA) with the energy industry players being the leading participants. There is equally rapid growth in Asia and Europe. (Douglas-Jones, 2002) Closer

home, in Africa, businesses in the Agriculture and related industries have developed interest in the weather derivatives. Agriculture is largely unsubsidized and the energy industry is hugely regulated. Maize and wheat growers, silo owners, transport companies, sugar industry, fishing as well as insurance companies in South Africa have taken advantage of this new instrument (Bolin, 2002). Agriculture, clothing, construction, hospitality and outdoor entertainment industries which are highly weather sensitive and whose revenues and productivity are closely correlated with weather conditions are the ones heavily affected by harsh weather in the region.

The hospitality, tourism and entertainment industries are mostly busy during summer, the same time that most of Africa receives its rainfall. Thus, the attendance figures plummet. The construction industry is heavily hit in financial terms for projects that run beyond completion deadlines since operation of heavy machinery and working outdoor during rainy conditions is rather difficult. The clothing industry is equally dictated by weather since weather conditions dictate what people buy and wear. For example, during winter, sweater and jacket products experience faster sales unlike during a milder than normal winter. In agriculture, weather conditions affect the quality and quantity of produce (Geysler, 2002).

The emergence of weather derivatives in the agriculture sector has particularly drawn considerable interest from international institutions like World Bank, International Finance Corporation (IFC) and International Monetary Fund (IMF). In the developing world, farmers are never covered by government sponsored insurance programs yet the weather risk is most prevalent in devastating scales. The weather derivatives can therefore provide a sure way of protecting them against risk of drought and poor harvest or any other weather related risk.

According to Cooper (2001) a large portion of South America's economy relates to growing commodities and selling them to the world market. In Brazil for instance,

coffee harvest can be adversely affected by bad weather conditions, which will in turn have considerable effects on the economy. Weather is an essential factor of production which is, however, uncontrollable. It therefore poses risks that are the major sources of uncertainty in production and income flows. Weather derivatives can bring added stability.

Weather derivatives have the advantage that they are not affected by moral hazard or adverse selection, which may be a serious problem for insurance companies. However, a considerable risk may remain with the producer when using weather derivatives, because individual yield variations in general are not completely correlated with the relevant weather variable (Oliver et al, 2006).

Available literature increasingly deals with the question if weather derivatives can also play a role as risk management tools in agriculture, (Richards et al, 2004; Turvey 2001).

Cao et al (2004) proposed a pricing model for rainfall based on daily rainfall in which they calculated a fair premium while ignoring market price of rainfall risk. Carmona and Diko (2005) proposed a Markov process model for the rainfall process for stochastic dynamics of the underlying precipitation. They assumed the existence of tradable rainfall assets and used utility indifference approach to price the derivatives.

Leobacher and Ngare (2011) on improving Carmona and Diko (2005) did construct a Markovian Gamma model for rainfall process with seasonal effects and gives utility indifference prices with exponential utility. Lee and Oren (2010), Hardle and Ospienko (2011) obtained equilibrium prices for weather derivatives on cumulative monthly rainfall by simulating market conditions of two types-farmers with profits exposed to weather and financial market investors aiming to diversify their financial portfolios. The aim of this paper is to develop a pricing framework of rainfall derivatives that takes into account market price of risk. We rely on the Esscher transform to construct an equivalent martingale measure with which we calculate the arbitrage free prices of European call options whose underlying is a rainfall process.

Classical arbitrage theory assumes that stocks can accurately replicate options on tradable assets. However, for derivatives on weather conditions like rainfall, temperature indices cannot rely on hedging principles since the underlying cannot be traded. Therefore, with the market being incomplete there ought to be many equivalent martingales to price rainfall derivatives. Moreover, these derivatives should be arbitrage free since they are indeed tradable.

The outline of the paper is as follows: Section 2 describes our method for pricing rainfall derivatives, includes the model for monthly rainfall and information on the Esscher transform. Section 3 includes the data applied to our approach to calculate theoretical prices for options on monthly rainfall with constant market price of risk from the

rainfall data. Section 4 provides a discussion and conclusion. All computations were carried out in R version 3.1.1.

2. Methods

2.1 General Framework.

The weather derivatives market is an incomplete market. The underlying weather process is not a tradable asset and thus cannot be replicated by other risk factors. In this paper we find arbitrage free prices whose underlying is a rainfall process by using an equivalent martingale measure via the Esscher transform with a constant market price of risk. We calculate the prices under the risk neutral probability measure Q which we consider equivalent to the physical probability measure P . Since the market is incomplete, there are many equivalent martingales Q . We need an extra parameter θ , the market price of risk. Since the assumed distribution is nonnormal, an esscher transform of the distribution is performed with constant market price of risk.

2.2 Daily Rainfall Model

Carmona and Diko (2000) proposed a time homogenous jump markov process to model the rainfall process. To price the derivatives, they assumed the existence of tradable asset whose price depended on rainfall and relied on the utility indifference method to price the derivatives. This model was later to be improved by Leobacher and Ngare (2011) who constructed a Markovian- Gamma model for rainfall process which accounts for the seasonal effects of rainfall and calculates utility indifference prices with exponential utility.

We intend to exploit this model together with the Escher transform with constant market price rainfall risk to calculate the prices using an equivalent martingale measure. To account for the seasonal effects of rainfall over a given period Leobacher and Ngare (2011) partitioned the period under consideration into equal sub-periods and separately modelled the total amount of rainfall within each sub-period.

By letting Y_0, Y_1, Y_2, \dots to be the sequence of total rainfall per sub-period, they assumed that in some sub-period k , the rainfall has a cumulative distribution function (CDF), $F_{k \text{ mod } m}$, $k \geq 0$ where F_k is a continuous function and strictly increasing such that the inverse, F_k^{-1} exists and that it is strictly increasing and continuous.

The assumptions above indicate that the sequence $(F_{k \text{ mod } m}(Y_k)), k \geq 0$ constitutes of generally dependent random variables U_k uniform on $(0, 1)$ which can generate a future rainfall sample path by setting $Y_k := F_{k \text{ mod } m}^{-1}(U_k), k \geq 0$.

The sequence $F_{k \text{ mod } m}(Y_k)$ is a discrete-time Markov process with state space $(0, 1)$ and therefore rainfall amounts of two consecutive months or even the days are not independent.

We assume that the rainfall within sub-period k follows a gamma distribution with shape and scale parameters α and

β respectively. Therefore, for some standard normal random variable Z ,

$$F_k(z) = \int_{-\infty}^z f(y) dy, \quad \text{where}$$

$$f_k(y) = \begin{cases} \frac{1}{\beta_k \Gamma \alpha_k} \left(\frac{y}{\beta_k}\right)^{\alpha_k-1} e^{-y/\beta_k}, & y > 0, \alpha_k, \beta_k > 0 \\ 0, & y < 0 \end{cases} \dots \dots (1)$$

2.3 Parameter Estimation

Suppose we are given data y_0, y_1, \dots, y_{n-1} of precipitation at a specific location with m observations per year. We want to fit our model to actual data, that is, we want to find a set of parameters $\alpha_0, \dots, \alpha_{m-1}, \beta_0, \dots, \beta_{m-1}$ such that if F_k is the CDF of a gamma distribution with parameters α, β for each k , then y_0, \dots, y_{n-1} has the maximum likelihood. If the observations are monthly then $m = 12$.

We are supposed to estimate (α_k, β_k) for every month using ordinary maximum likelihood estimation framework. We note that consecutive observations are hardly independent; no big error can be expected by assuming that consecutive Januaries are independent.

From this we would have estimated that the CDFs, F_0, F_1, \dots, F_{m-1} and thus can compute $\Phi^{-1}(F_{k \text{ mod } m}(y_k)) := z_k$. This view is also shared by Leobacher and Ngare (2011).

2.4 MLE for Gamma Distribution Using data Containing Zeros

Wilks (1990), Leobacher and Ngare (2011) constructed their models using the method below.

Suppose the given data set contains M_0 censored data points (recorded as zeros) of some censoring level for example 0.1 and M_v points with known values where $M = M_0 + M_v$. Then the likelihood function for the distribution parameters is given by:

$$Y(\alpha, \beta; y) = \prod_{j=1}^{M_0} G(A; \alpha, \beta) \prod_{i=1}^{M_v} g(y_i; \alpha, \beta) = [G(A; \alpha, \beta)]^{M_0} \prod_{i=1}^{M_v} \frac{1}{\beta \Gamma \alpha} \left(\frac{y_i}{\beta}\right)^{\alpha-1} e^{-y_i/\beta}, \text{ where} \quad (1)$$

$$G(A; \alpha, \beta) = \int_0^A g(y_i; \alpha, \beta) dy = \mathbb{P}[y_j \leq A]$$

If we assume that $M_0=0$ that is, all the data values are known, then the MLE of the parameters satisfy:

$$\log(\beta) + \varphi(\alpha) = \sum_{i=1}^{M_v} \frac{\log y_i}{M_v}$$

$$\alpha - \frac{1}{\beta} \sum_{i=1}^{M_v} \frac{y_i}{M_v} = 0$$

where $\varphi(\alpha) = \frac{d \log \Gamma \alpha}{d \alpha}$ is the digamma function. Hence the MLE for β and α can be determined. In the case where $M_0 \neq 0$, then

$$L(\alpha, \beta; y) = M_0 \log[G(A, \alpha, \beta)] - M_v [\alpha \log(\beta) + \log(\alpha)] + (\alpha - 1) \sum_{i=1}^{M_v} \log y_i - \frac{1}{\beta} \sum_{i=1}^{M_v} y_i \quad (2)$$

(2) can be evaluated numerically for the values of α and β using any of the available mathematical software like R.

2.5 The Market Model

We choose a market model given by the stochastic differential equation:-

$$dS_t = \mu(Y_t) S_t dt + \sigma(Y_t) S_t dZ_t \quad (3)$$

where $Z_t \sim iid N(0,1)$ and $\mu(Y_t)$ and $\sigma(Y_t)$ are measurable functions whose concrete form we are yet to determine.

However we can take $\sigma(Y_t)$ as a constant and evaluate $\mu(Y_t)$ as:

$$\mu(Y_t) = a \log(Y_t) + b, Y_t > 0 \quad (4)$$

The parameters a and b are estimated by the maximum likelihood estimation framework by combining both the market and rainfall data. That is, given the rainfall records $y_0, y_1, \dots, y_{t-1}, y_t$ and asset prices $S_0, S_1, \dots, S_{n-1}, S_n$ of some hypothetical asset, we can set $\mathfrak{Y}_t = \log(Y_t)$ almost surely and

$$dS_t = S_t - S_{t-1}, \text{ such that}$$

$$dS_t = (a \mathfrak{Y}_t + b) S_t dt + \sigma S_t dZ_t$$

$$dS_t = \mu S_t dt + \sigma S_t dZ_t$$

$$\int \frac{dS_t}{S_t} = \mu t + \sigma Z_t, Z_t \sim iid N(0,1) \quad (5)$$

Rewriting (5) gives:

$$\frac{\ln S_t - \mu t}{\sigma} = Z_t \sim N(0,1)$$

$$f(z, t) = \frac{1}{\sqrt{2\pi}} \exp - \frac{1}{2} [Z_t]^2$$

$\prod_{t=1}^n \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2} [P_t^2 - 2P_t \mu t + \mu^2 t^2]\right]$ (6)
 we maximize the log likelihoods of (6) to obtain the expressions below as solutions for a and b and σ :

$$a = \frac{\beta \delta - \gamma \alpha}{\varepsilon \delta - \gamma^2} \quad \text{and} \quad b = \frac{\alpha \varepsilon - \beta \gamma}{\varepsilon \delta - \gamma^2}$$

$$\sigma^2 = \tau + a^2 \varepsilon + 2ab\gamma + b^2 \delta - 2a\beta - 2ba$$

where: $n\alpha = \sum_{t=0}^{n-1} t P_t$, $n\varepsilon = \sum_{t=0}^{n-1} t^2 K_t^2$, $n\beta = \sum_{t=0}^{n-1} t P_t K_t$, $n\gamma = \sum_{t=0}^{n-1} t^2 K_t$, $n\delta = \sum_{t=0}^{n-1} t^2$, $n\tau = \sum_{t=0}^{n-1} P_t^2$, $P_t = \ln S_t = \log\left(\frac{S_t}{S_{t-1}}\right)$

2.6 Equivalent Martingale Measure Q Using the Esscher Transform

We study a risk neutral distribution, or equivalently, martingale measure associated with a Markovian gamma process. A discrete-time martingale is a stochastic process x_1, x_2, \dots that satisfies for all n ;

$$E(|X_1|) < \infty$$

and $E(|X_{n+1}| | X_1, X_2, \dots, X_n) = X_n$
 Gerber and Shiu (1994) proposed one approach for finding an equivalent martingale measure using the Esscher transform. Accordingly, given a statistical model \mathbf{P} , the Esscher transform induces an equivalent probability

measure \mathbf{Q} and a martingale process. The Esscher parameter is determined so that the discounted asset price is a martingale under the new probability measure \mathbf{Q} .

Letting

$$S_t = S_0 e^{X_t}, \quad (7)$$

where $\{X_t\}_{t \geq 0}$ is a process with stationary and independent increments and $X_0 = 0$ then for each t the random variable X_t has an infinitely divisible distribution with probability density given by:

$$f(x, t), t > 0 \quad (8)$$

In addition, the moment-generating function, assumed to exist, is defined as-

$$M(u, t) = E[e^{uX_t}] = \int_{-\infty}^{\infty} e^{ux} f(x, t) dx \quad (9)$$

Assuming that $M(u, t)$ is continuous at $t = 0$, then by infinite divisibility:-

$$M(u, t) = [M(u, 1)]^t \quad (10)$$

Let θ be a real number such that $M(\theta) = \int_{-\infty}^{\infty} e^{\theta x} f(x) dx$ exists, then the Esscher transform of $\{X_t\}_{t \geq 0}$ with parameter θ is defined as a process with stationary and independent increments where the new probability density of $X_t, t > 0$, is:

$$f(x, t; \theta) = \frac{e^{\theta x} f(x, t)}{\int_{-\infty}^{\infty} e^{\theta y} f(y, t) dy} = \frac{e^{\theta x} f(x, t)}{M(\theta, t)}$$

The modified distribution of X_t is the Esscher transform of the original distribution whose moment-generating function given by :-

$$M(u, t; \theta) = \int_{-\infty}^{\infty} e^{ux} f(x, t; \theta) dx = \frac{M(u + \theta, t)}{M(\theta, t)}$$

and

$$M(u, t, \theta) = [M(u, 1; \theta)]^t$$

Proposition 2.6.1

The Esscher measure of a gamma process has a MGF at $t = 1$ given by:

$$\left\{ \frac{1 - \theta\beta}{1 - (u + \theta)\beta} \right\}^\alpha$$

For detailed proof of the proposition 2.6.1 cf the appendix

The probability measure of the process has in fact changed and its exponential function is positive. Therefore, the modified probability measure is equivalent to the original probability measure, that is, they both have same null sets – sets of probability measure zero (Gerber and Shiu, 1994). The parameter θ is determined so that the modified probability measure \mathbf{Q} is an equivalent martingale measure to the original statistical probability measure \mathbf{P} . The aim is to find $\theta = \theta^*$, so that the discounted stock price process $\{e^{-rt} S_t\}_{t \geq 0}$ is a martingale with respect to the probability measure corresponding to θ^*

With the martingale condition that,

$$S_0 = E^Q[e^{-rt} S_t] = e^{-rt} E^Q[S_t],$$

the parameter θ^* is a solution to:-

$$S_0 = E^Q[e^{-rt} S_t] = e^{-rt} E^Q[S_0 e^{X_t}] \\ = e^{-rt} S_0 \frac{E^P[e^{(\theta+1)X(t)}]}{E^P[e^{\theta X(t)}]} = e^{-rt} S_0 \frac{M(\theta + 1, t)}{M(\theta, t)}$$

with r as the constant risk free rate of interest.

This is equivalent to:

$$1 = e^{-rt} E^Q[e^{X_t}] \text{ or } e^{rt} = M(1, t; \theta^*)$$

We note that the solution is independent of t and then by setting $t = 1$, we obtain:

$$e^r = M(1, 1; \theta^*)$$

And in logarithm form, the parameter θ is a solution to:-

$$r = \log[M(1 + \theta^*)] = \log[M(1 + \theta^*; 1)] - \log[M(\theta^*, 1)]$$

That is $r = \log\left[\left\{\frac{1-\theta\beta}{1-(u+\theta)\beta}\right\}^\alpha\right] = \alpha[\log(1 - \beta\theta) - \log(1 - \beta u + \theta)]$

We know that the parameter $\theta = \theta^*$ is chosen such that the process $\{e^{-rt} S_t\}_{t \geq 0}$ is a martingale with respect to the probability measure corresponding to θ^* .

Precisely, $S_0 = E[e^{-rt} S_t; \theta^*]$;

Hence $e^{rt} = E[e^{X_t}; \theta^*] = [M(1, 1, \theta^*)]^t$

that is, $r = \log[M(1, 1; \theta^*)]$

The Esscher measure corresponding to the parameter θ^* is the risk neutral Esscher measure.

The price of a derivative security, whose payments depend on $\{S(t)\}$ is calculated as a discounted expected value where the expectation is taken with respect to the risk -neutral Esscher measure.

The value of a European option, at time $t = 0$, whose exercise price and date are K and t respectively is given as :

$$E^Q[e^{-rt} (S_t - K)_+] = e^{-rt} \int_{\tau}^{\infty} [S_t e^x - K] f(x, t; \theta^*) dx \\ = e^{-rt} S_t \int_{\tau}^{\infty} e^x f(x, t; \theta) dx - e^{-rt} K [1 - F(\tau, t; \theta^*)], \text{ for } \\ \tau = \log\left[\frac{K}{S(0)}\right]$$

It therefore follows that:

$$e^x f(x, t; \theta) = \frac{e^{(\theta^*+1)x} f(x, t)}{M(\theta^*, t)} = \frac{M(\theta^* + 1, t)}{M(\theta^*, t)} f(x, t; \theta^* + 1) \\ = M(1, t; \theta) f(x, t; \theta^* + 1) = e^{-rt} f(x, t; \theta^* + 1)$$

Letting $I(\cdot)$ denote the indicator function and as above $\tau = \log\left[\frac{K}{S(0)}\right]$, the price of the option at $t = 0$ is :-

$$e^{-rt} E[(S_t - K)I(S_t > K); \theta^*] \\ = e^{-rt} E[S_t I(S_t > K; \theta^*)] - e^{-rt} K [I(S_t > K; \theta^*)]$$

The expectation on the right hand side is equivalent to

$$\Pr[S_t > K; \theta^*] = 1 - F(\tau, t; \theta^*)$$

Thus, the price of a European call option with exercise price K and date t can be given as:

$$P_{EC} = S_t [1 - F(\tau, t; \theta^* + 1) - e^{-rt} K (1 - F(\tau, t; \theta))] \quad (11)$$

Accordingly (15) can be written as

$$S_0 \Pr[S_\tau > K; \theta^* + 1] - e^{-rt} K \Pr[S_\tau > K; \theta^*] \quad (12)$$

3. Results and Discussion

We apply our method developed in this paper to price a hypothetical derivative whose underlying is a markovian gamma rainfall process at a particular station in Kenya. We use the share price of the main government owned electricity producer. We remark that most of Kenya's electricity is hydro generated.

First we estimate all the parameters including the esscher parameter as described earlier in section 2. The derivatives

being modelled here are European call options for monthly rainfall. The contracts are based on a single site, the Dagorreti weather in Nairobi, Kenya. And the rainfall is the total daily are recorded by the Kenya Meteorological Authority.

In this article we use daily prices and rainfall with monthly averages. The rainfall data used consist of monthly averages in mm for Nairobi city from January 2002 to December 2012. and the stock market share price is the daily share price for (Kenya Power (KPLC) for the period January 2002 to December 2012

By assuming a constant MPR risk, we estimate the esscher transform of the gamma distribution. In fact, we observe that the esscher distribution also transforms the mean and variance of the distribution.

Theoretical prices of the monthly rainfall options under Q are estimated by using the $MPR \neq 0$ in the simulated rainfall process. This approach advantageous since we can pick any non-zero values for θ .

The monthly values for θ are obtained from the parameters estimates of the monthly rainfall and our choice of interest rate which in our case is $r = 0.11$ which is a bond market rate in Kenya.

We remark that we are not able to compare the theoretical prices with the actual market prices since derivative do not trade in the Kenyan market. We observe from the Table 1.0 that, with increase in the strike price K, there is a corresponding increase in the payoff.

4. Conclusion

In this article, we have presented a method on how to calculate risk-neutral prices of rainfall derivatives. A standard rainfall model is used to simulate the rainfall process. Then the process distribution is shifted by the Esscher transform to obtain neutral prices.

Rainfall derivatives do not trade in the Kenyan market. Therefore the reported prices are actually hypothetical prices since they are not from actual trading. Hopefully, in the near future, when derivative trading get established, similar approaches can be used to investigate the behaviour of rainfall derivatives and the nature of the market price of risk. Our calculation can be used for daily trading to analyse temporal behaviour of market price of risk and spatial behaviour among different regions in the country.

Table 1: EsscherPrices, $t=0.25, r=0.11, S(0)=100$

| K/Month | Jan | Feb | Mar | Apr | May | Jun |
|---------|--------|--------|--------|--------|--------|--------|
| 80 | 35.724 | 35.724 | 52.224 | 35.724 | 35.724 | 35.724 |
| 90 | 30.908 | 30.908 | 58.658 | 33.158 | 30.908 | 30.908 |
| 100 | 32.093 | 32.093 | 65.093 | 35.093 | 32.093 | 26.093 |
| 110 | 42.277 | 43.777 | 75.277 | 45.277 | 43.777 | 24.350 |

| | | | | | | |
|-----|--------|--------|--------|--------|--------|--------|
| 120 | 45.717 | 46.467 | 82.461 | 50.217 | 46.467 | 29.450 |
| 130 | 50.775 | 50.803 | 80.646 | 55.974 | 50.804 | 36.635 |
| 140 | 58.247 | 59.094 | 77.330 | 65.557 | 59.094 | 39.733 |
| 150 | 69.495 | 69.717 | 77.811 | 74.837 | 69.638 | 46.918 |

| K/Month | Jul | Aug | Sept | Oct | Nov | Dec |
|---------|--------|--------|--------|--------|--------|--------|
| 80 | 37.974 | 35.724 | 35.724 | 35.724 | 35.724 | 35.724 |
| 90 | 38.408 | 30.927 | 33.908 | 33.908 | 33.158 | 30.908 |
| 100 | 50.093 | 26.264 | 43.343 | 43.343 | 35.093 | 26.103 |
| 110 | 54.282 | 24.107 | 47.527 | 47.527 | 45.277 | 25.137 |
| 120 | 58.533 | 24.531 | 53.217 | 53.217 | 47.999 | 31.054 |
| 130 | 71.856 | 38.337 | 58.275 | 58.275 | 53.845 | 36.770 |
| 140 | 78.098 | 66.449 | 73.136 | 73.136 | 59.958 | 40.483 |
| 150 | 79.737 | 00.000 | 74.906 | 74.906 | 71.487 | 49.081 |

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Appendix

Proof for proposition 2.6.1

$$\begin{aligned} \text{for } M(u + \theta) &= \int_0^\infty e^{(u+\theta)y} \frac{1}{\beta^\alpha \Gamma_\alpha} y^{\alpha-1} e^{-\frac{y}{\beta}} dy \\ &= \frac{1}{\beta^\alpha \Gamma_\alpha} \int_0^\infty y^{\alpha-1} e^{-\left(\frac{1-\beta(u+\theta)}{\beta}\right)y} dy = \frac{1}{\beta^\alpha \Gamma_\alpha} \int_0^\infty y^{\alpha-1} e^{-\left(\frac{1-\beta(u+\theta)}{\beta}\right)y} dy * \left[\frac{1-\beta(u+\theta)}{\beta} \right]^{\alpha-1} \\ &= \left(\frac{\beta}{1-\beta(u+\theta)} \right)^{\alpha-1} \frac{1}{\beta^\alpha \Gamma_\alpha} \int_0^\infty \left(\frac{1-\beta(u+\theta)y}{\beta} \right)^{\alpha-1} e^{-\left(\frac{1-\beta(u+\theta)}{\beta}\right)y} dy \quad (a) \end{aligned}$$

Now let $y_* = \left(\frac{1-\beta(u+\theta)}{\beta} \right) y$

Therefore $dy = \left(\frac{\beta}{1-\beta(u+\theta)} \right) dy_*$

Equation (a) becomes

$$\left(\frac{\beta}{1-\beta(u+\theta)} \right)^{\alpha-1} \frac{1}{\beta^\alpha \Gamma_\alpha} \int_0^\infty (y_*)^{\alpha-1} e^{-y_*} \left(\frac{\beta}{1-\beta(u+\theta)} \right) dy_* \quad (b)$$

Recall: $\Gamma_\alpha = \int_0^\infty y^{\alpha-1} e^{-y} dy$

$$\begin{aligned} \text{This leads (12) to} &\left(\frac{\beta}{1-\beta(u+\theta)} \right)^{\alpha-1} \frac{1}{\beta^\alpha \Gamma_\alpha} \left(\frac{\beta}{1-\beta(u+\theta)} \right) \Gamma_\alpha \\ &= \left(\frac{1}{1-\beta(u+\theta)} \right)^\alpha \quad (c) \end{aligned}$$

And for $M(\theta, 1) = \int_0^\infty e^{\theta y} \frac{1}{\beta^\alpha \Gamma_\alpha} y^{\alpha-1} e^{-\frac{y}{\beta}} dy$

$$= \frac{1}{\beta^\alpha \Gamma_\alpha} \int_0^\infty y^{\alpha-1} e^{-\frac{(1-\beta\theta)y}{\beta}} dy$$

$$= \frac{1}{\beta^\alpha \Gamma_\alpha} \int_0^\infty y^{\alpha-1} e^{-\frac{(1-\beta\theta)y}{\beta}} dy * \left(\frac{1-\beta\theta}{\beta} \right)^{\alpha-1}$$

$$= \frac{1}{\beta^\alpha \Gamma_\alpha} \left(\frac{\beta}{1-\beta\theta} \right)^{\alpha-1} \int_0^\infty \left(\frac{1-\beta\theta}{\beta} y \right)^{\alpha-1} e^{-\frac{(1-\beta\theta)y}{\beta}} dy$$

Let $y_* = \frac{y(1-\beta\theta)}{\beta}$, then $dy = \left(\frac{\beta}{1-\beta\theta} \right) dy_*$

Which now gives rise to, as above,

$$= \frac{1}{\beta^\alpha \Gamma_\alpha} \left(\frac{\beta}{1-\beta\theta} \right)^{\alpha-1} \int_0^\infty (y_*)^{\alpha-1} e^{-y_*} \left(\frac{\beta}{1-\beta\theta} \right) dy_*$$

$$= \left(\frac{\beta}{1-\beta\theta} \right)^{\alpha-1} \frac{1}{\beta^\alpha \Gamma_\alpha} \left(\frac{\beta}{1-\beta\theta} \right) \Gamma_\alpha = \left(\frac{1}{1-\beta\theta} \right)^\alpha \quad (d)$$

Dividing (c) by (d) completes the proof.